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A Fresh Look at Gauge Coupling Unification

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The apparent unification of gauge couplings around 10^{16} GeV is one of the strong arguments in favor of Supersymmetric extensions of the Standard Model (SM). In this contribution a new analysis, using the latest experimental data, is performed. The strong coupling α_s emerges as the key factor for evaluating the results of the fits, as the experimental and theoretical uncertainties in its measurements are substantially higher than for the electromagnetic and weak couplings. The present analysis pays special attention to numerical and statistical details. The results, combined with the current limits on the supersymmetric particle masses, favor a value for the SUSY scale $\lesssim$ 150 GeV and for $\alpha_s = 0.118 - 0.119$.

Keywords: Gauge Coupling Unification; Supersymmetry; SUSY and GUT Scales.

1. Introduction

By 1991 the weak coupling was measured with much higher precision than the strong one at LEP. In a renowned paper [1] the famous plot was produced, showing that in contrast to the SM the Minimal Supersymmetric Standard Model (MSSM) leads to a single unification scale of a Grand Unified Theory (GUT), if we let the couplings run according to the MSSM theory:

$$M_{SUSY} = 10^{3.0\pm1.0}\mathrm{GeV}\ ;\ M_{GUT} = 10^{16.0\pm0.3}\mathrm{GeV}\ ;\ 1/\alpha_{GUT} = 25.7 \pm 1.7$$

where M_{SUSY} is a single generic SUSY scale where the spectrum of supersymmetric particles starts to play a role, M_{GUT} is the scale of grand unification where the electromagnetic, weak and strong coupling come together as an unified coupling α_{GUT}. In 1991, at the Z mass scale M_Z, the relative errors in $\alpha(M_Z)$, $\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}}(\mathrm{M_Z})$ and $\alpha_s(M_Z)$ were 0.24, 0.77 and 4.6 % respectively.

From Review of Particle Properties [2] (RPP) 2004 the relative error in $\alpha(M_Z)$, $\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}}(\mathrm{M_Z})$ and $\alpha_s(M_Z)$ is 0.014, 0.065 and 1.7 %, so we have improved in the last decade by more than an order of magnitude except for the strong coupling (less than 3 times). So the time is ripe for new analyses, and one is presented here. [a]

A word of caution: it is amazing that we are trying to extrapolate from 10^2 to 10^{16} GeV. From experiments we now that even interpolation or modest extrapolations can be non-trivial. We measure the "offsets", i.e. the value of the three

[a] Some recent analyses are [3,4].

couplings, around the Z peak and rely on theory to give us the "slopes" of the running couplings - without errors - up to the GUT scale. This may be an illusion e.g. extra dimensions could modify the running already at TeV scales, so the "unification" point may be imaginary.

2. Experimental Inputs

From Review of Particle Properties 2004:

$$1/\alpha(M_Z) = 127.918 \pm 0.018 \ ; \ \sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}}(\mathrm{M_Z}) = 0.23120 \pm 0.00015.$$

The strong coupling $\alpha_s(M_Z)$ is a more complicated story, characterized by much larger statistical errors and theory uncertainties. Different measurements and world averages typically fall in two groups which we will call "high" and "low" values:
$\alpha_s(M_Z) = 0.1224 \pm 0.0038$ *from* Γ_h/Γ_μ *at Z peak* (*RPP* 2004)
$\alpha_s(M_Z) = 0.1213 \pm 0.0018$ *Global Electroweak Fit* (*RPP EW Section*)
$\alpha_s(M_Z) = 0.1187 \pm 0.0020$ *RPP QCD section*
$\alpha_s(M_Z) = 0.1183 \pm 0.0027$ *S.Bethke World aver. LEP/SLC, Deep Inel. Scatt.* [5]

3. Analysis Technique

We perform a χ^2 minimization with `MINUIT` for three parameters: M_{SUSY}, M_{GUT} and $1/\alpha_{GUT}$. A strong correlation (> 0.999) between M_{GUT} and $1/\alpha_{GUT}$ is observed. The minimization problem can be re-factored with two parameters, taking $1/\alpha_{GUT}$ as the weighted average of the three couplings at any given scale. The results for the parameter values are numerically the same, except for the error on the GUT coupling. Even so the correlation $M_{SUSY} - M_{GUT}$ is > 0.96.

