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HIGGSLESS DECONSTRUCTION WITHOUT BOUNDARY CONDITION*

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Deconstruction is a powerful means to explore the rich dynamics of gauge theories in four and higher dimensions. We demonstrate that gauge symmetry breaking in a compactified higher dimensional theory can be formulated via deconstructed 4D moose theory with *spontaneous symmetry breaking* and *without boundary condition.* The proper higher-D boundary conditions are automatically induced in the continuum limit rather than being imposed. We identify and analyze the moose theories which exhibit *delayed unitarity violation* (effective unitarity) as a *collective effect* of many gauge groups, without resorting to any known 5D geometry. Relevant phenomenological constraints are also addressed.

Keywords: Deconstruction, Unitarity, Higgsless Electroweak Symmetry Breaking

1. Advantageous Deconstruction

Deconstruction[1] is a powerful means to explore the rich dynamics of gauge theories in four and higher dimensions. A compactified higher dimensional theory may be deconstructed into a proper moose representation[2] or the equivalent transverse Wilson lattice[3]. An essential advantage of the deconstruction is that it allows us to formulate the often involved higher dimensional gauge symmetry breaking (without/with gauge group rank reduction) in terms of the conventional 4D gauged nonlinear sigma model à la CCWZ[4], where no extra boundary condition (BC) is required *a priori.* Furthermore, deconstruction allows us to identify and analyze the general 4D moose theories with arbitrary inputs of gauge couplings g_j and Goldstone decay constants f_k or with only a few Kaluza-Klein (KK)[5] modes. Such theories need not resemble any known higher-D geometry, but as we will show, they can still exhibit *delayed unitarity violation* (effective unitarity) as a *collective effect* of many participating gauge groups, which was originally revealed for the deconstruction of specified 5D geometries[6,7]. The effective unitarity in the compactified or deconstructed 5D Yang-Mills theories is ensured by *the presence of spin-1 vector-bosons (gauge KK modes)*[8,6,7] rather than the conventional Higgs scalar[9,10], which is the key for seeking realistic Higgsless models of electroweak symmetry breaking (EWSB) in the

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recent literature[11,12]. These new vector bosons also provide discovery signatures at the CERN LHC.

2. Eaten Goldstone Bosons in General Higgsless Moose Theory

Consider a most general linear moose theory depicted in Fig. 1,[12] consisting of the replicated gauge group $G_A^{N+1} \otimes G_B^{M+1}$ which is *spontaneously broken* down to the diagonal subgroup G_D by the link fields U_j transforming as bi-fundamentals under the two adjacent gauge groups. Without losing generality, we will set $G_A = SU(2)$ and $G_B = U(1)$,[12] for the convenience of analyzing the EWSB, so the residual gauge symmetry is $G_D = U(1)_{\rm em}$ with a massless photon.

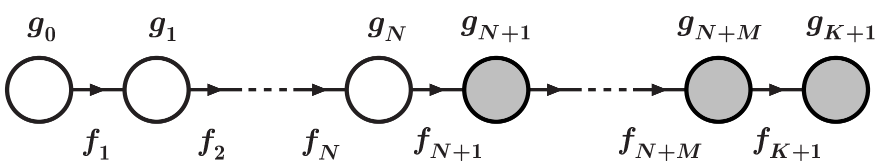


Fig. 1. A generic Higgsless deconstruction of linear moose theory with arbitrary product gauge group $G_A^{N+1} \otimes G_B^{M+1}$, where the most general inputs of gauge couplings and Goldstone decay constants $\{g_j,\, f_k\}$ $(j = 0, 1, \cdots, K+1;\ k = 1, 2, \cdots, K+1;\ K \equiv N+M)$ are allowed. The open circles represent the group G_A and shaded circles the group G_B.

The Lagrangian for this linear moose theory is

$$\mathcal{L} \;=\; \sum_{j=0}^{K+1} -\frac{1}{2}{\rm Tr}\left(F_{j\mu\nu}F_j^{\mu\nu}\right) \;+\; \sum_{j=1}^{K+1}\frac{f_j^2}{4}{\rm Tr}\left[(D_\mu U_j)^\dagger (D^\mu U_j)\right], \tag{1}$$

with $D^\mu U_j = \partial^\mu U_j - i g_{j-1}\mathbf{A}_{j-1}^\mu U_j + i g_j U_j \mathbf{A}_j^\mu$, and $U_j = \exp\left[i2\boldsymbol{\pi}_j/f_j\right]$, where $\mathbf{A}_j^\mu \equiv A_j^{a\mu}T^a \in SU(2)_j$ for $j = 0, 1, \cdots, N$, and $\mathbf{A}_j^\mu \equiv A_j^\mu T^3 \in U(1)_j$ $(j = N+1, \cdots, K+1)$. Also, $\boldsymbol{\pi}_j \equiv \pi_j^a T^a$ $(1 \leqslant j \leqslant N+1)$ and $\boldsymbol{\pi}_j \equiv \pi_j^3 T^3$ $(N+2 \leqslant j \leqslant K+1)$. The $(N+1)\times(N+1)$ mass matrix of charged gauge bosons is given by[12]

$$\mathbb{M}_W^2 = \frac{1}{4}\left(\begin{array}{c|c|c|c|c}
g_0^2 f_1^2 & -g_0 g_1 f_1^2 & & & \\ \hline
-g_0 g_1 f_1^2 & g_1^2(f_1^2+f_2^2) & -g_1 g_2 f_2^2 & & \\ \hline
 & -g_1 g_2 f_2^2 & g_2^2(f_2^2+f_3^2) & -g_2 g_3 f_3^2 & \\ \hline
 & & \ddots & \ddots & \ddots \\ \hline
 & & & -g_{N-1} g_N f_N^2 & g_N^2(f_N^2+f_{N+1}^2)
\end{array}\right), \tag{2}$$

which may be denoted as $\mathbb{M}_W^2 = \mathbb{M}_{[0,N+1)}^2$ by using the notation of open/closed intervals[12]. Similarly the $(K+2)\times(K+2)$ mass matrix of neutral gauge bosons is $\mathbb{M}_Z^2 = \mathbb{M}_{[0,K+1]}^2$ and can be obtained from the matrix $\mathbb{M}_{[0,N+1)}^2$ with simple replacements, $N \to K+1$ and $f_{K+2} \to 0$. The eigenvalues of $\mathbb{M}_W^2$ are denoted by M_W^2 for light $W_0^\pm \equiv W^\pm$ boson and M_{Ww}^2 $(w = 1, 2, \cdots, N)$ for heavy "KK" states $W_w^\pm$, while $\mathbb{M}_Z^2$ has eigenvalues 0 (M_Z^2) for photon A_γ^0 (light $Z_0^0 \equiv Z^0$ boson) and M_{Zz}^2 $(z = 1, 2, \cdots, K)$ for heavy "KK" states Z_z^0. With the diagonal matrices $\mathcal{F}_{n+1} = \text{diag}\left(f_1, f_2, \cdots, f_{n+1}\right)/2$ and $\mathcal{G}_{n+1} = \text{diag}\left(g_0, g_1, \cdots, g_n\right)$, it is convenient to define the notations, $\mathbb{Q}_W = \mathcal{F}_{N+1}\mathcal{D}_W\mathcal{G}_{N+1}$ and $\mathbb{Q}_Z = \mathcal{F}_{K+1}\mathcal{D}_Z\mathcal{G}_{K+2}$, where the $(N+1)\times(N+1)$ and $(K+1)\times(K+2)$ non-diagonal matrices $\mathcal{D}_W$ and $\mathcal{D}_Z$ are

$$\mathcal{D}_W = \begin{pmatrix} 1 & -1 & & & \\ & 1 & -1 & & \\ & & \ddots & \ddots & \\ & & & 1 & -1 \\ & & & & 1 \end{pmatrix}, \qquad \mathcal{D}_Z = \begin{pmatrix} 1 & -1 & & & & \\ & 1 & -1 & & & \\ & & \ddots & \ddots & & \\ & & & 1 & -1 & \\ & & & & 1 & -1 \end{pmatrix}. \tag{3}$$

So the mass matrices $(\mathbb{M}_W^2, \mathbb{M}_Z^2)$ and their duals $(\widetilde{\mathbb{M}}_W^2, \widetilde{\mathbb{M}}_Z^2)$ can be written as[12]

$$\mathbb{M}_W^2 = \mathbb{Q}_W^T\mathbb{Q}_W\,, \quad \mathbb{M}_Z^2 = \mathbb{Q}_Z^T\mathbb{Q}_Z\,; \quad \widetilde{\mathbb{M}}_W^2 = \mathbb{Q}_W\mathbb{Q}_W^T\,, \quad \widetilde{\mathbb{M}}_Z^2 = \mathbb{Q}_Z\mathbb{Q}_Z^T\,; \tag{4}$$

where the dual mass matrices $\widetilde{\mathbb{M}}_W^2$ and $\widetilde{\mathbb{M}}_Z^2$ share the same nonzero eigenvalues as $\mathbb{M}_W^2$ and $\mathbb{M}_Z^2$, respectively. This is because $\mathbb{M}_a^2$ and $\widetilde{\mathbb{M}}_a^2$ $(a = W, Z)$ are real symmetric matrices and $\mathbb{Q}_a$ can be diagonalized by the bi-orthogonal rotations,

$$\widetilde{\mathbb{R}}_a^T\,\mathbb{Q}_a\,\mathbb{R}_a = \mathbb{Q}_a^{\text{diag}}\,, \tag{5}$$

which leads to

$$\begin{aligned} \mathbb{R}_a^T\,\mathbb{M}_a^2\,\mathbb{R}_a &= (\mathbb{Q}_a^{\text{diag}})^T(\mathbb{Q}_a^{\text{diag}}) \equiv (\mathbb{M}_a^2)^{\text{diag}}\,, \\ \widetilde{\mathbb{R}}_a^T\,\widetilde{\mathbb{M}}_a^2\,\widetilde{\mathbb{R}}_a &= (\mathbb{Q}_a^{\text{diag}})(\mathbb{Q}_a^{\text{diag}})^T \equiv (\widetilde{\mathbb{M}}_a^2)^{\text{diag}}\,. \end{aligned} \tag{6}$$

