International Journal of Modern Physics A

STABILITY OF BRANONIUM

JAMES ELLISON

Department of Physics and Astronomy, University of Sussex,
Falmer, Brighton BN1 9QJ, United Kingdom

ANDRÉ LUKAS

Rudolf Peierls Centre for Theoretical Physics,
1 Keble Road, Oxford OX1 3NP,United Kingdom

Received (25th October 2004)

We investigate the time-dependent behaviour of brane-antibrane orbital systems, mainly concentrating on the stability of the “Branonium” configuration.

Keywords: brane; orbit;supergravity.

Introduction

In recent years, it has become clear that the construction of realistic particle physics models from string theory typically involves D-branes as part of the vacuum configuration. Moreover, these branes are often capable of moving in the compact extra dimensions. Therefore, when considering the cosmology of such models the motion of these branes is an interesting, generic feature which deserves systematic study [1–8].

An interesting illustration of this is the planetary “Branonium” configuration of Burgess et al [9]. This model involves a light anti D6-brane moving in the BPS background created by a large stack of coincident D6 brane sources, and is so named by virtue of its similarity to positronium. It transpires that the fully relativistic worldvolume action for the antibrane allows for the existence of non-precessing elliptical orbits similar to those admitted for a non-relativistic particle experiencing a central potential. This seems to indicate that substantial inflationary epochs can be induced by the encircling antibranes. However, it should be noted that the analysis of Ref. [9] was carried out in a strictly static background geometry.

It is known from related systems [7] that the inclusion of time-dependent gravitational moduli, such as internal radii, is necessary for consistency and qualitatively alters the evolution of the fields. Therefore, we should now investigate how the inclusion of gravitational moduli affects systems of Dp-$\overline{\mathrm{D}}p$ branes and, in particular, the orbital motion found in the D6-$\overline{\mathrm{D}}$6 case.

Bulk action and p-brane solutions

First, consider a stack of heavy p-branes acting as a "stellar" source. To solve for the fields produced by the stack we consider the following bulk action in D dimensions

$$S_{\rm bulk} = \frac{1}{2\kappa^2}\int d^D x\sqrt{-g}\left[R(g) - \frac{1}{2}\nabla_A\Phi\nabla^A\Phi - \frac{1}{2(d+1)!}e^{a\Phi}F^2_{[d+1]}\right] \tag{1}$$

where g is the Einstein-frame metric, Φ is the dilaton, $F_{[d+1]}$ is the $d+1$-form field strength for a d-form potential $A_{[d]}$, and a is a constant. Splitting the space-time coordinates into the two sets as $(x^A) = (x^\mu, y^m)$, where $\mu = 0,..,p$ and $m = d,..,D-1$, an elementary, "electric" p-brane solution to the action (1) is given by [10]

$$\begin{aligned} ds^2 &= e^{2\nu_0}h^{-\tilde{\gamma}}\eta_{\mu\nu}dx^\mu dx^\nu + e^{2\beta_0}h^{\gamma}\delta_{mn}dy^m dy^n \\ e^{\Phi} &= e^{\phi_0}h^{\sigma} \\ A_{\mu_1\ldots\mu_d} &= \epsilon_{\mu_1\ldots\mu_d}\, e^{d\nu_0 - \frac{1}{2}a\phi_0}\zeta h^{-1} \end{aligned} \tag{2}$$

where ν_0, β_0, ϕ_0 are arbitrary integration constants and

$$\gamma = \frac{4d}{\Delta(D-2)}, \quad \tilde{\gamma} = \frac{4\tilde{d}}{\Delta(D-2)}, \quad \sigma = \frac{2a}{\Delta}, \quad \zeta = \frac{2}{\sqrt{\Delta}}, \quad \Delta = a^2 + \frac{2d\tilde{d}}{D-2}$$

with $\tilde{d} = D - d - 2$. The harmonic function h is given by

$$h = 1 + \frac{k_0}{r^{\tilde{d}}}\, e^{-\frac{1}{2}a\phi_0 - \tilde{d}\beta_0}$$

where r is the isotropic radial coordinate defined by $r^2 = \delta_{mn}y^m y^n$, and k_0 is a dimensionful constant.

