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Comments on Supersymmetry Algebra and Contact Term in Matrix String Theory

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We write down two supercharges in the matrix string theory at the leading order, following previous works relating the matrix string theory to the light-cone superstring field theory. We check the supersymmetry algebra at this order and also discuss the problems of higher order contact terms.

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1. Introduction

String field theory is a standard off-shell formalism to describe fundamental string interactions in the perturbative way. Recently its importance is recognized in various contexts, such as tachyon condensation and the PP-wave/SYM correspondence. However, in the case of the light-cone type, our understanding of the higher order contact terms is still very restricted. Although there is a canonical method to discuss the contact terms using the supersymmetry algebra, various questions related to its general structure remain unanswered. The main difficulty consists in its notorious complexity of the algebra.

Last summer, Dijkgraaf and Motl[1] revisited the matrix string theory, another proposal to describe the fundamental string interactions. They related very extensively the matrix string theory to the light-cone superstring field theory in the flat space and also came up with an interesting proposal on the contact terms. Since on the string field theory side we have a systematic method to discuss the contact terms using the supersymmetry algebra, it is very tantalizing to apply this method to the matrix string theory. We hope we can avoid the complexity of the algebra on the string field theory side in this way and present a systematic calculation to prove or disprove the contact term proposal of Ref. 1. We will report on some progress we made in this direction[2].

2. Light-Cone String Field Theory

In this section we shall explain the systematic method to discuss the contact terms in the light-cone superstring field theory. The light-cone superstring field theory is constructed by generalizing the light-cone quantization of string theory in Green-Schwarz formalism. Hence, the start point is the covariant string worldsheet action with the spacetime supersymmetries. Since we cannot quantize the action covariantly, we have to choose the light-cone gauge. After the gauge-fixing, we have the light-cone Hamiltonian H_0 as well as two supercharges $Q_0^{\dot\alpha}$ and $\tilde{Q}_0^{\dot\alpha}$. These charges satisfy the supersymmetry algebra

$$\{Q_0^{\dot\alpha}, Q_0^{\dot\beta}\} = \{\tilde{Q}_0^{\dot\alpha}, \tilde{Q}_0^{\dot\beta}\} = 2H\delta^{\dot\alpha\dot\beta},\ [Q_0^{\dot\alpha}, H_0] = [\tilde{Q}_0^{\dot\alpha}, H_0] = 0, \tag{1}$$

with two supercharges being independent:

$$\{Q_0^{\dot\alpha}, \tilde{Q}_0^{\dot\beta}\} = 0. \tag{2}$$

So far, we only have the free part. To construct the light-cone superstring field theory, we have to include the interaction part

$$H = H_0 + g_s H_1 + \cdots,\ Q^{\dot\alpha} = Q_0^{\dot\alpha} + g_s Q_1^{\dot\alpha} + \cdots,\ \tilde{Q}^{\dot\alpha} = \tilde{Q}_0^{\dot\alpha} + g_s \tilde{Q}_1^{\dot\alpha} + \cdots, \tag{3}$$

respecting the full supersymmetry algebra: We require that (1) and (2) still hold with the indices 0 dropped.

At the leading non-trivial order, H_1, $Q_1^{\dot\alpha}$ and $\tilde{Q}_1^{\dot\alpha}$ were solved in Ref. 3. Since these interaction vertices relate the two-string Hilbert space to the one-string Hilbert space, it is best to represent them as ket states:

$$|H_1\rangle = \bar{Z}^i Z^j v^{ij}(\Lambda)|V\rangle,\ |Q_1^{\dot\alpha}\rangle = \bar{Z}^i s^{i\dot\alpha}(\Lambda)|V\rangle,\ |\tilde{Q}_1^{\dot\alpha}\rangle = Z^i \tilde{s}^{i\dot\alpha}(\Lambda)|V\rangle. \tag{4}$$

Here $|V\rangle$ represents the geometrical overlapping of two strings and one string. The quantities Z^i, $\bar{Z}^i$ and Λ^a are, roughly speaking, the renormalized operators of holomorphic part of the bosonic momentum $P^i - X^{i\prime}$, anti-holomorphic part $P^i + X^{i\prime}$ and the fermionic momentum λ^a evaluated at the interaction point respectively and v^{ij}, $s^{i\dot\alpha}$ and $\tilde{s}^{i\dot\alpha}$ are some complicate polynomials of Λ^a.