Expanding the Lagrangian (1) gives the gauge-Goldstone boson mixing term,[12]

$$\begin{aligned} \mathcal{L}_{\text{GB}}^{\text{mix}} &= \left[-\mathbb{A}_\mu^{+\,T}\mathbb{Q}_W^T\partial^\mu\Pi^- + \text{h.c.}\right] - \mathbb{A}_\mu^{3\,T}\mathbb{Q}_Z^T\partial^\mu\Pi^0 \\ &= \left[-\mathbb{W}_\mu^{+\,T}\mathbb{M}_W^{\text{diag}}\partial^\mu\widetilde{\Pi}^- + \text{h.c.}\right] - \mathbb{Z}_\mu^{0\,T}\widetilde{\mathbb{M}}_Z^{\text{diag}}\partial^\mu\widetilde{\Pi}^0\,, \end{aligned} \tag{7}$$

with

$$\begin{aligned} \mathbb{A}^{\pm\mu} &= (A_0^{\pm\mu},\, A_1^{\pm\mu},\, \cdots,\, A_N^{\pm\mu})^T, & \mathbb{A}^{3\mu} &= (A_0^{3\mu},\, A_1^{3\mu},\, \cdots,\, A_{K+1}^{3\mu})^T; \\ \mathbb{W}^{\pm\mu} &= (W_0^{\pm\mu},\, W_1^{\pm\mu},\, \cdots,\, W_N^{\pm\mu})^T, & \mathbb{Z}^{0\mu} &= (Z_0^{0\mu},\, Z_1^{0\mu},\, \cdots,\, Z_K^{0\mu})^T; \\ \Pi^\pm &= (\pi_1^\pm,\, \pi_2^\pm,\, \cdots,\, \pi_{N+1}^\pm)^T, & \Pi^0 &= (\pi_1^0,\, \pi_2^0,\, \cdots,\, \pi_{K+1}^0)^T; \\ \widetilde{\Pi}^\pm &= (\widetilde{\pi}_0^\pm,\, \widetilde{\pi}_1^\pm,\, \cdots,\, \widetilde{\pi}_N^\pm)^T, & \widetilde{\Pi}^0 &= (\widetilde{\pi}_0^0,\, \widetilde{\pi}_1^0,\, \cdots,\, \widetilde{\pi}_K^0)^T; \end{aligned} \tag{8}$$

where $(\mathbb{W}^{\pm\mu}, \mathbb{Z}^{0\mu})$ are mass-eigenbasis fields, while $(\widetilde{\Pi}^\pm, \widetilde{\Pi}^0)$ are "eaten" Goldstone fields, which are connected to the site Goldstone states $(\Pi^\pm, \Pi^0)$ via

$$\widetilde{\Pi}^\pm = \widetilde{\mathbb{R}}_W^T\,\Pi^\pm\,, \qquad \widetilde{\Pi}^0 = \widetilde{\mathbb{R}}_Z^T\,\Pi^0\,. \tag{9}$$

Hence, the "eaten" Goldstone bosons are exactly aligned with the "gauge boson" mass-eigenstates of the dual moose. The gauge-Goldstone mixing (7) can be removed by the familiar R_ξ gauge-fixing term,

$$\begin{aligned}\mathcal{L}_{\rm gf} &= \sum_{n=0}^{N} -\frac{1}{\xi_W} F_n^+ F_n^- + \sum_{n=0}^{K+1} -\frac{1}{2\xi_Z}(F_n^0)^2 \,, \\ F_n^a &= \partial_\mu V_n^{a\mu} + \xi_a M_{an}\widetilde{\pi}_n^a \,,\end{aligned} \tag{10}$$

where $V_n^{a\mu} \in (W_n^{\pm\mu},\, Z_n^{0\mu})$, $M_{an} \in (M_{Wn}, M_{Zn})$ and $\xi_a \in (\xi_W\,, \xi_Z)$. Also, $Z_{K+1}^{0\mu} \equiv A_\gamma^{0\mu}$, $M_{Z(K+1)} \equiv M_\gamma = 0$ and $\widetilde{\pi}_{K+1}^0 \equiv 0$. From Eq. (10), the "eaten" Goldstone boson mass is given by $M^2_{\widetilde{\pi}_n^a} = \xi_a M_{an}^2$.

3. 4D Higgs Mechanism and Geometrization in 5D Continuum

Corresponding to the gauge-fixing (10), we derive Faddeev-Popov ghost term,

$$\mathcal{L}_{\rm FP} = \sum_{n=0}^{N} \left[\overline{c}_n^+ \widehat{s} F_n^- + \overline{c}_n^- \widehat{s} F_n^+\right] + \sum_{n=0}^{K+1} \overline{c}_n^Z \widehat{s} F_n^0 \,, \tag{11}$$

which ensures BRST[13] invariance at quantum level. Following [6, 7], we directly extend Appendix-A of the fourth paper in [14] to derive a 4D Slavnov-Taylor (ST) identity for the moose theory (1) with replicated gauge group,

$$\langle 0|T F^{a_1 n_1}(x_1) F^{a_2 n_2}(x_2) \cdots F^{a_\ell n_\ell}(x_\ell)\Phi_\alpha|0\rangle = 0 \,, \tag{12}$$

which is due to the *manifest gauge invariance* of the moose action $\int dx^4 \mathcal{L}$ from (1) or the equivalent BRST invariance of the effective action $\int dx^4 \left[\mathcal{L} + \mathcal{L}_{\rm gf} + \mathcal{L}_{\rm FP}\right]$. In (12) Φ_α denotes other possible amputated physical fields. We stress that (12) generally holds for *any* possible form of the gauge-fixing function F^{an}. Amputating the external lines in (12), we arrive at a matrix identity for S-matrix elements,[23]

$$\mathcal{T}\Big[\,\overline{\mathbb{F}}^{a_1}(p_1),\, \overline{\mathbb{F}}^{a_2}(p_2),\, \cdots,\, \overline{\mathbb{F}}^{a_\ell}(p_\ell),\, \Phi_\alpha\Big] = 0 \,, \tag{13}$$

where $\overline{\mathbb{F}}^a(p) \equiv p^\mu \mathbb{V}_\mu^a - \widetilde{\mathbb{M}}_a^{\rm diag}\mathbf{C}^a\widetilde{\Pi}^a = \widetilde{\mathbb{M}}_a^{\rm diag}(\mathbb{V}_S^a - \mathbf{C}^a\widetilde{\Pi}^a)$ with $\mathbb{V}_S^a = \epsilon_S^\mu \mathbb{V}_\mu^a$ ($\epsilon_S^\mu \equiv p^\mu/\widetilde{\mathbb{M}}_a^{\rm diag}$), $\mathbb{V}^{a\mu} = \mathbb{W}^{\pm\mu}, \mathbb{Z}^{0\mu}$, and $\mathbf{C}^a = -i[1 + O(\text{loop})]$. The identity (13) states that in the S-matrix element the unphysical eaten "KK" Goldstones $\widetilde{\Pi}^a$ and the unphysical "KK" scalar gauge-components $\mathbb{V}_S^a$ are *confined,* so they together have no net contribution to any physical process — a quantitative formulation of the 4D Higgs mechanism at the S-matrix level, which holds even without a physical Higgs boson such as in the present moose theory with replicated gauge groups. Under high energy expansion, Eq. (13) results in a *generalized form* of the equivalence theorem (ET)[14,6], which connects the high energy longitudinal gauge boson scattering amplitude to that of the "eaten" Goldstone bosons,

$$\mathcal{T}\big[V_L^{a_1 n_1}, V_L^{a_2 n_2}, \cdots\big] = C_{\rm mod}^{n_1 m_1, n_2 m_2 \cdots}\, \mathcal{T}[\widetilde{\pi}^{a_1 m_1}, \widetilde{\pi}^{a_2 m_2}, \cdots] + O\!\left(\frac{M_{an}}{E_n}\right), \tag{14}$$

where $V_n^a = W_n^{\pm},\, Z_n^0$ $(n = 0, 1, 2, \cdots)$, and the indices $(m_1, m_2, \cdots)$ are summed up. The exact expression of $O(M_{an}/E_n)$ term is obtained from directly expanding ET identity (13) as we did in [14]. The radiative modification factor is[23]

$$C_{\rm mod}^{n_1 m_1, n_2 m_2 \cdots} \equiv \mathbf{C}^{a_1}_{n_1 m_1} \mathbf{C}^{a_2}_{n_2 m_2} \cdots = (-i)^{\ell} \left[(\delta_{n_1 m_1} \delta_{n_2 m_2} \cdots) + O(\text{loop}) \right]. \qquad (15)$$

Analyzing both sides of (14), we observed [6] that the non-canceled leading E^2-terms in the usual nonlinear gauged sigma model are now suppressed by large N due to the *collective effect* of many "KK" modes in the deconstruction theory.