Branonium

Having identified the form of these BPS p-brane solutions, we now consider the worldvolume action (in the Einstein frame) for a d-dimensional "planetary" probe,

$$S_{\rm probe} = -T\int d^d\xi\sqrt{-\hat{g}}\, e^{-\frac{1}{2}a\hat{\Phi}} + Q\int \hat{A}_{[d]} \tag{3}$$

where T is the bare positive tension of the probe, Q is its bare charge and ξ^μ are intrinsic coordinates on the worldvolume. The quantities $\hat{g}, \hat{\Phi}, \hat{A}_{[d]}$ are simply pullbacks of the corresponding bulk fields, and the X^A are embedding coordinates that locate the probe in the ambient space-time. Exploiting the ξ^μ coordinate reparameterisation freedom, one can always choose a gauge such that

$$X^\mu = \xi^\mu \qquad , \qquad X^m = Y^m(\xi^\mu)$$

leaving Y^m as the physical degrees of freedom that locate the probe in the space transverse to the source. Inserting the background solutions (2) into the probe action, and momentarily setting $\nu_0 = \beta_0 = 0$, one finds the result

$$S_{\text{probe}} = -\mathcal{M} \int d^d x \left[h^{-\eta} \sqrt{1 + h^\omega \sum (\partial Y^n)^2} - q\zeta h^{-1} \right] \tag{4}$$

with

$$\eta = \frac{2[a(\tilde{d} - d) + \tilde{d}d]}{\Delta(D-2)}, \qquad \omega = \frac{4}{\Delta}, \qquad q = \pm 1, \qquad \mathcal{M} = Te^{-\frac{1}{2}a\phi_0}$$

where $\mathcal{M}$ corresponds to the physical ADM mass density of the probe measured at infinity where there is a constant dilaton vev ϕ_0. In the special case of a probe anti D6 brane circling a stack of source D6's in ten dimensions, one finds $\eta = \omega = \zeta = \tilde{d} = -q = 1$ and the worldvolume action simplifies to

$$S_{\text{probe}} = -\mathcal{M} \int d^7 x \left[\sqrt{1 + h \sum (\partial Y^n)^2} + 1 \right] h^{-1} \tag{5}$$

After assuming the moduli Y^n are only functions of time, six of the dimensions can be integrated out to give an effective action for a point-particle moving in three spatial dimensions. In both the relativistic and non-relativistic limits this reduced action then implies the existence of closed elliptical orbits.

Time-dependent backgrounds

We now recall that the BPS background has been assumed to be strictly static whilst the antibrane moves. In general there is no reason to presume that this is a valid assumption, since the antibrane has non-standard kinetic terms that act as sources for the bulk fields. We should, therefore, now add back in the bulk action Eq.(1) and analyse the coupled equations of motion of the brane and bulk moduli.

To this end, we promote ν_0, β_0, ϕ_0 to slowly-varying functions of the longitudinal coordinates x^μ, and insert the p-brane ansatz for the metric and fields into the action (1). After explicitly integrating over the $D - d$ dimensions transverse to the central stack, we can combine the d-dimensional bulk and probe actions to yield

$$\begin{aligned} S =& \frac{V}{2\kappa^2}\int d^dx\sqrt{-\tilde{g}}\left\{\tilde{R}-\frac{1}{2}(\partial\phi)^2-\frac{(\tilde{d}+2)(\tilde{d}+d)}{(d-2)}(\partial\beta)^2\right. \\ & \left.-\frac{2\kappa^2 T}{V}\left[\frac{1}{2}h^{\omega-\eta}e^{-\frac{1}{2}a\phi-\tilde{d}\beta}\sum_n(\partial Y^n)^2+e^{-\frac{1}{2}a\phi-\frac{d(\tilde{d}+2)}{(d-2)}\beta}\left(h^{-\eta}-q\zeta h^{-1}\right)\right]\right\} \end{aligned} \quad (6)$$

Consistency of the dimensional reduction also requires the following restriction on the moduli space:

$$\epsilon = k_0 e^{-\frac{1}{2}a\phi-\tilde{d}\beta} \ll 1$$

An important feature of this action is that the kinetic terms for the brane position moduli Y^n are non-trivial, depending on exponentials of the bulk moduli ϕ and β. Hence, whenever the brane moves and the volume V of the transverse space is kept finite the gravitational moduli will also evolve in time even if they are static at some initial time. Only in the decompactification limit $V \to \infty$ can the bulk moduli ϕ and β be set to constants consistently.

If now focus on the particular case of an anti D6-brane in the background of a stack of D6 branes in ten dimensions, with parameters

$$a = -3/2, \quad \sigma = -3/4, \quad \Delta = 4, \quad \zeta = \omega = \eta = \tilde{d} = -q = 1\ .$$

then the above action specialises to

$$\begin{aligned} S = \frac{V}{2\kappa^2}\int d^7x\sqrt{-\tilde{g}}&\left\{\tilde{R}-\frac{1}{2}(\partial\phi)^2-\frac{24}{5}(\partial\beta)^2\right. \\ &\left.-\frac{2\kappa^2 T}{V}\left[\frac{1}{2}e^{\frac{3}{4}\phi-\beta}\sum_{n=1}^{3}(\partial Y^n)^2+2e^{\frac{3}{4}\phi-\frac{21}{5}\beta}h^{-1}\right]\right\} \end{aligned} \quad (7)$$

