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QUANTUM CORRECTIONS TO NEWTON'S LAW IN RESUMMED QUANTUM GRAVITY

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We present the elements of resummed quantum gravity, a new approach to QG based on the work of Feynman using the simplest example of a scalar field as the representative matter. We show that we get a UV finite quantum correction to Newton's law.

Keywords: Quantum Gravity; Newton; Resummation

1. Introduction

Newton's law, one of the most basic laws in physics, is a special case of the solutions of the classical field equations of Albert Einstein's general theory of relativity. Successful tests of Einstein's theory in classical physics are described in Refs. [1–3]. Quantum mechanics, as formulated by Heisenberg and Schroedinger, following Bohr,has explained, in the Standard Model(SM) [4], all empirically established quantum phenomena except the quantum treatment of Newton's law. This obtains even with the tremendous progress in quantum field theory, superstrings [5, 6], loop quantum gravity [7], etc. In this paper, we address this issue by using a new approach [8, 9] to quantum gravity(QG), building on previous work by Feynman [10, 11], to get a minimal union of Bohr's and Einstein's ideas.

From the view of the generic approaches [12] to the attendant bad UV behavior of QG, our approach, based on YFS methods [13, 14], is a new version of the resummation approach [12] and allows us to make contact with both the extended theory [12] and the asymptotic safety [15, 16] approaches and to discuss issues in black hole physics, some of which relate to Hawking [17] radiation.

The description of our new resummed QG theory is already presented in these *Proceedings* in Ref. [9], to which we refer the reader. Here, we go directly to the issue of the quantum corrections to Newton's law.

2. Quantum Corrections to Newton's Law in Resummed QG

The model which we use [8, 9] is gravity coupled to a scalar field as formulated in Refs. [10, 11]. In our resummed QG theory, the graphs in Fig. 1 become finite as we explain in Refs. [8, 9]. In this way, we get a UV finite quantum correction to

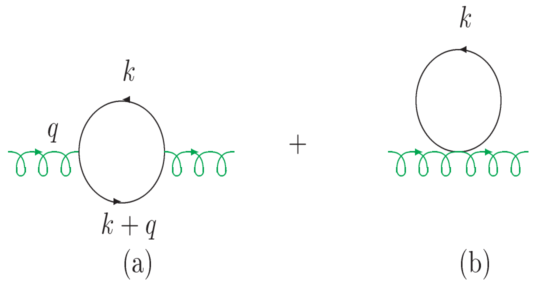

Fig. 1. The scalar one-loop contribution to the graviton propagator. q is the 4-momentum of the graviton.

Newton's law *without modifying Einstein's theory.* In Refs. [8], we show that this UV finiteness holds for all orders in the loop expansion.

Specifically, introducing the YFS resummed propagators as derived in Refs. [8, 9] into Fig. 1 yields , by the standard methods [8], that the graviton propagator denominator, $q^2 + \frac{1}{2}q^4\Sigma^{T(2)} + i\epsilon$, is specified by

$$-\frac{1}{2}\Sigma^{T(2)} \cong \frac{c_2}{360\pi M_{Pl}^2} \tag{1}$$

for $c_2 = \int_0^\infty dx x^3(1+x)^{-4-\lambda_c x} \cong 72.1$ where $\lambda_c = \frac{2m^2}{\pi M_{Pl}^2}$. This implies the Newton potential

$$\Phi_N(r) = -\frac{G_N M_1 M_2}{r}(1 - e^{-ar}) \tag{2}$$

where $a = 1/\sqrt{-\frac{1}{2}\Sigma^{T(2)}} \simeq 3.96 M_{Pl}$ where for definiteness we set $m \cong 120\text{GeV}$ [18]. We note that $c_2 \cong \ln\frac{1}{\lambda_c} - \ln\ln\frac{1}{\lambda_c} - \frac{\ln\ln\frac{1}{\lambda_c}}{\ln\frac{1}{\lambda_c} - \ln\ln\frac{1}{\lambda_c}} - \frac{11}{6}$. Without resummation, $\lambda_c = 0$, and c_2 is infinite and, as this is the coefficient of q^4 in the inverse propagator, **no renormalization of the field and/or of the mass could remove such an infinity**. In our new approach, this infinity is absent.

We can make a cross check of our gauge invariant [8] analysis with the gauge invariant analysis of Ref. [19] where the complete result of the one-loop divergences of our scalar field coupled to Einstein's gravity have been computed. Since c_2 diverges without our resummation, it follows [8] that we need to make the correspondence between the poles in n, the dimension of space-time, at $n = 4$ calculated in Ref. [19] and the leading log $\ln\frac{1}{\lambda_c}$. This implies [8]

$$\frac{1}{(2 - n/2)} \leftrightarrow c_2. \tag{3}$$

so that, if we look at the limit $q^2 \to 0$, we find that the coefficient of q^4 in the graviton propagator denominator above is $3/(2 - n/2)$ times the coefficient of c_2 on the right-hand side of (1), in complete agreement with the result that is implied by eq.(3.40) in Ref. [19] , for example.

Sub-Planck scale physics is accessible to point particle field theory so that current superstring theories may be phenomenological models for a more fundamental theory (TUT=The Ultimate Theory) just as the old string theory [20] is such a model for QCD. Other types of correspondences are not excluded here [21]. Our deep Euclidean studies are complementary to the low energy studies of Ref. [22]. The effective cut-off which we generate dynamically is at M_{Pl} so that, while renormalizable quantum field theory (QFT) below M_{Pl} is unaffected, some non-renormalizable QFT's are given new life here – other problems notwithstanding.

Some phenomenological implications of (2) are presented in Ref. [8, 9]. To sum up, it appears that we may have indeed found a minimal union of the ideas of Bohr and Einstein.

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