To take the 5D continuum limit we redefine the lattice link field (Wilson line),

$$U_j(x) = U(x, x_j^5) = \exp\left[i \int_{x_{j-1}^5}^{x_j^5} dx^5 g_j \mathbf{A}_5(x, x^5) \right] = \exp\left[i\, \mathbf{a} g_j \mathbf{A}_5(x, x_j^5) \right] \qquad (16)$$

where $\mathbf{a} = \Delta x_j^5$ is lattice spacing and $A_5^a(x, x_j^5)$ is related to the site-Goldstone boson $\pi_j^a(x)$ in (1), $A_5^a(x, x_j^5) = \pi_j^a(x)(2/g_j f_j \mathbf{a})$. So π_j^a will geometrize as $\widehat{V}_5^a$ component of 5D gauge field $\widehat{A}_J^a = (\widehat{A}_\mu^a,\, \widehat{A}_5^a)$, where $J, I, \ldots \in (\mu, 5)$. Ignoring $U(1)^{M+1}$ in Fig. 1, we define position-dependent couplings $g_j = g_5 \kappa_j / \sqrt{\mathbf{a}}$ and decay constants $f_j = h_j f$. Taking continuum limit of (1), we reproduce a $SU(2)_{\rm 5D}$ theory,

$$S_5 \;=\; \int d^5\widehat{x}\, \sqrt{-\widehat{g}}\;\, \widehat{g}^{IK} \widehat{g}^{JL}\, \frac{-1}{4\kappa^2(x^5)}\, \widehat{F}^a_{IJ} \widehat{F}^a_{KL}\,, \qquad (17)$$

defined on a general 5D background,

$$ds^2 \;=\; [\kappa(x^5) h(x^5)]^2 \eta_{\mu\nu} dx^\mu dx^\nu - dx^5 dx^5\,, \qquad (18)$$

where $\widehat{g}_{IK} = \text{diag}\left(\eta_{\mu\nu}(\kappa h)^2,\, -1\right)$. For $\kappa = h = 1$, it reduces to flat 5D geometry, while for $\kappa = 1$ and $h = \exp[-k|x^5|]$ it reduces to the familiar warped RS1[15]. Various extended continuum models from (17) can be derived from (1) by including $U(1)^{M+1}$ and/or proper folding(s) of the moose chain in Fig. 1. These together with *induced* 5D BCs and possible brane terms will be analyzed in Sec. 4. Here we note that since 5D Lorentz symmetry is reduced to 4D Lorentz symmetry at boundaries $(x^5 = 0, L)$, it is natural for a brane term to take the form of 4D Yang-Mills Lagrangian under the residual gauge symmetry at the boundary, *e.g.,*

$$S_{\rm BT} \;=\! \int d^5\widehat{x}\; \delta(x^5 \!-\! y)\, \frac{-\zeta}{4} \widehat{F}^{a\mu\nu} \widehat{F}^a_{\mu\nu}\,, \qquad (y = 0, L)\,. \qquad (19)$$

The brane term (19) may vary, depending on the residual symmetry at the boundary. We stress that the action $S_5 + S_{\rm BT}$ is invariant under the 5D gauge symmetry supplemented by BCs [*induced* from the continuum limit of (1), cf. Sec. 4].

Analyzing (17) we construct an R_ξ gauge-fixing to remove $\widehat{A}^a_\mu$-$\widehat{A}^a_5$ mixing,

$$\widehat{\mathcal{L}}_{\rm gf} \;=\; -\frac{1}{2\xi} (\widehat{F}^a)^2\,, \qquad \widehat{F}^a = \kappa^{-1} \partial_\mu \widehat{A}^{a\mu} + \xi\kappa\, \partial_5 (h^2 \widehat{A}^{a5})\,. \qquad (20)$$

The corresponding ghost term is given by $\widehat{\mathcal{L}}_{\rm FP} \;=\; \widehat{\bar{c}}^a \widehat{s} \widehat{F}^a$. Using 5D gauge invariance of the action $S_5 + S_{\rm BT}$, we derive an ST identity[7] in parallel to (12),

$$\langle 0 | T \widehat{F}^{a_1}(\widehat{x}_1) \widehat{F}^{a_2}(\widehat{x}_2) \cdots \widehat{F}^{a_\ell}(\widehat{x}_\ell) \widehat{\Phi}_\alpha | 0 \rangle \;=\; 0\,. \qquad (21)$$

With the 5D equation of motion (EOM) for free field $\widehat{V}^a_{\mu,5}$,

$$\begin{aligned}
&\left(\partial_\mu^2 - \kappa^2\partial_5 h^2\partial_5\right)\widehat{A}^a_\nu - \left(1-\xi^{-1}\right)\partial_\nu\partial_\mu\widehat{A}^{a\mu} \;=\; 0\,,\\
&\partial_5(\kappa^2\partial_5 h^2\widehat{A}^a_5) - \xi^{-1}\partial_\mu^2\widehat{A}^a_5 \;=\; 0\,,
\end{aligned} \tag{22}$$

and the KK expansions,

$$\widehat{A}^{a\mu}(\widehat{x}) = \frac{1}{\sqrt{L}}\sum_{n=0}^{\infty} V_n^{a\mu}(x)X_n(x^5)\,, \qquad \widehat{A}^{a5}(\widehat{x}) = \frac{1}{\sqrt{L}}\sum_{n=0}^{\infty} V_n^{a5}(x)\widetilde{X}_n(x^5)\,, \tag{23}$$

we can decompose the 5D gauge-fixing function $\widehat{F}^a$ as [23]

$$\begin{aligned}
\widehat{F}^a(\widehat{x}) \;&=\; \frac{1}{\sqrt{L}}\sum_{n=0}^{\infty} F^{an}(x)\,X_n(x^5)/\kappa(x^5)\,,\\
F^{an}(x) \;&=\; \partial_\mu V_n^{a\mu}(x) - \xi M_{an}V_n^{a5}(x) \;\equiv\; \underline{K}^T_{an}\underline{V^a_n}\,,
\end{aligned} \tag{24}$$

where $\underline{K}_{an} = (\partial_\mu,\, -\xi M_{an})^T$ and $\underline{V^a_n} = (V_n^{a\mu},\, V_n^{a5})^T$. Thus we obtain a 4D ST identity from (21),

$$\langle 0|TF^{a_1n_1}(x_1)F^{a_2n_2}(x_2)\cdots F^{a_\ell n_\ell}(x_\ell)\Phi_\alpha|0\rangle \;=\; 0\,. \tag{25}$$

Deriving the BRST transformations for KK fields and relevant ST identities, we amputate the external fields in (25) and obtain an identity for S-matrix elements,[23]

$$\mathcal{T}\left[\overline{F}^{a_1n_1}(p_1),\, \overline{F}^{a_2n_2}(p_2),\, \cdots,\, \overline{F}^{a_\ell n_\ell}(p_\ell), \Phi_\alpha\right] \;=\; 0\,, \tag{26}$$

where $\overline{F}^{an} = p^\mu V_\mu^{an} - M_{an}\widehat{C}^a_{nm}V_5^{am} = M_{an}(V_S^{an} - \widehat{C}^a_{nm}V_5^{am})$, $V_S^{an} = \epsilon_S^\mu V_\mu^{an}$ $[\epsilon_S^\mu \equiv k^\mu/M_{an} = \epsilon_L^\mu - v^\mu,\ v^\mu = O(M_{an}/E_n)]$, and $\widehat{C}^a_{nm} \equiv i(M_{am}/M_{an})[(\mathbf{1} + \widetilde{\Delta}^a(p^2))(\mathbf{1}+\Delta^a(p^2))^{-1}]_{mn}$. Similar to (13) for the deconstruction theory, our ET identity (26) shows that the unphysical scalar-KK-component V_S^{an} and the fifth gauge-KK-component V_5^{an} (or its linear composition $\widehat{C}^a_{nm}V_5^{am}$) are *confined* at the S-matrix level, so they together have zero contribution to any physical process. This is just the quantitative S-matrix formulation of the 5D *geometric Higgs mechanism* (GHM)[7,8], where V_5^{an}'s serve as the would-be Goldstone bosons and get converted to the longitudinal gauge KK-modes V_L^{an}'s. Expanding the ET identity (26), we thus derive a *general Kaluza-Klein ET* (KK-ET[8]) at asymptotic energy,

$$\mathcal{T}\left[V_L^{a_1n_1},\, V_L^{a_2n_2},\cdots\right] \;=\; \widehat{C}_{\rm mod}^{n_1m_1,n_2m_2\cdots}\,\mathcal{T}\left[V_5^{a_1m_1},\, V_5^{a_2m_2},\cdots\right] \;+\; O\!\left(\frac{M_{an}}{E_n}\right), \tag{27}$$

where $\widehat{C}_{\rm mod}^{n_1m_1,n_2m_2\cdots} = \widehat{C}^{a_1}_{n_1m_1}\cdots\widehat{C}^{a_\ell}_{n_\ell m_\ell} = i^\ell[(\delta_{n_1m_1}\delta_{n_2m_2}\cdots) + O(\text{loop})]$, and the external momenta are put on-shell ($p_n^2 = M_{an}^2$). The exact expression of the suppressed term $O(M_{an}/E_n)$ in (27) is tedious and is directly obtained from expanding the ET identity (26) by $\epsilon_S^\mu = \epsilon_L^\mu - v^\mu$, as we did in [14]. We stress that *our KK-ET (27) and its KK-ET identity (26) are valid for arbitrary geometry with any consistent BC and possible brane term such as (19) (cf. Sec. 4).* The KK-ET (27) is the manifestation of the 5D geometric Higgs mechanism[7,8] at the S-matrix level, where the conversion $V_5^{an} \Longrightarrow V_L^{an}$ is realized. An essential advantage of this 5D formulation is that we start with the well-defined 4D moose theory which is manifestly gauge-invariant without any BC. The consistent BCs and possible brane terms are *automatically induced* by taking proper continuum limits (cf. Sec. 4), which also ensure the gauge-invariance of 5D action at the boundaries.

