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Geometric Transitions, Non-Kahler Geometries and String Vacua

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We summarize an explicit construction of a duality cycle for geometric transitions in type II and heterotic theories. We emphasize that the manifolds with torsion constructed with this duality cycle are crucial for understanding different phenomena appearing in effective field theories.

Keywords: Geometric Transition; Non-Kahler Manifolds; Superpotentials.

1. Introduction

The connections between string theory and realistic supersymmetric gauge theories have been extensively studied in the last years. One of the approaches is the one taken by Vafa [1] that is based on the duality between open topological strings on a Lagrangian submanifold and closed topological strings on a resolved conifold. This has been extended to the type IIB theory in [2,3] where the open string side is described by D5 branes wrapping a two cycle of a resolved conifold and the closed string side is a warped deformed conifold with fluxes. The open string side captures the far IR behavior of the gauge theory. The full picture which studies the UV as well as IR behavior was described in reference [4] where the cascading from the UV to the IR is shown to arrive from an infinite sequence of flop transitions.

The first goal of this note is to describe the type IIA transition in detail, by considering D6 branes wrapped on three cycles inside a non-Kähler deformation of the deformed conifold [5]. This corresponds to the open string picture. The closed string dual is a compactification with RR fluxes on another non-Kähler manifold with $dJ \neq 0$ and $d\Omega \neq 0$ and with a superpotential

$$W_{IIA} = \int (J + iB) \wedge d\Omega. \tag{1}$$

The second goal is to extend the cycle of geometric transitions to type I and heterotic strings. The dual corresponds to either a type I string compactified on a non-Kähler but complex manifold (observe that the type IIA manifolds were non-complex) with the superpotential [6,7]

$$W_I = \int (H_{RR} + idJ) \wedge \Omega, \tag{2}$$

or to the heterotic string compactified on a non-Kähler but again complex manifold with the superpotential [6,7]

$$W_{het} = \int (H + idJ) \wedge \Omega. \tag{3}$$

Here H is the usual three form of the heterotic theory that satisfies $dH = \text{tr } R \wedge R - \frac{1}{30}\text{tr } F \wedge F$ and H_{RR} is the RR three form that is S-dual to H.

One important result of our work is that we identify a geometric transition for both the type I and heterotic string theories. We will be able to specify the backgrounds in the type I and heterotic theories on both sides of the duality. This suggests the possibility of having a gravity description for wrapped D5 branes (for type I) or wrapped NS5 branes (for heterotic). Our work also provides an alternative picture to the Landscape distribution of string vacua [8,9].

2. Type IIA Superstrings and Non-Calabi-Yau Manifolds

Geometric transitions are examples of generalized AdS/CFT correspondences which relate D-branes in the open string picture and fluxes in the closed string picture. There are several types of geometric transitions depending on the framework in which we formulate them. The type IIB geometric transition starts with $D5$ branes wrapping a P^1 of a resolved conifold. The corresponding metric is given in [10] as

$$ds^2 = (dz + \Delta_1 \cot\theta_1 \ dx + \Delta_2 \cot\theta_2 \ dy)^2 + |dz_1|^2 + |dz_2|^2, \tag{4}$$

where we have replaced the metric of two spheres of the resolved conifold by two tori with complex structures τ_1 and τ_2. The complex one forms $dz_i, \ i = 1, 2$ are therefore defined as

$$dz_1 = dx - \tau_1 \ d\theta_1, \qquad dz_2 = dy - \tau_2 \ d\theta_2. \tag{5}$$

We now want to apply the result of [11] which tells us that for a manifold admitting a T^3 structure, and in the limit of a large complex structure the mirror manifold is obtained by performing three T-dualities on the T^3 torus [a]. In our case, the large complex structure is obtained by boosting [b]

$$dz_i \to dz_i + f_i \, d\theta_i, \quad f_i \to \infty, \quad i = 1, 2. \tag{6}$$

In the presence of a NS field with components $b_{x\theta_1}$ and $b_{y\theta_2}$, we have explicitly performed the mirror tranformation in [5]. The outcome after the mirror transformation is the following metric [c]:

$$\begin{aligned} ds^2_{IIA} = \;& g_1 \, [dz + \Delta_1 \cot \hat{\theta}_1 \, (dx - b_{x\theta_1} \, d\theta_1) + \Delta_2 \cot \hat{\theta}_2 \, (dy - b_{y\theta_2} \, d\theta_2)]^2 \\ &+ g_2 \, [d\theta_1^2 + (dx - b_{x\theta_1} d\theta_1)^2] + g_3 \, [d\theta_2^2 + (dy - b_{y\theta_2} d\theta_2)^2] + g_4 \sin \psi \, [(dx - b_{x\theta_1} d\theta_1) d\theta_2 \\ &+ (dy - b_{y\theta_2} d\theta_2) d\theta_1] + \; g_4 \cos \psi \, [d\theta_1 d\theta_2 - (dx - b_{x\theta_1} d\theta_1)(dy - b_{y\theta_2} d\theta_2)]. \end{aligned} \tag{7}$$

This metric (7) is exactly a non-Kähler deformation of the metric for D6 branes on the three cycle of a deformed conifold. The non-Kähler deformation can be seen from the presence of $b_{x\theta_1}$ in $d\hat{x} = dx - b_{x\theta_1}$ and of $b_{y\theta_2}$ in $d\hat{y} = dy - b_{y\theta_2}$. This implies that the Kähler form J and the 3-form Ω are not-closed. Even though the manifold does not have an SU(3) holonomy it has an SU(3) structure, so that supersymmetry is preserved.

