Weak phases from Amplitude Parametrization

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We determine the weak phases and the amplitudes by proposing an parametrization to each topologies of amplitudes in power of Wolfenstein parameter $\lambda \sim 0.22$. Comparing this parametrization with the experimental data, we can obtain the phase ϕ_3 from the $B \to K\pi$ data up to theoretical uncertainty of $O(\lambda^3) \sim 5\%$. We find that these solutions indicate a large color-suppressed tree amplitude. The extraction of the phase ϕ_2 and ϕ_3 are consistent with the global unitarity triangle fit.

Keywords: B decay; CP violation; Weak Phases.

One of the major missions in B physics is to determine the weak phases in the Kobayashi-Maskawa ansatz for CP violation [1] . In general, one can either explore the problem with the theoretically clean modes, which are usually experimentally difficult, or use the mode with higher feasibility, which, however, require theoretical inputs. Recently, we propose the counting rules for various topologies of amplitudes [2] in two-body nonleptonic B meson decays in terms of powers of the Wolfenstein parameter $\lambda \sim 0.22$ [3]. The power counting rules are supported by the known QCD theories [4,5,6,7], and slightly different from those postulated in [3]. We drop the topologies with higher powers of λ until the number of free parameters are equal to the number of available measurements. The weak phases and the decay amplitudes can then be solved exactly by comparing the resultant parametrization with experimental data. Afterwards, it should be examined whether the solved amplitudes obey the power counting rules. If they do, the extracted weak phases suffer only the theoretical uncertainty from the neglected topologies. If not, the inconsistency could be regarded as a warning to QCD theories for two-body nonleptonic B meson decays.

The most general topological-amplitude parametrization of the $B \to K\pi$ and $B \to \pi\pi$ decay amplitudes have been shown in [2]. The relative power of each amplitude can also be found in [2]. By dropping the $O(\lambda^3)$ and $O(\lambda^4)$ terms, the phase ϕ_3 can be determined from the $B \to K\pi$ data. In principle, the theoretical uncertainty from the ignored amplitudes is around $O(\lambda^3) \sim 5\%$, the same as in the extraction based on the isospin symmetry [8]. The weak phase ϕ_1 is set to 23.5^o from the time-dependent $B \to J/\psi K^{(*)}$ measurement. Assuming unitarity, the phase ϕ_2 can be determined from $\phi_2 = 180^o - \phi_1 - \phi_3$. We then use this ϕ_2 as an input in the $B \to \pi\pi$ modes and solve for the parametrized amplitudes. Finally, we compare the amplitudes from both case and look for a consistent solutions. The uncertainty of determined amplitudes in $B \to \pi\pi$ case will also be $O(\lambda^3) \sim 5\%$.

From recent experimental data[9],we can solve the simutanious equations of $B \to K\pi$ modes. Up to $O(\lambda)$, we have,

$$\frac{|T'|}{P'} = 0.23\ , \quad \delta_{T'} = -13^o\ , \frac{|P'_{ew}|}{P'} = 0.50\ , \quad \delta_{ew'} = -88^o\ . \tag{1}$$

The above result of T'/P' is in agreement with the PQCD prediction, $T'/P' \sim 0.20\exp(-27^o i)$ [4,10,11,12], while the central value of $|P'_{ew}|/P'$ is larger than the PQCD prediction, $|P'_{ew}|/P' \sim 0.1$. The ratio $|P'_{ew}|/P' = 0.5$ and the nearly 90^o phase between P'_{ew} and P' are consistent with the speculation made in [13]. We also derive the allowed ranges $0.06 < |T'|/P' < 0.72$ and $0.22 < |P'_{ew}|/P' < 0.70$, implying that the extracted ratios $|T'|/P'$ and $|P'_{ew}|/P'$ are roughly consistent with the power counting rules in[2]. That is, the $B \to K\pi$ data have not yet provided a strong hint of new physics, contrary to the conclusion in [14,15]. This can be even more clearer in the $O(\lambda^2)$ solutions. To $O(\lambda^2)$, one can get several solutions:

$$\frac{T'}{P'} = 0.26e^{-168^o i}\ , \quad \frac{P'_{ew}}{P'} = 0.17e^{34^o i}\ , \quad \frac{C'}{T'} = 1.01e^{-18^o i}\ , \quad \phi_3 = 61^o\ , \tag{2}$$

$$\frac{T'}{P'} = 0.28e^{-11^o i}\ , \quad \frac{P'_{ew}}{P'} = 0.37e^{98^o i}\ , \quad \frac{C'}{T'} = 0.94e^{171^o i}\ , \quad \phi_3 = 118^o\ , \tag{3}$$

$$\frac{T'}{P'} = 0.62e^{-5^o i}\ , \quad \frac{P'_{ew}}{P'} = 0.49e^{-89^o i}\ , \quad \frac{C'}{T'} = 0.42e^{-149^o i}\ , \quad \phi_3 = 117^o\ . \tag{4}$$