The gauge interactions of V_5^{an} arise from $\int d^5\widehat{x}\frac{-h^2}{2}\widehat{F}^a_{\mu 5}\widehat{F}^{a\mu 5}$ under KK expansion. Examining this[8,7] and applying power counting[16], we find that scalar amplitude $\mathcal{T}[V_5^{a_1n_1}, V_5^{a_2n_2}, \cdots\cdots] = O(E^0)$ generally holds[7], *independent of the detail of any particular compactification/BC and possible brane term.* Because of this and Eq. (27), we conclude that the longitudinal amplitude $\mathcal{T}[V_L^{a_1n_1}, V_L^{a_2n_2}, \cdots\cdots] = O(E^0)$, generally guaranteeing the E-power cancellations down to a constant.

Finally, we note that the nonlinearly realized gauge symmetry in the moose theories may be regarded as a (spontaneously) broken phase formulation, but the crucial difference from an arbitrary or random *explicit symmetry breaking* is that our formalism (including the continuum 5D action) is manifestly gauge-invariant or BRST-invariant in which the would-be Goldstone bosons and the corresponding "Higgs" mechanism are consistently embedded, so the various Slavnov-Taylor and Ward-Takahashi identities can be derived even in the broken phase[17]. It is such *general identities and their resulting ET (14) or KK-ET (27)* that have directly guaranteed the relevant E-cancellations in *all* $2 \to n$ $(n \geqslant 2)$ longitudinal gauge boson scatterings for *any* 4D or compactified higher-D theory, leading to the effective unitarity.[6,8,7]

4. Deconstruction, (Non)Geometric Gauge Symmetry Breaking and Induced Boundary Conditions

Various compactified 5D theories can be derived by taking proper continuum limit of our general moose theory (Fig. 1). The bulk gauge symmetry and its breaking at the boundary are essentially determined by the structure of the 4D moose and its spontaneous symmetry breaking pattern. In comparison with the traditional 4D Higgs mechanism[9] which spontaneously breaks the gauge symmetry by nontrivial vacuum, the 5D gauge symmetry can be spontaneously broken by the proper compactification at boundaries which arise from the "boundary" groups (sites) of the moose theory (Fig. 1) under possible folding(s). This provides us a conceptually clean and elegant formulation of the gauge symmetry breaking (without/with gauge-group rank reduction) in compactified 5D theories, without inputting the BCs *a priori* or relying on the technique of adding extra boundary Higgs fields[18]. Our general moose analysis below also reveals that the *induced* 5D BCs are actually *independent of any particular choice of bulk geometry,* as we expect.

We start by considering the special case of Fig. 1 with $M = 0$ and $f_{N+1} = 0$, whose gauge symmetry breaking pattern is $SU(2)^{N+1} \to SU(2)_D$, as shown in Fig. 2. The $(N+1)\times(N+1)$ gauge boson mass-matrix $\mathbb{M}_a^2 = \mathbb{M}^2_{[0,N]}$ is given in (2) with $f_{N+1} = 0$, and its orthonormal eigenvectors are denoted as $\mathbb{X}_n =$

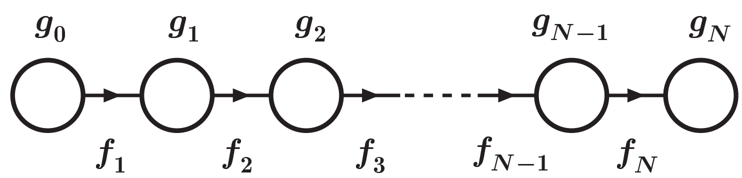


Fig. 2. Higgsless Moose-A: a special case of Fig. 1 with $M = 0$ and $f_{N+1} = 0$. The gauge symmetry breaking pattern is $SU(2)^{N+1} \to SU(2)_D$, with massless zero modes.

$(X_{0n}, X_{1n}, \cdots, X_{Nn})^T$ $(n = 0, 1, 2, \cdots, N)$. So the gauge-eigenbasis field $\mathbb{A}^{a\mu}$ and mass-eigenbasis field $\mathbb{V}^{a\mu}$ are connected by $\mathbb{A}^{a\mu} = \mathbb{R}_a \mathbb{V}^{a\mu}$ with rotation $\mathbb{R}_a = (\mathbb{X}_0, \mathbb{X}_1, \cdots, \mathbb{X}_N)$, i.e.,

$$A_j^{a\mu} = \sum_{n=0}^{N} V_n^{a\mu} X_{jn}\,, \quad \Longrightarrow \quad \widehat{A}^{a\mu}(x, x^5) = \sum_{n=0}^{N} V_n^{a\mu}(x) X_n(x^5)\,, \tag{28}$$

where the second relation is the corresponding continuum KK-expansion under $N \to \infty$ and $j\,\mathbf{a} \to x^5$. Analyzing the eigenvalue equation $\mathbb{M}_a^2 \mathbb{X}_n = \lambda_n \mathbb{X}_n$, we find that the eigenvectors satisfy the following consistency relations,

$$g_0 X_{0,n} - g_{-1} X_{-1,n} = 0\,, \qquad g_{N+1} X_{N+1,n} - g_N X_{N,n} = 0\,. \tag{29}$$

The consistency of geometry requires the "extra" couplings (g_{-1}, g_{N+1}) such that $g_{-1} = g_0$ and $g_{N+1} = g_N$. Since $g_j = \kappa(x_j^5) g_5/\sqrt{\mathbf{a}}$ and $\kappa(x^5)$ is a continuous function, this requirement corresponds to defining $\partial_5 \kappa(x^5)|_{x^5=0^-} = 0$ and $\partial_5 \kappa(x^5)|_{x^5=L^+} = 0$, which are always allowed. So, from (29) we derive the *induced boundary conditions (BCs)* of Neumann type in the 5D continuum limit,

$$\partial_5 \widehat{A}_\mu^a \Big|_{x^5=0} = 0\,, \qquad \partial_5 \widehat{A}_\mu^a \Big|_{x^5=L} = 0\,, \tag{30}$$

where $L = N\mathbf{a}$ is the length of 5D. This compactified 5D theory has an unbroken $SU(2)$ gauge symmetry survived in 4D (corresponding to the residual diagonal group $SU(2)_D$ of the Moose-A in Fig. 2), which ensures the massless zero KK-modes. So, the Neumann BCs (30) break a 5D $SU(2)$ into a 4D $SU(2)$ (with massive gauge KK towers as well as massless zero modes), and preserve the rank of 5D gauge group, precisely matching the symmetry breaking structure of its parent Moose-A in Fig. 2.

We can further derive the BCs for $\widehat{A}^{a5}$, by analyzing the Goldstone mass matrix $\xi_a \widetilde{\mathbb{M}}_a^2$ (or the gauge boson mass matrix $\widetilde{\mathbb{M}}_a^2$ in the dual moose)[23]. Inspecting the eigenvalue equation $\widetilde{\mathbb{M}}_a^2 \widetilde{X}_n = \lambda_n \widetilde{X}_n$, we find that the eigenvectors must satisfy the following consistency conditions,[23]

$$\widetilde{X}_{0,n} = 0\,, \qquad \widetilde{X}_{N+1,n} = 0\,, \tag{31}$$

which *induce* the 5D BCs for $\widehat{A}_5^a$ in the continuum limit,

$$\widehat{A}_5^a \Big|_{x^5=0} = 0\,, \qquad \widehat{A}_5^a \Big|_{x^5=L} = 0\,. \tag{32}$$

They are both of Dirichlet type, unlike the Neumann BCs for $\widehat{A}^{a\mu}$ in (30).

We then turn to a different gauge symmetry breaking structure. This is most cleanly described by the Moose-B in Fig. 3 (which we called "enlarged moose" in 2001[7]). This is a special case of our general moose theory in Fig. 1 with $M = 0$ and $g_{N+1} = 0$, which has the gauge group breaking pattern, $SU(2)^{N+1} \to \text{nothing}$. As we will show, it *induces* the Dirichlet type BCs in 5D continuum limit and reduces the rank of 5D gauge group.

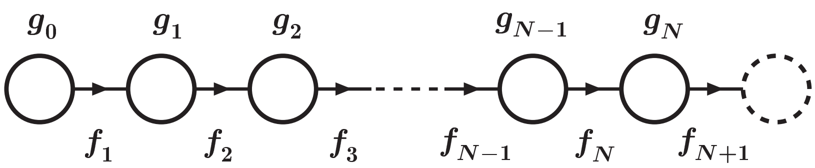


Fig. 3. Higgsless Moose-B: a special case of Fig. 1 with $M = 0$ and $g_{N+1} = 0$. The gauge symmetry breaking pattern is $SU(2)^{N+1} \to$ nothing, without massless "zero modes".

The gauge boson mass matrix $\mathbb{M}_a^2 = \mathbb{M}^2_{[0,N+1)}$ is just given by Eq. (2), which has massive eigenvalues only. Analyzing the eigenvalue equation $\mathbb{M}_a^2 \mathbb{X}_n = \lambda_n \mathbb{X}_n$, we observe that the eigenvectors satisfy the consistency relations,

$$g_0 X_{0,n} - g_{-1} X_{-1,n} = 0\,, \qquad X_{N+1,n} = 0\,. \tag{33}$$

Setting the extra coupling $g_{-1} = g_0$ as before and taking the continuum limit, we derive the induced BCs,

$$\partial_5 \widehat{A}^a_\mu \Big|_{x^5=0} = 0\,, \qquad \widehat{A}^a_\mu \Big|_{x^5=L} = 0\,, \tag{34}$$

where we see that similar to (30) the BC at $x^5 = 0$ remains the Neumann type, but the new condition at $x^5 = L$ is a Dirichlet BC. The moose representation of Fig. 3 clearly shows that it is this induced Dirichlet BC at $x^5 = L$ that completely breaks the 5D $SU(2)$ gauge group and makes all zero KK-modes massive. The above formulation based on the Moose-A and -B can be directly generalized to any 5D gauge theory with an arbitrary simple group G_A.