Apparently, there are many questions related to this construction. Does this expansion end at this order? If not, how do the terms of the next order H_2, $Q_2^{\dot\alpha}$ and $\tilde{Q}_2^{\dot\alpha}$ look like? Can we compute all the higher order terms? However, the present formalism is complicated and we do not have a satisfactory answer to these questions. We will see how these questions can be discussed in the matrix string theory.

3. Matrix String Theory

Let us turn to the matrix string theory. The matrix string theory proposal was motivated by compactifying M(atrix) theory[4] on one dimension. Accordingly, the definition of the matrix string theory is the low energy effective action of the two-dimensional super Yang-Mills theory. When the theory flows to the infrared (IR) point, it will look like a free theory of string bits propagating in the moduli space with the Hamiltonian and the supercharges given similar to those of the light-cone superstring field theory.

To incorporate the interaction terms, we have to return slightly from the complete IR limit and consider the least irrelevant operators. The leading non-trivial order of the Hamiltonian in matrix string theory is conjectured to be[5]

$$H_1 = \sum_{m<n} \int_0^{2\pi} \frac{d\sigma}{2\pi} ([\tau^i \bar{\tau}^j] \Sigma^i \bar{\Sigma}^j)_{mn}. \tag{5}$$

Here all the fields are twist fields or spin fields exchanging m-th and n-th string bits. If we take $[\sigma\bar{\sigma}]$ to be the twist field in the bosonic sector, which corresponds to the anti-periodic ground state $|0\rangle_{\rm AP}$, $[\tau^i\bar{\tau}^j]$ is the excited twist field, corresponding to $\alpha^i_{-1/2}\bar{\alpha}^j_{-1/2}|0\rangle_{\rm AP}$. The spin fields in the fermionic sector can be classified by $\mathbf{8_v}$ and $\mathbf{8_c}$, therefore we denote them as Σ^i and $\Sigma^{\dot{\alpha}}$. One evidence for this interaction term is the dimension counting. The conformal dimension of the least irrelevant operator (5) is (3/2,3/2). This fact corresponds beautifully to the fact that the first non-trivial interaction should be proportional to the string coupling g_s.

In Ref. 1 the relation between the matrix string theory and the light-cone superstring field theory was made explicit. They found the quantities v^{ij}, $s^{i\dot{\alpha}}$ and $\tilde{s}^{i\dot{\alpha}}$ appearing in the first non-trivial order of the light-cone superstring field theory (4) are related respectively to

$$v^{ij} \Leftrightarrow \Sigma^i \bar{\Sigma}^j,\ s^{i\dot{\alpha}} \Leftrightarrow \Sigma^{\dot{\alpha}} \bar{\Sigma}^i,\ \tilde{s}^{i\dot{\alpha}} \Leftrightarrow \Sigma^i \bar{\Sigma}^{\dot{\alpha}}, \tag{6}$$

in the matrix string theory. Comparing (4) with (6), it is very plausible to write down the supercharges at the first non-trivial order as

$$Q^{\dot{\alpha}}_1 = -\sqrt{2} \sum_{m<n} \int_0^{2\pi} \frac{d\sigma}{2\pi} ([\sigma\bar{\tau}^i] \Sigma^{\dot{\alpha}} \bar{\Sigma}^i)_{mn},\ \tilde{Q}^{\dot{\alpha}}_1 = -\sqrt{2} \sum_{m<n} \int_0^{2\pi} \frac{d\sigma}{2\pi} ([\tau^i\bar{\sigma}] \Sigma^i \bar{\Sigma}^{\dot{\alpha}})_{mn}, \tag{7}$$

where $[\tau^i\bar{\sigma}]$ and $[\sigma\bar{\tau}^i]$ correspond respectively to $\alpha^i_{-1/2}|0\rangle_{\rm AP}$ and $\bar{\alpha}^i_{-1/2}|0\rangle_{\rm AP}$.

After writing down these two supersymmetry charges, we can proceed to the supersymmetry algebra. In Ref. 2 we have evaluated several necessary operator product expansions and checked the supersymmetry algebra at the leading order. For higher order contact terms the situation is much more subtle. There are three types of terms in $\{Q^{\dot{\alpha}}_1, Q^{\dot{\beta}}_1\}$ depending on relative interaction points. While one of them looks similar to the one conjectured in Ref. 1 and one of them gives no contribution, there also seems to be a divergent contribution which requires appropriate regularization.

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