An important point is how to compute the running couplings. We perform the analysis two times:

- 1-loop Renormalization Group (RG) running - can solve analytically; the coefficients for the 3 couplings are independent, given by the SM or MSSM:
$$1/\alpha(\nu) = 1/\alpha(\mu) - (b_i/2\pi) \cdot ln(\nu/\mu).$$
- 2-loop-RG running: additional terms so the 3 couplings depend on each other - solved numerically; the errors depend on the scale (for typical GUT scales they grow by 4, 12 and 6 % respectively for the three couplings). Here a threshold correction to $\alpha_s(M_{GUT})$ is applied in order to meet the GUT boundary condition. [2]

4. Results and Discussion

The results of the fits are summarized in Table 1. The MSSM still can provide coupling unification at a GUT scale well below the Planck scale for the full set of precise 2004 measurements. The strong coupling is a key for interpreting the results: the "high" $\alpha_s(M_Z)$ values require an uncomfortably low SUSY scale ~ 10 GeV - way below the experimental lower limit ~ 100 GeV. The "low" $\alpha_s(M_Z)$ values favor

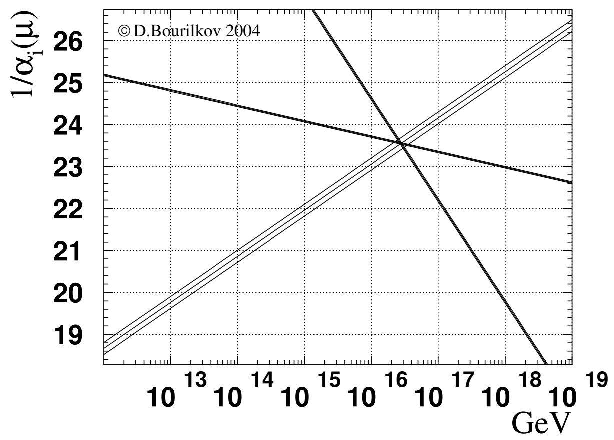

Fig. 1. Example of SUSY fit with 1-loop-RG running. Perfect unification is still possible.

values for the SUSY scale just above the present limits, e.g. for $\alpha_s(M_Z) = 0.1187$ we get $M_{SUSY} < 143(508)$ GeV at one-sided 95 % CL for threshold corrections of -4(-3) % - well in the LHC (even FNAL or LEP2 e.g. by measuring precisely α_s above the Z peak) direct discovery range. The results for 2-loop-RG-running with correction -4 % give similar values for the SUSY scale as 1-loop-RG. The GUT scale is higher: $M_{GUT} = 10^{16.7\pm0.1}$GeV, pushing the proton lifetime ($\sim 10^{36}$ years) well beyond the experimental limits ($\sim$ 5 x 10^{33} years). In contrast a threshold correction of 0 % brings the SUSY scale to $\sim$ 3 TeV.

Table 1. Fit results - all for 2-loop-RG running except the first row.

Inputs	Threshold correction [%]	M_{SUSY} [GeV]	M_{GUT} [GeV]	$1/\alpha_{GUT}$
	Low $\alpha_s(M_Z) = 0.118 - 0.119$			
RPP 2004 QCD; 1-loop-RG	$\pm$ 0	$10^{1.5\pm0.6}$	$10^{16.5\pm0.2}$	23.5 ± 1.0
RPP 2004 QCD	- 4	$10^{1.5\pm0.4}$	$10^{16.7\pm0.1}$	22.3 ± 0.7
RPP 2004 QCD	- 3	$10^{2.05\pm0.4}$	$10^{16.5\pm0.1}$	23.3 ± 0.7
RPP 2004 QCD	$\pm$ 0	$10^{3.5\pm0.3}$	$10^{16.0\pm0.1}$	25.8 ± 0.5
S. Bethke 2002-2004	- 4	$10^{1.55\pm0.5}$	$10^{16.55\pm0.15}$	22.4 ± 0.9
	High $\alpha_s(M_Z) = 0.121 - 0.122$			
RPP 2004 EW Global Fit	- 4	$10^{1.1\pm0.3}$	$10^{16.8\pm0.1}$	21.6 ± 0.55
Γ_h/Γ_μ *at Z peak*	- 4	$10^{0.95\pm0.6}$	$10^{16.85\pm0.2}$	21.3 ± 1.1

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