We have also derived the BCs for $\widehat{A}^{a5}$ in the Moose-B. Analyzing the eigenvalue equation $\widetilde{\mathbb{M}}_a^2 \widetilde{\mathbb{X}}_n = \lambda_n \widetilde{\mathbb{X}}_n$, we deduce the consistency relations,[23]

$$\widetilde{X}_{0,n} = 0\,, \qquad f_{N+1} \widetilde{X}_{N+1,n} - f_{N+2} \widetilde{X}_{N+2,n} = 0\,. \tag{35}$$

We are free to choose the extra decay constant f_{N+2} as $f_{N+2} = f_{N+1}$ for the consistency of geometry. With the relation $f_j = h(x_j^5) f$, this corresponds to defining the derivative $\partial_5 h(x^5)|_{x^5=L^+} = 0$. Thus, we derive the *induced BCs* from (35) in the 5D continuum limit,[23]

$$\widehat{A}^a_5 \Big|_{x^5=0} = 0\,, \qquad \partial_5 \widehat{A}^a_5 \Big|_{x^5=L} = 0\,. \tag{36}$$

Another advantage of our general deconstruction formalism is that we can also derive proper brane kinetic terms in the continuum limit. Considering our Moose-B for instance, we quantify how a brane term at $x^5 = 0$ is reconstructed from localizing the boundary group $SU(2)_0$ at site $j = 0$. Setting $f_1^2 \gg f_j^2$ $(j \geqslant 2)$ for taking continuum limit (which prevents site-0 from joining 5D geometry), we examine the equation of motion (EOM) for $A_0^{a\mu}$ and derive a relation,

$$A_0^{a\mu} \;=\; (g_1/g_0)\, A_1^{a\mu}\,. \tag{37}$$

In the continuum theory, this localizes $SU(2)_0$ as a brane kinetic term at $x^5 = 0$,

$$S_{\rm BT} \;=\int d^5\widehat{x}\;\delta(x^5 - 0^+)\,\frac{-(\widehat{g}/g_0)^2}{4}\widehat{F}^a_{\mu\nu}\widehat{F}^{a\mu\nu}\,, \tag{38}$$

where $\widehat{g} \equiv g_5$ denotes the 5D gauge coupling. We note that the above brane term is actually localized at the *right-neighborhood* of $x^5 = 0$ at which our BC $\partial_5\widehat{A}^a_\mu|_{x^5=0} = 0$ is imposed. This is because our deconstruction procedure of deriving this BC from (33) must *include* boundary site $j = 0$ itself plus its left-neighborhood.

Next, we analyze the Moose-C in Fig. 5, which is a special case of Fig. 1 with $M = 0$ and $g_{N+1} \neq 0$. As a slight variation of Moose-B, Moose-C has gauged its subgroup $U(1)_{N+1}$ and has the symmetry breaking pattern $SU(2)^{N+1} \otimes U(1) \to U(1)_{\rm em}$, as a straightforward extension of Ref. [19] which studied the nontrivial minimal models $SU(2)^3 \otimes U(1) \to U(1)_{\rm em}$ $(N+1 = 3)$ and $SU(2)^2 \otimes U(1)^2 \to U(1)_{\rm em}$ $(N+1 = M+1 = 2)$ with general inputs $(g_j,\, f_k)$.

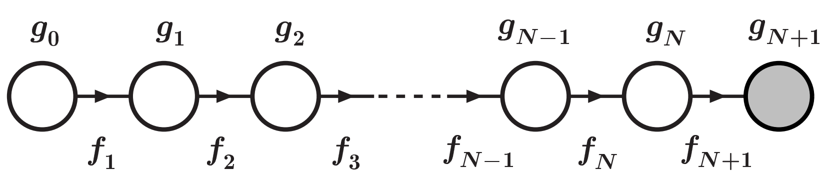


Fig. 4. Higgsless Moose-C: a special case of Fig. 1 with $M = 0$ and $g_{N+1} \neq 0$. The gauge symmetry breaking pattern is $SU(2)^{N+1} \otimes U(1) \to U(1)_{\rm em}$, extended from Ref. [19].

The charged gauge boson mass matrix $\mathbb{M}_W^2 = \mathbb{M}^2_{[0,N+1)}$ is the same as in Moose-B, while the neutral gauge boson mass matrix $\mathbb{M}_Z^2 = \mathbb{M}^2_{[0,N+1]}$ can be obtained from $\mathbb{M}_Z^2$[Moose-A] by the replacement $N \to N+1$. So we find the consistency relations for eigenvectors $\{\mathbb{X}_n^{\pm},\, \mathbb{X}_n^3\}$ to be a mixture of Moose-A and -B in (29) and (33),

$$\begin{aligned} &g_0 X^{\pm}_{0,n} - g_{-1}X^{\pm}_{-1,n} \;=\; 0\,, \qquad X^{\pm}_{N+1,n} \;=\; 0\,;\\ &g_0 X^{3}_{0,n} - g_{-1}X^{3}_{-1,n} \;=\; 0\,, \qquad g_{N+2}X^3_{N+2,n} - g_{N+1}X^3_{N+1,n} \;=\; 0\,. \end{aligned} \tag{39}$$

So, with the same reasoning as before, we obtain the induced 5D BCs,

$$\partial_5\widehat{A}^{\pm}_\mu\Big|_{x^5=0} = 0\,, \quad \partial_5\widehat{A}^{3}_\mu\Big|_{x^5=0} = 0\,; \quad \widehat{A}^{\pm}_\mu\Big|_{x^5=L} = 0\,, \quad \partial_5\widehat{A}^{3}_\mu\Big|_{x^5=L} = 0\,. \tag{40}$$

In the continuum limit, the boundary group $U(1)_{N+1}$ will be localized at $x^5 = L$, similar to the brane term which we reconstructed in (38). So, we can localize $U(1)_{N+1}$ by setting $f_{N+1}^2 \gg f_j^2 \ (j \leqslant N)$ for the continuum limit. We quantify this by examining the EOM for $A_{N+1}^{3\mu}$ under large f_{N+1}^2 limit. Thus we derive,

$$A_{N+1}^{3\mu} \;=\; \left(g_N/g_{N+1}\right) A_N^{3\mu}\,, \tag{41}$$

which generates the localized brane kinetic term at $x^5 = L$,

$$S_{\rm BT} \;=\int d^5\widehat{x}\;\delta(x^5 - L^-)\frac{-(\widehat{g}\,/g_{N+1})^2}{4}\widehat{F}^3_{\mu\nu}\widehat{F}^{3\mu\nu}\,. \tag{42}$$

This brane term is localized at left-neighborhood of $x^5 = L$ at which the BC $\partial_5\widehat{A}^3_\mu|_{x^5=L} = 0$ is imposed, because our deconstruction procedure of deriving this BC from (39) has to include boundary site $j = N+1$ plus its right-neighborhood.

Then, we consider Moose-D in Fig. 6 which is obtained from Fig. 1 by folding $SU(2)$ lattice chain once at the middle with renumbering $N \to 2N$ and setting $M = 0$. The symmetry breaking structure is $SU(2)_L^{N+1} \otimes SU(2)_R^N \otimes U(1) \to U(1)_{\rm em}$, where $SU(2)_L^{N+1}$ has inputs $\{g_j,\, f_k\}$ and $SU(2)_R^N \otimes U(1)$ has inputs $\{\widetilde{g}_j,\, \widetilde{f}_k\}$.

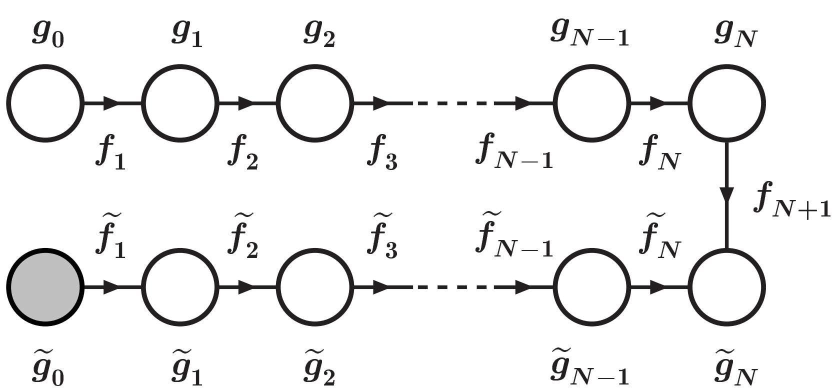


Fig. 5. Higgsless Moose-D: Folding the $SU(2)^{N+1}$ moose chain once produces a variation of Higgsless deconstruction from Fig. 1 with $M = 0$ and by renumbering $N \to 2N$. The gauge symmetry breaking pattern is $SU(2)_L^{N+1} \otimes SU(2)_R^N \otimes U(1) \to U(1)_{\rm em}$, whose continuum limit has a bulk gauge group $[SU(2)_L \otimes SU(2)_R]_{\rm 5D}$.