Furthermore one can easily show that any B_{NS} field appearing on the type IIA side after mirror is a gauge artifact. This is most transparent if one chooses integrable complex structures for the two tori in type IIB theory from the very beginning. There are also non-trivial one form fluxes from the D6 branes sources. The three form field vanishes and the coupling constant is equal to the type IIB coupling constant.

This is the starting point of the type IIA transition. In order to go to the closed string side, we need to first lift the geometry to M theory to perform a flop and then dimensionally reduce again to the type IIA theory [15]. The fact that the type IIA theory is compactified on an SU(3)-structure manifold implies that M theory is compactified on a G_2 structure manifold. The absence of a G_2 holonomy is due to the non-closure of $\Phi = J \wedge e^7 + \Omega_+$ and its Hodge dual. It was shown in [5] that the identification of the one forms and the performance of a flop can be done using the methods of [15] [d]. After doing so and descending to the type IIA theory, the result

[a]This is a different type of T-duality than the one considered in [12,13,14] where the result of a single T-duality was a brane configuration in type IIA.
[b]This can also be viewed as if we had introduced new complex structures on the two tori. There is a subtlety related to whether these complex structures are integrable or not. Considering only the integrable complex structures leads us very close to the right mirror metric, which can nevertheless be obtained by choosing non-integrable complex structures. This discussion has appeared in [5].
[c]There is a subtlety that we should mention here. The mirror rules of [11] tell us that we should take a limit of large complex structures. On the other hand geometric transitions occur in exactly the opposite limit. Therefore naively applying [11] we do not get the right answer. The correct answer was derived in [5] by performing a set of coordinate transformations.
[d]As expected, the one forms that we would now require will be different from the ones chosen by [15]. Indeed this is what we get. The one forms that we use to specify the M-theory manifolds reduce to the one forms of [15] when we turn off the non-Kählerity of the type IIA theory.

we get is

$$ds^2 = h_1\ [d\theta_1^2 + (dx - b_{x\theta_1}\ d\theta_1)^2] + h_2\ [d\theta_2^2 + (dy - b_{y\theta_2}\ d\theta_2)^2] \\ + h_3\ [dz + \Delta_1\ \cot\hat{\theta}_1\ (dx - b_{x\theta_1}\ d\theta_1) + \Delta_2\ \cot\hat{\theta}_2\ (dy - b_{y\theta_2}\ d\theta_2)]^2, \tag{8}$$

which is precisely the metric of a resolved conifold when we switch off $b_{x\theta_1}$ and $b_{y\theta_2}$. This is thus the closed string background with no D6 branes but only sources. The manifold is non-Kähler as well as non-complex. The identification between the open string side and the closed string side was made by mapping the expectation value of the gluino condensate on the stack of D6 branes and the volume of the resolution two cycle on the resolved conifold side. This map requires the term $J \wedge B_{(4)}$ on the flux side [1]. Our proposal is that the presence of $B_{(4)}$ is due to the fact that $d\Omega \neq 0$ and that the type IIA superpotential contains a term $J \wedge d\Omega$ [e].

Making a further mirror transformation to this background we obtain the closed string side of the type IIB geometric transition [17]. The type IIB manifold turns out to be a Kähler deformed conifold with RR and NS three forms. This is exactly what was expected from the results of [1].

To summarize, non-Kähler manifolds play a crucial in the gauge theory/ string theory duality. They provide important contributions to the superpotentials which are crucial for a correct description of the corresponding effective field theories.

3. Type I/Heterotic Strings and Non-Calabi-Yau Manifolds

Even though non-Kähler manifolds were never studied in the traditional string theory literature in much detail, their importance has become evident in recent times due to the large amount of new results in the area of string compactifications with fluxes. We will now extend the calculation done in the previous section to other type of models.

To do so, we start again from the type IIB compactification with NS and RR fluxes and go to its orientifold limit which will contain D7 branes and O7 planes. In order to obtain a metric, we consider the metric of (4) and analyze which terms are invariant under the O7 action. We consider the directions x and y to be transverse to the O7 planes. The metric should then be invariant under the orientifold action, should preserve some number of supersymetries (i.e $\mathcal{N} = 1$), should have a form close to the original type IIB metric and should allow wrapped D5 branes along with some number of D7 branes and O7 planes. Finally, after two T-dualities, it's form should closely resemble the metric obtained after T-dualizing the resolved conifold.