The last one, showing a large P'_{ew}/P', is close to that obtained from the $O(\lambda)$ analysis [2], which is valid for a smaller C'. The first two with larger C'/T' are new, and can not be derived at $O(\lambda)$.

The $O(\lambda)$ solutions for $B \to \pi\pi$ case is rather ambigious [2]. We got four available solutions which indicate that either C is large and constructive in order to enhance the $B^\pm \to \pi^\pm\pi^0$ modes, or P is large and destructive (after including the weak phase ϕ_2) in order to suppress the $B_d^0 \to \pi^\pm\pi^\mp$ modes [2,16]. When calculate the $O(\lambda^2)$ solutions, we need to use the the ϕ_3 values from $B \to K\pi$ modes, and look for solutions satisfying the approximate SU(3) symmetry. Substituting $\phi_2 = 180^o - \phi_1 - \phi_3 \sim 40^o$ from Eqs. (3) and (4), we find no solution, indicating that Eqs. (3) and (4) can not be the consistent solutions for both the $\pi\pi$ and $K\pi$ modes. In fact, ϕ_2 must be greater than 60^o in order for a solution to exist. That is, the current $B \to \pi\pi$ data have imposed a constraint on the allowed range of ϕ_2. The phase $\phi_2 \sim 90^o$ corresponding to Eq. (2) gives three solutions,

$$\frac{P}{T} = 0.38e^{150^o i}\ , \quad \frac{C}{T} = 0.81e^{-58^o i}\ , \quad \frac{P_{ew}}{P} = 0.22e^{10^o i}\ , \tag{5}$$

$$\frac{P}{T} = 0.38e^{150^o i}\ , \quad \frac{C}{T} = 0.72e^{-47^o i}\ , \quad \frac{P_{ew}}{P} = 0.25e^{-174^o i}\ , \tag{6}$$

$$\frac{P}{T} = 0.38e^{150^o i}\ , \quad \frac{C}{T} = 0.48e^{61^o i}\ , \quad \frac{P_{ew}}{P} = 2.5e^{49^o i}\ . \tag{7}$$

It is easy to observe that only Eq. (5) obeys the approximate SU(3) relations to Eq. (2). For Eq. (6), the direction of P_{ew}/P is almost opposite to that of P'_{ew}/P' in Eq. (2). For Eq. (7), the magnitude of P_{ew}/P is too much larger than that of

P'_{ew}/P', and C/T also dramatically differs from C'/T'. We emphasized that this consistent solutions does not necessarily exits. A consistency like the one between Eqs. (2) and (5) means that the $B \to \pi\pi$, $K\pi$ data are really agree with each other! Note that Eqs. (2) and (5) correspond to the central values of the data. If considering the allowed range, the two solutions can be even closer.

We have performed an $O(\lambda^2)$ analysis based on the topological-amplitude parametrizations for the $B \to \pi\pi$, $K\pi$ decays. Combining the recent $\pi\pi$, $K\pi$ data, such an investigation is allowed. We do not rely on the SU(3) flavor symmetry, but only require the extracted amplitude ratios from the $\pi\pi$, $K\pi$ modes to satisfy the approximate SU(3) relations. We have found that an exact solution from the $B \to \pi\pi$ data and an exact solution from the $B \to K\pi$ data, obeying this weaker and more reasonable requirement, indeed exist. These solutions show a large color-suppressed tree amplitude constructive to the tree amplitude, and a small electroweak penguin amplitude. The corresponding weak phases $\phi_2 \sim 90^o$ and $\phi_3 \sim 60^o$ should be convincing due to the consistency between the $\pi\pi$, $K\pi$ data. Compared to the predictions from the PQCD approach, the extracted P/T, T'/P', and $P^{(')}_{ew}/P^{(')}$ are all understandable. Only $C^{(')}/T^{(')}$, larger than the PQCD predictions, demands more study. This discrepancy may be attributed to the different definitions of $C^{(')}$ in this work and in the PQCD approach as stated before.

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