The continuum limit of Moose-D has bulk gauge group $[SU(2)_L \otimes SU(2)_R]_{\rm 5D}$ (with 5D couplings $\widehat{g}_L$ and $\widehat{g}_R$), plus a $U(1)_0$ brane term localized at $x^5 = 0$ [similar to (42)]. Other brane terms at $x^5 = 0$ for $SU(2)_L$ [similar to (38)] and at $x^5 = L$ for $SU(2)_D$ are also possible. So, at $x^5 = 0$ the BCs for $SU(2)_L$ $[SU(2)_R]$ are essentially the same as those of Moose-C at $x^5 = 0$ $[x^5 = L]$, i.e.,

$$\partial_5\widehat{A}_L^{a\mu}\Big|_{x^5=0} = 0\,; \qquad \widehat{A}_R^{\pm\mu}\Big|_{x^5=0} = 0\,, \qquad \partial_5\widehat{A}_R^{3\mu}\Big|_{x^5=0} = 0\,. \tag{43}$$

To derive the new BCs at $x^5 = L$, we order the sites $j = 0, 1, \cdots, N, \widetilde{N}, \widetilde{N+1}, \cdots, \widetilde{1}, \widetilde{0}$. So, the charged gauge boson mass matrix is $\mathbb{M}_W^2 = \mathbb{M}^2_{[0,\widetilde{0})}$ and the neutral gauge boson mass matrix is $\mathbb{M}_Z^2 = \mathbb{M}^2_{[0,\widetilde{0}]}$. Inspecting their eigenvalue equations we derive consistency relations for eigenvectors $\mathbb{X}_n \in SU(2)_L$ and $\widetilde{\mathbb{X}}_n \in SU(2)_R$, related to the boundary $x^5 = L$,

$$\begin{aligned} g_{N+1}X^a_{N+1} - \widetilde{g}_N\widetilde{X}^a_N &= 0\,, \\ \widetilde{g}_N\left[\frac{g_{N+1}}{g_N}X^a_{N+1} - X^a_N\right] + \zeta^2 g_N\left[\frac{\widetilde{g}_{N+1}}{\widetilde{g}_N}\widetilde{X}^a_{N+1} - \widetilde{X}^a_N\right] &= 0\,, \end{aligned} \tag{44}$$

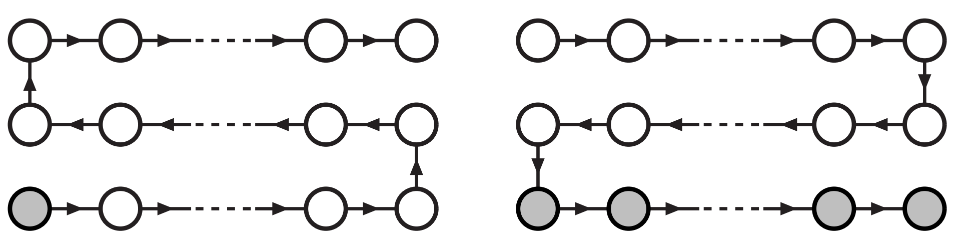
where $\zeta = (\widetilde{g}_N\widetilde{f}_{N+1})/(g_N f_{N+1})$ is the ratio of two types of lattice spacing defined from the $SU(2)_L$ chain and $SU(2)_R$ chain. In the continuum limit, the consistency of 5D geometry requires the ratio $\zeta = 1$ and the extra couplings $(g_{N+1}, \widetilde{g}_{N+1}) = (g_N, \widetilde{g}_N)$. So, we derive the following induced 5D BCs at $x^5 = L$,

$$\left(\widehat{g}_L\widehat{A}^{a\mu}_L - \widehat{g}_R\widehat{A}^{a\mu}_R\right)\Big|_{x^5=L} = 0\,, \qquad \partial_5\left(\widehat{g}_R\widehat{A}^{a\mu}_L + \widehat{g}_L\widehat{A}^{a\mu}_R\right)\Big|_{x^5=L} = 0\,. \tag{45}$$

We may also reconstruct a brane term at $x^5 = L$. This can be done by taking $f^2_N \gg f^2_{j<N}$ and $\widetilde{f}^2_N \gg \widetilde{f}^2_{j<N}$ together with $f^2_{N+1} \to \infty$. This will localize the diagonal group $SU(2)^D_N$ at $x^5 = L$. Setting $\widehat{g}_L/g_N = \widehat{g}_R/\widetilde{g}_N \equiv \eta$, we write the brane coupling of $SU(2)^D_N$ as $g_D = 1/\sqrt{1/g^2_N + 1/\widetilde{g}^2_N} = \eta^{-1}\widehat{g}_L\widehat{g}_R/\sqrt{\widehat{g}^2_L + \widehat{g}^2_R}$. The brane gauge field is $A^{a\mu}_{N,D} = (\widetilde{g}_N A^{a\mu}_N + g_N\widetilde{A}^{a\mu}_N)/\sqrt{g^2_N + \widetilde{g}^2_N}$. In the continuum limit, it becomes $A^{a\mu}_{N,D} = \eta(\widehat{g}_R\widehat{A}^{a\mu}_L + \widehat{g}_L\widehat{A}^{a\mu}_R)/\sqrt{\widehat{g}^2_L + \widehat{g}^2_R}$ at $x^5 = L$, and leads to the Yang-Mills term $-\frac{1}{4}(F^{a\mu\nu}_{N,D})^2 = -\frac{\eta^2}{4(\widehat{g}^2_L + \widehat{g}^2_R)}[\widehat{g}_R\widehat{F}^{a\mu\nu}_L + \widehat{g}_L\widehat{F}^{a\mu\nu}_R]^2$. So we derive the brane localized kinetic term at $x^5 = L$,

$$S_{\rm BT} = \int d^5\widehat{x}\;\delta(x^5 - L^-)\frac{-\eta^2}{4(\widehat{g}^2_L + \widehat{g}^2_R)}\left[\widehat{g}_R\widehat{F}^{a\mu\nu}_L + \widehat{g}_L\widehat{F}^{a\mu\nu}_R\right]^2. \tag{46}$$

Possible brane terms at $x^5 = 0$ can be similarly reconstructed as in Moose-B,C.



(a) (b)

Fig. 6. Higgsless Moose-E with double-folding from the general moose theory in Fig. 1. (a). Moose-E1 in the continuum limit has gauge group $[SU(2)^3]_{\rm 5D}\otimes U(1)_0$. (b). Moose-E2 in the continuum limit has gauge group $[SU(2)_L\otimes SU(2)_R\otimes U(1)_X]_{\rm 5D}$. They are both broken to $U(1)_{\rm em}$.

A further extension of the Moose-D is to fold the $SU(2)$-chain twice, called Moose-E1 in Fig. 7(a). This leads to a 5D continuum Higgsless theory with symmetry $[SU(2)^3]_{\rm 5D}\otimes U(1)_0$ whose BCs can be similarly derived as in Moose-D.

(a)

The gauge group breaks as $[SU(2)^3]_{5\text{D}} \otimes U(1)_0 \to U(1)_{\text{em}}$ under compactification. Another variation is to add many $U(1)$'s to Moose-D and form a separate $U(1)$-chain, or, from Fig. 1 we can fold the $SU(2)$-chain once in its middle as well as folding the moose at the intersection of the $SU(2)$ and $U(1)$ chains, which we call Moose-E2 in Fig. 7(b). Its 5D continuum model has the bulk gauge group $[SU(2)_L \otimes SU(2)_R \otimes U(1)_X]_{5\text{D}}$ which, for the case of warped geometry, gives precisely the 5D Higgsless model proposed by Csáki et al [11]. Possible brane terms can be generated in the continuum limit as before. With the above method, we have also derived[21] all the *induced* BCs in the continuum limit of Moose-E2, and found they agree to the BCs of Csáki et al (based on adding extra boundary Higgs[18]).

5. Effective Unitarity of Higgsless Deconstruction Without Geometry

In this section we will demonstrate a *conceptual* point that the delayed unitarity violation (effective unitarity) in the general moose theory can be realized without resorting to any known 5D geometry. We observe that the delay of unitarity violation is essentially a *collective effect* due to the participation in the EWSB from many gauge groups whose own symmetry breaking scales $\{f_j\}$ are higher than the SM EWSB scale, $v = (\sqrt{2}G_F)^{-1/2} \simeq 246\,\text{GeV}$ and whose gauge couplings $\{g_j\}$ are larger than $g \in SU(2)_{\text{SM}}$. Such a collective effect *does not* necessarily require any exact 5D geometry, and can be realized in general *non-geometric* moose settings.

5.1. *Formulation of the Effective Unitarity*

To analyze unitarity it is enough to take $g_{N+1} \to 0$ limit, so the nontrivial part of Fig. 1 reduces to our Moose-B (Fig. 3), corresponding to the custodial symmetry limit. Other settings such as Moose-C,D,E also reduce to Moose-B at zeroth order of g_{N+1}. From Eq. (1) we first derive the relevant unitary-gauge Lagrangian,

$$\begin{aligned}\mathcal{L} &= \sum_{j=0}^{N} -\frac{1}{4}F^a_{j\mu\nu}F^{a\mu\nu}_j + \frac{1}{2}\mathbb{A}^{a\,T}_\mu \mathbb{M}^2_W\,\mathbb{A}^{a\mu} \\ &= \sum_{j=0}^{N} -\frac{1}{4}\overline{W}^a_{j\mu\nu}\overline{W}^{a\mu\nu}_j + \frac{1}{2}\mathbb{W}^{a\,T}_\mu \mathbb{M}^{2\ \text{diag}}_W\,\mathbb{W}^{a\mu} + \mathcal{L}^{\text{int}}_G\end{aligned} \tag{47}$$

with the Yang-Mills interactions,

$$\begin{aligned}\mathcal{L}^{\text{int}}_G &= \sum_{j=0}^{N}\left[-\frac{g_j}{2}C^{abc}\overline{F}^{a\mu\nu}_j A^b_{j\mu}A^c_{j\nu} - \frac{g_j^2}{4}C^{abc}C^{ade}A^{b\mu}_j A^{c\nu}_j A^d_{j\mu}A^e_{j\nu}\right] \\ &= -\frac{G_3^{kmn}}{2}C^{abc}\overline{W}^{a\mu\nu}_k W^b_{m\mu}W^c_{n\nu} - \frac{G_4^{k\ell mn}}{4}C^{abc}C^{ade}W^{b\mu}_k W^{c\nu}_\ell W^d_{m\mu}W^e_{n\nu}\,,\end{aligned} \tag{48}$$

where $\overline{F}^{a\mu\nu}_j \equiv \partial^\mu A^{a\nu}_j - \partial^\nu A^{a\mu}_j$, $\overline{W}^{a\mu\nu}_j \equiv \partial^\mu W^{a\nu}_j - \partial^\nu W^{a\mu}_j$, and $\mathbb{W}^{a\mu} = (W^{a\mu}_0, \cdots, W^{a\mu}_N)^T$ denotes the mass-eigenbasis fields. Also the sums over repeated indices $k, \ell, m, n = (0, 1, \cdots, N)$ are implied. In (48), the effective couplings are

$$G_3^{kmn} = \sum_{j=0}^{N} g_j\, \mathbb{R}^a_{jk}\, \mathbb{R}^b_{jm}\, \mathbb{R}^c_{jn}\,, \qquad G_4^{k\ell mn} = \sum_{j=0}^{N} g_j^2\, \mathbb{R}^b_{jk}\, \mathbb{R}^d_{j\ell}\, \mathbb{R}^c_{jm}\, \mathbb{R}^e_{jn}\,, \tag{49}$$

with $\mathbb{R}^a = \mathbb{R}_W$ determined from the diagonalization $\mathbb{R}_W^T \mathbb{M}_W^2 \mathbb{R}_W = (\mathbb{M}_W^2)^{\rm diag}$.

