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METHOD OF IMPROVING THE TEACHING OF PARTICLE PHYSICS IN A NONCALCULUS COURSE OF PHYSICS

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The Klein Gordon equation–which describes mesons–can be reformulated to suit students who have no calculus background. The method is arrived at from a review and a reinterpretation of the mechanics of small oscillations. It may serve as a model for the design of new instructions for other areas of particle physics.

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1. Introduction

It is obvious that to popularize particle physics we must teach the subject in the early stage of the students' physics education. However to do so satisfactorily would require the use of quantum mechanics.(QM) This seems impossible since the usual formulation of QM requires at a minimun mathematical knowledge of parrtial differential equations. Incidentally this problem is not confined to particle physics but is common to all modern physics.

The author believes a solution can be found. It is inspired by this fact: In the text books of high school and beginning college physics*s*[1,2], we find teachings of the physics of the vibration of strings. Its mathematics, we note, is a one dimentional wave equation which *is* a partial differential equation. Thus a relatively abstract mathematics is avoided and replaced by the teaching of a concrete physical object. It suggests that a similar approach is possible for wave mechanics. This turns out to be true as will be explained below.

2. Mechanics of Beaded String Reviewed

Let us consider a vibrating string which is fixed at its end points. This system can be replaced by N particles connected by a weghtless string.–a "beaded string" system. N will finally be allowed to approach infinity, The solution of the eigen value problem are given in many text*s*[3] We shall refer to Ref. 3 and use the same notations for the various quantities. The picture is that of Fig. 8-25, p,215, Ref.3.

Let T be the tension of the string, a the separation between two neighboring particles, Y_ν the transverse displacement of the νth particle, being 0 for ν equal to 0 or$N+1$. Assume:

$$Y_\nu = A_\nu \cos wt. \tag{1}$$

By the second law of Newton, the total force acting on the νth particle is:

$$m(d^2Y_\nu/dx^2) = (T/a)(Y_{\nu-1} - 2Y_\nu + Y_{\nu+1}) \tag{2}$$

(The first equation (1), p.215, Ref.3)

The meaning of (1) implies that all the particles move as if each is acted on by a linear restoring force with the same spring constant k equal to $w^2 m$, in lieu of the force due to the string. Let the force on the right side of (2) be denoted by F_ν. At a time $t = t_ø$ when $\cos wt$ equals its maximun value 1, $Y_\nu = A_\nu$ and $F_\nu = k\ A_\nu$. This leads to :

$$F_\nu/A_\nu = \text{constant} \tag{3}$$

independent of ν.

We can adopt (3) as an alternative (but equivalent) definition of the eigen state. The forces in (3) can be represented and studied by graphical method which is familiar to the students.

We can give a simple example. Consider the case of $N = 3$. For the lowest eigen state:

$A_\nu = C \sin[\nu\pi/(N+1)]$

(From p.217, Ref.3, setting α =1. C is a constant.) This yields:

$A_1 = A_3 = C/\sqrt{2}$, $A_2 = C$

We also have:

$A_0 = A_4 = 0$

Together, they yield:

$F_1/A_1 = F_2/A_2 = F_3/A_3 = 2 - \sqrt{2}$

verifying (3)

3. Extension to Klein Gordon Equation

The one dimensional Klein Gordon equation is known as "string-in-membrane" (term used by Morse and Feshback), with properties similar to the "string". We can treat the problem by discretizing the it, leading to a "beaded-string and rubber-strips"system: namely having each bead acted on by a rubber strip that exert a linear restoring force F_ν in the transverse direction. (in addition to the string force) The solution to the eigen value problem is similar leading to an analogue of (3)

$$F_\nu/A_\nu + k_\nu = \text{constant} \tag{4}$$

where k_ν is the spring constant of the νth rubber strip.(Acting on the νth particle.) Just as in (3), the constant k is interpretable as the spring constant of a set of equivalent simple harmonic motions.

A simple example is: Set N=2, $A_1 = 1/2 + \sqrt{3}/2$ and A_2=1. Then $F_1/A_1 = 2F_2/A_2$ and setting k_1=0, $k_2 = F_2/A_2$ will satisfy (4). Such a state is physically quite simple and can be readily explained to the students with the help of some diagrams.

It can be shown that the important features of the one dimensional wave function can be explained in terms of physical concepts and elementary mathematics known to the students. For an example, to obtain the nonrelativistic portion of the energy, it is only a matter of expanding the square of $w + \Delta w$ assuming $w \gg \Delta w$, and using $k = w^2 m$. One obtain Δw in terms of k and Δk, the latter being expressed in terms of c,$\hbar$... Thus only elementary algebra is used here.

4. Comments on the Nature of the Method

Having obtained the above results, the author made the following observation: The Klein Gordon equation on which the method is based is a real wave equation. It is therefore related to the real wave equation that Schroedinger originally derived[4] and on which the author had done some research[5,6]. Schroedinger noted that his real wave equation, Eqn.4 of Ref.4, is valid in the case of a conservative system. This is consistent with the present result since, to be able to obtain the eigen functions, as we did above, requires that the system is conservative. In the mean time, it turns out that some issues encountered in Ref.5,6 can be clarified considerably. The author hopes to be able to address these questions in the future.

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