Cosmology

We now perform a numerical simulation of the D6-$\overline{\mathrm{D}}$6 orbital system. We assume that all moduli are time-dependent only, and adopt a simple Friedmann Robertson Walker ansatz for the seven dimensional geometry of the form

$$d\tilde{s_7}^2 = -dt^2 + e^{2\alpha}\delta_{ij}dx^i dx^j$$

where $\alpha \equiv \alpha(t)$ denotes the scale-factor for the spatial part of the metric $\tilde{g}$ and $i, j = 1, .., 6$ index the corresponding coordinates. For the antibrane moduli we assume that initially $Y_3 = \dot{Y}_3 = 0$ so that there is some chance of orbital motion in the (Y^1, Y^2) phase plane. We then set $\dot{\beta}_0 = \dot{\phi}_0 = 0$ so that all bulk moduli fields have zero initial kinetic energy, while, of course, the initial value of the Hubble parameter $\dot{\alpha}$ is given by the Friedmann equation.

The results are shown in Fig. 1. The necessarily small value of $1/V$ allows the antibrane to orbit in elliptical fashion for many hundreds of revolutions, but these orbits fail to close exactly due to a progressive diminution of the radial force. Eventually the antibrane breaks free of the potential well, and the motion becomes unbounded.

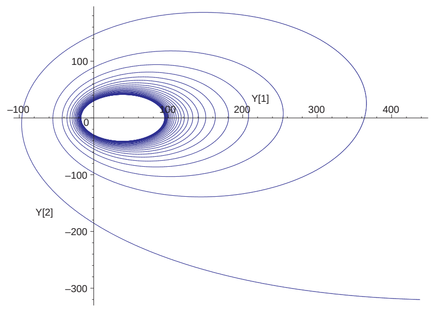

Fig. 1. *The (Y^1, Y^2) phase-plane for $V = 10^9$ and $\dot{\beta}_0 = \dot{\phi}_0 = 0$, showing the diminishing effect of the radial force as the background evolves.*

This overall behaviour is easily explicable, once we notice that there is no longer a conserved Runge-Lenz vector **A** after the moduli have been promoted to functions of time. Recall that **A** usually lies in the plane of the orbit, is directed toward the perihelion point, and has a magnitude equal to the eccentricity e of the elliptical trajectory. Its conservation thus guarantees the existence of non-precessing elliptical orbits with fixed e. That it was conserved in the original branonium analysis can be ascribed to the special, static form for h, which satisfied

$$\frac{\partial h}{\partial R} R^2 = \text{constant} \tag{8}$$

This simply states that, in the absence of external forces, the flux arriving at any spherical surface surrounding a $1/R^2$ source is a constant, since the surface area over which the field extends scales up as R^2. This property is obviously lost once the moduli and h become time-dependent. Indeed, from the antibrane's perspective these variations in β and ϕ modify the force it experiences *as if* the source charge is changing due to some external influence.

Conclusion

We have derived the effective action for an anti Dp-brane moving in the background generated by a stack of heavy Dp branes, taking into account the gravitational moduli. For the case of finite transverse volume we find that these additional moduli cannot be consistently set to constants once the anti Dp-brane moves. This implies that the elliptical orbits do not exactly close, and that the antibrane will either fall in and annhilate or spiral out and escape. This inability to switch off the evolution of the background means that the the slow-roll conditions are always violated, and inflation does not occur.

References

1. G. Dvali and S. H. Tye, "Brane inflation," Phys. Lett. B **450** (1999) 72 [hep-ph/9812483].
2. A. Kehagias and E. Kiritsis, JHEP **9911** (1999) 022 [hep-th/9910174].
3. E. Papantonopoulos, "Brane inflation from mirage cosmology," [hep-th/0011051.
4. J. Khoury, B. A. Ovrut, P. J. Steinhardt and N. Turok, "The ekpyrotic universe: Colliding branes and the origin of the hot big bang," [hep-th/0103239].
5. C. P. Burgess, M. Majumdar, D. Nolte, F. Quevedo, G. Rajesh and R. J. Zhang, "The inflationary brane-antibrane universe," [hep-th/0105204].
6. R. Kallosh, L. Kofman, A. Linde and A. Tseytlin, "BPS Branes in Cosmology," [hep-th/0106241].
7. E. J. Copeland, J. Gray and A. Lukas, "Moving five-branes in low-energy heterotic M-theory," Phys. Rev. D **64** (2001) 126003 [hep-th/0106285].
8. E. J. Copeland, J. Gray, A. Lukas and D. Skinner, "Five-dimensional moving brane solutions with four-dimensional limiting behaviour," Phys. Rev. D **66** (2002) 124007 [hep-th/0207281].
9. C. P. Burgess, P. Martineau, F. Quevedo and R. Rabadan, "Branonium," JHEP **0306** (2003) 037 [hep-th/0303170].
10. M. J. Duff, R. R. Khuri and J. X. Lu, "String solitons," Phys. Rept. **259** (1995) 213 [hep-th/9412184].