The longitudinal scattering $W^a_{mL} W^b_{nL} \to W^c_{kL} W^d_{\ell L}$ has the following structure,

$$\begin{aligned} \mathcal{T}^{ab,cd}_{mn,k\ell} &= \mathcal{T}^{ab,cd}_{mn,k\ell[c]} + \mathcal{T}^{ab,cd}_{mn,k\ell[s]} + \mathcal{T}^{ab,cd}_{mn,k\ell[t]} + \mathcal{T}^{ab,cd}_{mn,k\ell[u]} \\ &= G_4^{mnk\ell} \mathcal{A}^{abcd}_{[c]} + \sum_{j=0}^{N} \left[\frac{G_3^{mnj} G_3^{k\ell j}}{s - M^2_{Wj}} \mathcal{A}^{abcd}_{[s]} + \frac{G_3^{mkj} G_3^{n\ell j}}{t - M^2_{Wj}} \mathcal{A}^{abcd}_{[t]} + \frac{G_3^{m\ell j} G_3^{nkj}}{u - M^2_{Wj}} \mathcal{A}^{abcd}_{[u]} \right], \end{aligned} \tag{50}$$

which is a sum of possible diagrams from the contact interactions (denoted as $[c]$) and the gauge boson exchanges via (s,t,u)-channels (denoted as $[s],[t],[u]$). Here the quantities $\mathcal{A}^{abcd}_{[c],[s],[t],[u]}$ contain an overall factor of relevant $SU(2)$ structure constants, and are functions of the energy $\sqrt{s}$ and scattering angle θ, whose analytical structures are identical to that of the (Higgsless) SM (with $g' \simeq 0$) up to a simple overall factor g^2. Taking the asymptotic energy $s \gg M^2_{Wj}$ and expanding the amplitude (50), we observe that all $O(E^4)$ terms exactly cancel as enforced by the ET (14), while the nonzero $O(E^2)$ terms are *suppressed* by large $N+1$, leading to the *delayed unitarity violation*[6] in deconstruction theories.

To understand the suppressed $O(E^2)$ terms, it is very useful to examine the pure Goldstone interactions derived from (1), with general inputs for Moose-B,

$$\mathcal{L}^{\rm int}_{\widetilde{\pi}} = \frac{\mathcal{C}_{k\ell mn}}{6v^2} \left[(\widetilde{\pi}^a_k \partial_\mu \widetilde{\pi}^a_\ell)(\widetilde{\pi}^b_m \partial^\mu \widetilde{\pi}^b_n) - (\widetilde{\pi}^a_k \widetilde{\pi}^a_\ell)(\partial_\mu \widetilde{\pi}^b_m \partial^\mu \widetilde{\pi}^b_n) \right] + O\big(\widetilde{\pi}^6_j\big)\,, \tag{51}$$

where $(k,\ell,m,n) = 0,1,2,\cdots,N$ are summed up and the quartic $\widetilde{\pi}^a_n$ coupling is

$$\mathcal{C}_{k\ell mn} = \sum_{j=1}^{N+1} \frac{v^2}{f_j^2} \widetilde{\mathbb{R}}^a_{jk}\, \widetilde{\mathbb{R}}^a_{j\ell}\, \widetilde{\mathbb{R}}^b_{jm}\, \widetilde{\mathbb{R}}^b_{jn} \leqslant 1\,, \tag{52}$$

with $\widetilde{\mathbb{R}}^a = \widetilde{\mathbb{R}}_W$ determined from the diagonalization $\widetilde{\mathbb{R}}_W^T \widetilde{\mathbb{M}}_W^2 \widetilde{\mathbb{R}}_W = (\widetilde{\mathbb{M}}_W^2)^{\rm diag}$. For the special case of the Higgsless SM ($N+1 = M+1 = 1$), the coupling $\mathcal{C}^{\rm SM}_{0000} \equiv 1$. Computing the leading amplitude of $\widetilde{\pi}^a_n \widetilde{\pi}^b_n \to \widetilde{\pi}^c_n \widetilde{\pi}^d_n$ $(n \geqslant 0)$, we arrive at

$$\mathcal{T}[\widetilde{\pi}^a_n \widetilde{\pi}^b_n \to \widetilde{\pi}^c_n \widetilde{\pi}^d_n] = \frac{\mathcal{C}_{nnnn}}{v^2} \left[s\, \delta^{ab}\delta^{cd} + t\, \delta^{ac}\delta^{bd} + u\, \delta^{ad}\delta^{bc} \right]. \tag{53}$$

So, we obtain a *delayed* unitarity violation scale relative to the Higgsless SM,

$$\mathscr{D}_{\rm U} \equiv \frac{E^\star}{E^\star_{\rm SM}} = \frac{1}{\sqrt{\mathcal{C}_{nnnn}}} > 1\,, \qquad \text{with} \qquad \mathcal{C}_{nnnn} = \sum_{j=1}^{N+1} \frac{v^2}{f_j^2} \widetilde{\mathbb{R}}^{a\,4}_{jn}\,. \tag{54}$$

The original analysis [6] found that for a flat-geometry input $f_0 = \cdots = f_N \equiv f$ and $g_0 = \cdots = g_N \equiv \widetilde{g}$, the delay factor is $\mathscr{D}_{\rm U} \simeq (f/v)\sqrt{N+1}$ for large $N+1$, where f scales like[6] $\sqrt{N+1}$ when holding g and $1/R$. This means that the delay factor $\mathscr{D}_{\rm U}$ scales as $N+1$ for large $N+1$.

To analyze essential features of the delay factor $\mathscr{D}_{\rm U}$, we first consider the simplest flat version of Moose-B (Fig. 3) with inputs $f_0 = \cdots = f_N = f_{N+1} \equiv f$ and $g_0 = \cdots = g_N \equiv \widetilde{g}$. The Goldstone decay constants f_j are connected to the EWSB vacuum expectation value (VEV) v via low energy four-fermion interactions,[12]

$$\left[\sum_{j=p+1}^{N+1} \frac{1}{f_j^2}\right]^{-\frac{1}{2}} = v = \left(\sqrt{2}G_F\right)^{-\frac{1}{2}} \simeq 246\,\mathrm{GeV}\,, \tag{55}$$

where site $j = p \in [0, N]$ is the location of left-handed SM fermions. Thus, inputting v makes one of $\{f_j\}$'s non-independent, and for the flat (equal) f_j 's we deduce $f = v\sqrt{N+1-p}$. The symmetry breaking pattern also imposes a relation $g_0^{-2} + \cdots + g_N^{-2} = g^{-2}$ with $g \in SU(2)_W$, so for the flat (equal) g_j 's we have $g_j \equiv \widetilde{g} = g\sqrt{N+1}$. With these we exactly solve all the mass eigenvalues and eigenvectors. We find that $\Pi^a = (\pi_1^a, \cdots, \pi_{N+1}^a)^T$ are connected to $\widetilde{\Pi}^a = (\widetilde{\pi}_0^a, \cdots, \widetilde{\pi}_N^a)^T$ by the orthogonal rotation $\Pi^a = \widetilde{\mathbb{R}}^a \widetilde{\Pi}^a$, where $\alpha_n \equiv \left(n + \frac{1}{2}\right)\pi / \left(N + \frac{3}{2}\right)$ and

$$\widetilde{\mathbb{R}}_{jn}^a = \sqrt{\frac{2}{N+\frac{3}{2}}}\sin\left[(j+1)\alpha_n\right], \qquad (\,j, n = 0, 1, \cdots, N\,)\ . \tag{56}$$

Substituting (56) and $f = v\sqrt{N+1-p}$ into (54), we compute, for a flat Moose-B,

$$\mathscr{D}_{\rm U} = \frac{f}{v}\left[\sum_{j=0}^{N} \widetilde{\mathbb{R}}_{jn}^{a\ 4}\right]^{-\frac{1}{2}} = \left[\frac{2}{3}\left(N + \frac{3}{2}\right)(N+1-p)\right]^{\frac{1}{2}}\,, \tag{57}$$

which scales as $N/\sqrt{1.5}$ for large $N{+}1$ and small $p = 0 - O(1)$. For large $p \sim N$, the factor $\mathscr{D}_{\rm U}$ will be reduced, scaling like $\sim\sqrt{N}$.

