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B meson wave function in k_T factorization

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We construct a valid definition of the k_T-dependent B meson wave function, and then calculate next-to-leading-order corrections. We show that the B meson wave function remains normalizable after taking into account renormalization-group evolution effects, contrary to the observation derived from the collinear factorization theorem.

Keywords: normalizability; B-meson wave function; k_T factorization.

1. Introduction

Most of currently available theories for exclusive B decays are based on collinear factorization theorem; only PQCD approach is based on k_T factorization theorem. The corresponding nonperturbative quantities are the so-called light-cone distribution amplitude(LCDA) and wave function(WF), respectively.

The behavior of B-meson LCDA, ϕ_+, certainly matters, and has been proposed and concluded differently in various approaches recently[1,2]. The asymptotic behavior of ϕ_+ was also extracted from an renormalization-group (RG) evolution equation and a non-normalizable B-meson LCDA was concluded [3]. This striking feature has been confirmed in a QCD sum rule analysis [4].

A non-normalizable B-meson LCDA does not cause a problem in practice [5] since only the first inverse moment is relevant [6,7] in a leading-order collinear factorization formula. However, the non-normalizability does introduce an ambiguity in defining the B meson decay constant f_B. We shall show that the above undesirable feature is a consequence of adopting the collinear factorization theorem [8].

2. Definition of A Wave Function and $O(\alpha_{\rm S})$ Corrections

The definition of the B-meson WF, $\tilde{\Phi}_+(v\cdot y, b, \mu)$, is nontrivial at all. A gauge-invariant definition is given via the nonlocal matrix element.

$$\langle 0|\bar{q}(y)W_y(n_-)^\dagger I_{n_-;y,0}W_0(n_-)\ \not{n}_-\Gamma h(0)|\bar{B}(v)\rangle\ , \tag{1}$$

as a naive extension of B meson LCDA with $y=(0,y^-,\mathbf{b})$. In Eq. (1), h is the rescaled b quark field characterized by the velocity v, Γ represents a Dirac matrix,

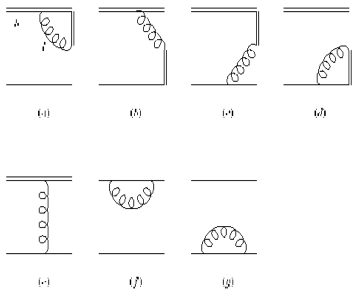

Fig. 1. **FIG. 1** $O(\alpha_s)$ diagrams for the B meson wave function.

the factor $W_y(n_-)$ denotes the Wilson line operator, and the two Wilson lines $W_y(n_-)$ and $W_0(n_-)$ are connected by a link $I_{n_-;y,0}$ at infinity in this case.

As pointed out in [9], Eq. (1) contains additional collinear divergences from the region with a loop momentum parallel to n_-. These light-cone divergences, cancelling each other as $b \to 0$, do not cause a problem in the collinear factorization theorem. In the k_T factorization theorem, however, they must be subtracted in a gauge-invariant way. Two modified definitions have been proposed in [9]:

$$\langle 0|\bar{q}(y)W_y(u)^\dagger I_{u;y,0}W_0(u)\ \not{n}_-\Gamma h(0)|\bar{B}(v)\rangle\ , \tag{2}$$

$$\frac{\langle 0|\bar{q}(y)W_y(n_-)^\dagger I_{n_-;y,0}W_0(n_-)\ \not{n}_-\Gamma h(0)|\bar{B}(v)\rangle}{\langle 0|W_y(n_-)^\dagger W_y(u')I_{n_-;y,0}I^\dagger_{u';y,0}W_0(n_-)W_0(u')^\dagger|0\rangle}\ . \tag{3}$$

Both the above modifications with the appropriate Wilson links are gauge-invariant. By investigating the $O(\alpha_s)$ diagrams in Fig. 1 according to both modifications, we found that Eq. (2) would change the ultraviolet structure of the quark-Wilson-line vertex correction in Eq. (1). In this work we shall adopt Eq. (3), and demonstrate that the freedom in choosing the vector u' allows a correct RG evolution of the B meson wave function.

The lowest-order evolution kernel for $\Phi_+(k^+, b, \mu)$ is written as

$$K^{(0)}(k^+, k'^+, b, \mu) = \delta(k^+ - k'^+)\ . \tag{4}$$

As indicated in Eq. (4), we perform k_T factorization in the conjugate b space. We then calculate the $O(\alpha_s)$ corrections to Eq. (4) in dimensional regularization. As suggested in [9], a small plus component is added to the null vector n_- in Eq. (3) at the intermediate step of the calculation and then we take the $u \to n_-$ limit eventually.

To highlight the difference between the collinear and k_T factorizations, we present the loop integrals associated with Figs. 1(a) and 1(b).

$$N_a^{(1)} + N_b^{(1)} = -ig^2 C_F \mu^{2\epsilon} \int \frac{d^{4-2\epsilon}l}{(2\pi)^{4-2\epsilon}} \frac{u\cdot v}{(v\cdot l + i\epsilon)(l^2 - m_g^2 + i\epsilon)(u\cdot l + i\epsilon)}$$
$$\times \left[\delta(k^+ - k'^+) - \delta\left(k^+ - k'^+ + l^+\right)\exp(-i\mathbf{l}_T\cdot\mathbf{b})\right]\ . \tag{5}$$

In the collinear factorization theorem the Fourier factor disappears, corresponding to the $b \to 0$ limit. Moreover, for $u = n_-$, we obtain, from Eq. (5), the counterterm identical to that found in the collinear factorization theorem [3],

$$-\frac{\alpha_s C_F}{4\pi}\frac{2}{\epsilon}\left[\frac{k^+}{k'^+}\frac{\theta(k'^+ - k^+)}{(k'^+ - k^+)_+} + \frac{\theta(k^+ - k'^+)}{(k^+ - k'^+)_+}\right] . \tag{6}$$

The resultant anomalous dimension contributes to the splitting kernel in the RG evolution equation, that determines the asymptotic behavior of the B meson distribution amplitude [3]. The ultraviolet pole $1/\epsilon$ arises from the integration over the transverse loop momentum l_T up to infinity. It is then expected that Eq. (6) will be absent in the k_T factorization theorem due to the suppression in the large l_T region from $\exp(-i\mathbf{l}_T \cdot \mathbf{b})$.

After evaluating the total one-loop correction to Eq. (4), and the treatment of different types of logarithms in the corrections, the evolution of the B meson wave function from Eq. (3) is then given by

$$\Phi_+(k^+, b, \mu) = S(k^+, b, \nu)\phi_+(k^+, b, \mu) , \quad \phi_+(k^+, b, \mu) = R(b, \mu, \nu)\phi_+(k^+, b, \mu = 1/b) \tag{7}$$

The factors S and R in Eq. (7) are referred to [10,11].

At last, we discuss the normalization of the B meson wave function $\Phi_+(k^+, b, \mu)$ in the k_T factorization theorem, which is defined as

$$\int_0^\infty dk^+ \lim_{b\to 1/k^+} \Phi_+(k^+, b, \mu) = \int_0^\infty dk^+ \phi_+(k^+, b = 1/k^+, \mu) . \tag{8}$$

The Sudakov factor in Eq. (7) becomes identity in the limit $b \to 1/k^+$, which approaches zero in the heavy quark limit for a fixed momentum fraction $x \equiv k^+/(m_B v^+)$. It is then obvious from Eq. (7) that the normalizability of the B meson distribution amplitude is not spoiled by the RG evolution effect, when evaluated according to Eq. (8).

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