From the above, we see that a sizable delay factor $\mathscr{D}_{\rm U} > 1$ is essentially due to the *collective effect* of the many gauge groups participating in the EWSB via **(i)** the composition of the eaten Goldstone state $\widetilde{\pi}_n^a = \sum_j \widetilde{\mathbb{R}}_{jn}^a \pi_j^a$ in terms of all $N{+}1$ site-Goldstones $\{\pi_j^a\}$; and **(ii)** many *comparable* symmetry breaking scales $f_j > v$ under the condition (55). We observe that the realization of this collective effect does not necessarily require $(g_j,\, f_k)$ to obey any exact 5D geometry. In fact, for the general parameters $(g_j,\, f_k)$, we can achieve a visible delay factor $\mathscr{D}_{\rm U} > 1$ as long as $N{+}1$ is reasonably large (implying that many site-Goldstones participate in the collective symmetry breaking) and all relevant $(g_j,\, f_k)$ are significantly larger than $(g,\, v)$ of the SM. With these two requirements satisfied, we find that, even if the precise pattern $(g_j,\, f_k)$ are very non-geometric and random-like, a nearly maximal $\mathscr{D}_{\rm U}$ can be reached for each given p, which roughly scales like

$$\mathscr{D}_{\rm U} \sim \mathscr{D}_{\rm U}^{\max} \longrightarrow O(1)\left[(N+1)(N+1-p)\right]^{\frac{1}{2}}\ . \tag{58}$$

5.2. *Explicit Analysis of Effective Unitarity Without Geometry*

Next, we demonstrate how the effective unitarity can be realized in various minimal moose theories without geometry. For the explicit analysis, we will focus on the zero-mode amplitude $\mathcal{T}[W^a_{0L}W^b_{0L} \to W^c_{0L}W^d_{0L}]$ which is to be tested at the LHC. The unitarity requires its s-wave, $\left|a^0_{0000}[ab,cd]\right| < 1/2$. We know that the naive Higgsless SM ($N+1 = M+1 = 1$) predicts the unitarity violation scale in the $W_{0L}W_{0L}$ scattering as, $E^\star_{\rm SM} \simeq \sqrt{8\pi}v \simeq 1.2\,\mathrm{TeV}$[10,20]. This is significantly lower than the conventional cutoff scale $\Lambda \simeq 4\pi v \simeq 3\mathrm{TeV}$ as estimated by the consistency of chiral perturbation theory. So we will identify and analyze the minimal moose (MM) models that can exhibit an effective unitarity at least up to a scale $E^* \gtrsim 3\,\mathrm{TeV}$.

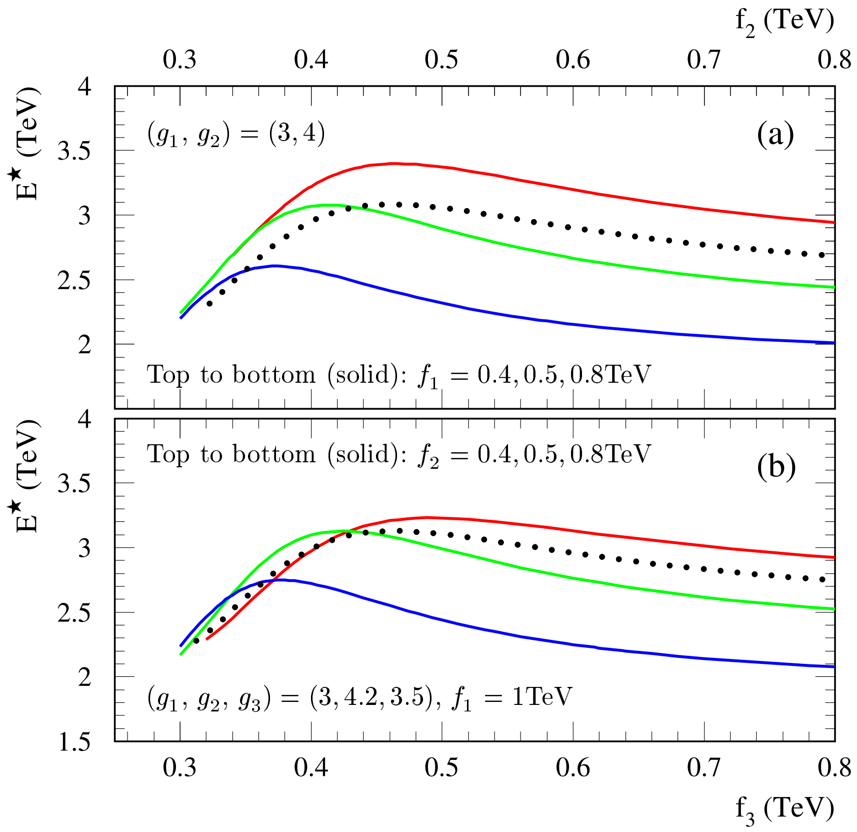


Fig. 7. Unitarity limit $E^\star$ is shown for the moose $SU(2)^3 \otimes U(1)$ (MM3) with $p = 0$ in plot (a) as a function of f_2, and for the moose $SU(2)^4 \otimes U(1)$ (MM4) with $p = 1$ in plot (b) as a function of f_3. The solid curves are shown for three sets of non-geometric inputs which appear random-like. As a comparison, the dotted curves come from two sets of typical geometric inputs: $g_1 = g_2 = 4$, $f_1 = \sqrt{3}v \simeq 0.43\,\mathrm{TeV}$ in plot (a); and $g_1 = g_2 = g_3 = 4$, $f_1 = 1\,\mathrm{TeV}$, $f_2 = \sqrt{3}v$ in plot (b).

We perform the numerical analysis at zeroth order of $U(1)$ coupling g_{N+1}. We ensure $M_W = M_{W0} \simeq 80\,\mathrm{GeV}$ and $M_{W1} \gtrsim 800\,\mathrm{GeV}$. We consider one minimal moose $SU(2)^3 \otimes U(1)$ (called MM3, $N+1 = 3$ and $M = 0$) with left(right)-handed fermions coupled to site $j = p = 0$ ($j = q = 3$) in Fig. 1, and another minimal moose (called MM4, $N+1 = 4$ and $M = 0$) with left(right)-handed fermions coupled to site $j = p = 1$ ($j = q = 4$). We compute unitarity limit $E^\star$ for each model by scanning its parameter space in Fig. 7, where the limits $E^\star$ with "non-geometric" inputs are shown by solid curves and are compared to the "geometric" inputs as depicted by dotted curves. We choose $W^+_{0L}W^-_{0L} \to W^+_{0L}W^-_{0L}$ in Fig. 7 for illustration.

Fig. 7 shows that either solid or dotted curves are generally quite flat and there exists no sharp "peak" when we vary f_2 in plot (a) and f_3 in plot (b). There is a sizable region on each curve to realize a nearly maximal scale $E^\star \sim E^\star_{\max}$. The inputs for dotted curves mimic closely for certain flat-geometry settings, but for realizing $E^\star \sim E^\star_{\max}$ they do not exhibit any real advantage over other non-geometric inputs (solid curves). We find $E^\star$ to be less sensitive to the variation of the pattern of gauge couplings. We have also analyzed other moose models with different inputs of $N{+}1$ and/or p and found similar features, although the limit $E^\star$ for each scattering generally increases for larger $N{+}1$ and smaller p, as expected.

6. Discussion and Conclusion

In conclusion, using powerful deconstruction approach has revealed that 4D gauge theories exhibit far richer dynamics than naive expectation. With this conceptually clean tool, we have formulated and classified the compactified 5D gauge symmetry breaking (without/with gauge group rank reduction) via spontaneous symmetry breaking in general gauge-invariant 4D moose theories. We demonstrate that various consistent 5D BCs as well as possible brane terms are automatically *induced* from taking proper continuum limits of the general gauge-invariant moose theory.

We observe that the *effective unitarity* (delayed unitarity violation) is essentially a *collective effect* due to the participation in the EWSB from many gauge groups whose symmetry breaking scales (f_j) are higher than the SM EWSB scale $v = (\sqrt{2}G_F)^{-1/2}$ and whose gauge couplings g_j are larger than the SM $SU(2)_W$ coupling g. We find that the effective unitarity can be naturally realized in a wide class of 4D moose theories even with a few extra gauge bosons, without resorting to any known 5D geometry. As shown in Sec. 5, an important advantage of the advocated non-geometric moose theories is that they contain much larger parameter space than the special geometric setting, so they open up much wider windows for accommodating experimental constraints while retaining effective unitarity. More elaboration including the effect of delocalized fermions will be given elsewhere[21].

The precision limits on our general moose (Fig. 1) with fermions localized at arbitrary sites $j = p\ (0 \leqslant p \leqslant N)$ and $j = q\ (N{+}1 \leqslant q \leqslant K{+}1)$ have been analyzed[12]. In particular, by requiring a sizable gap between the $M_{W,Z}$ and $M_{W1,Z1}$ ($\lesssim \sqrt{8\pi}\,v$), a sum rule for the precision parameters can be derived,[12]

$$\widehat{S} \,=\, \frac{1}{4s_Z^2}\left[\alpha S + 4c_Z^2\,(\Delta\rho - \alpha T) + \frac{\alpha\delta}{c_Z^2}\right] \geqslant \sum_{r=p+1}^{N} \frac{M_W^2}{M_r^2} \gtrsim \frac{M_W^2}{8\pi v^2} \simeq 4\times 10^{-3}, \qquad (59)$$

where $p \in [0, N)$ and $\{M_r^2\}$ are eigenvalues of mass matrix $\mathbb{M}^2_{(p,N+1)}$. Although the precision data seem not to favor (59), this phenomenological constraint may be evaded in a number of ways, *e.g.,* by smearing the fermion location out of a single site in the general moose theory,[21] similar to the use of delocalized bulk fermions in certain 5D models[22]. The full version of this presentation is available in [23].

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