A metric which satisfies these conditions was computed in [17]

$$ds^2 = a_1(dx^2 + |\tilde{\tau}_1|^2\ dy^2 + 2\mathrm{Re}\ \tilde{\tau}_1\ dx\ dy) + a_2(d\theta_1^2 + |\tilde{\tau}_2|^2\ d\theta_2^2) + a_3\ dz^2 + a_4\ dr^2. \tag{9}$$

After two T-dualities, this metric becomes a type I metric which takes the form

$$ds^2 = \alpha(1 + A^2)\ (dy - b_{y\theta_2}\ d\theta_2)^2 + \alpha(1 + B_1^2)\ (dx + b_{x\theta_1}\ d\theta_1)^2 + \gamma'\sqrt{H} dr^2 \tag{10}$$

[e]For half-flat manifolds this has also been proposed in [16].

$$+2\ \alpha AB\ (dx + b_{x\theta_1}\ d\theta_1)(dy - b_{y\theta_2}\ d\theta_2) + \alpha(1 + A^2)dz^2 + a_2|d\chi|^2.$$

The T-dualities were performed along the x and y directions. As y is the angular direction of the P^1 cycle on which the D5 branes are wrapped on, the D5 branes loose the y direction and gain the direction x, thus the final configuration is again with D5 branes wrapped on a P^1 cycle with the angular direction now being the x direction. Therefore, the starting point of the geometric transition is given by D5 branes wrapping on a two cycle inside a non-Kähler manifold.

After the transition we are again in the orientifold limit of some type IIB configuration. If we impose the same conditions as we did before the metric will take the form:

$$ds^2 = b_1\ |d\chi_1|^2 + b_2\ |d\chi_2|^2 + b_3\ dz^2 + b_4\ dr^2, \tag{11}$$

but now the complex structures will be different. We have Re $\tilde{\tau}_1 = 0$ and Re $\tilde{\tau}_2 \neq 0$ for this solution while earlier the complex structures satisfied Re $\tilde{\tau}_1 \neq 0$ and Re $\tilde{\tau}_2 = 0$. The final type I manifold can be shown to be a complex non-Kähler manifold [17].

One important aspect of these type I compactifications is that the metrics are all non-Kähler but complex. The integrability of the complex structures is related to the torsional constraints the metrics are required to satisfy, as well as the DUY equations for the vector bundles.

We can go one step further by performing an S-duality to go from the type I theory to the heterotic string. This means trading the RR flux of the type I theory with the NS flux of heterotic. A geometric transition will then take place between NS branes wrapped on some two cycles of a non-Kähler complex manifold and NS flux on another non-Kähler complex manifold.

One important check of our approach is the fact that there are two conditions which should be mapped into one another. These two conditions are the self-duality of the type IIB fluxes in the orientifold limit

$$H_{RR} = *H_{NS}, \tag{12}$$

and the torsional equation for the heterotic string compactifications

$$H_{NS} = *dJ \equiv i\ (\partial - \bar{\partial})J. \tag{13}$$

In [17] it was shown that this mapping holds up to some conjectured identification between constants which enter in the flux definitions. This gives a strong check on the consistencies of these results.

4. Conclusions

In this short note we summarized the complete duality cycle of the geometric transitions taking place in all string theories and M-theory. For the type II and M-theory transitions, these transitions are well accounted for with various direct and indirect checks. On the other hand the type I and heterotic cases are relatively new. Our conjecture here is that the world volume dynamics of wrapped branes in the type

I and heterotic theories on non-Kähler complex manifolds will have a description purely in terms of another non-Kähler complex manifold with branes replaced by fluxes. The manifolds predicted in the type I and heterotic theories are examples of new non-Kähler complex manifolds that complement the existing examples in the literature [6,18,19].

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References

1. C. Vafa, *Journal of Math. Phys.*, **42**, 2798 (2001).
2. F. Cachazo, K. A. Intriligator and C. Vafa, *Nucl. Phys.B* **603**, 3 (2001).
3. J. D. Edelstein, K. Oh and R. Tatar, *JHEP* **0105**, 009 (2001).
4. F. Cachazo, B. Fiol, K. A. Intriligator, S. Katz and C. Vafa, Nucl. Phys. B **628**, 3 (2002).
5. M. Becker, K. Dasgupta, A. Knauf and R. Tatar, hep-th/0403288, to appear in *Nucl. Phys. B.*
6. K. Becker, M. Becker, K. Dasgupta and P. S. Green, JHEP **0304**, 007 (2003); K. Becker, M. Becker, K. Dasgupta and S. Prokushkin, Nucl. Phys. B **666**, 144 (2003).
7. G. L. Cardoso, G. Curio, G. Dall'Agata and D. Lust, JHEP **0310**, 004 (2003); G. L. Cardoso, G. Curio, G. Dall'Agata and D. Lust, Fortsch. Phys. **52**, 483 (2004).
8. M. Becker and K. Becker, *Nucl. Phys. B* **477**, 155 (1996), K. Dasgupta, G. Rajesh and S. Sethi, JHEP **9908**, 023 (1999); S. B. Giddings, S. Kachru and J. Polchinski, *Phys. Rev. D* **66**, 106006 (2002); S. Kachru, R. Kallosh, A. Linde and S. P. Trivedi, *Phys. Rev. D* **68**, 046005 (2003).
9. M. R. Douglas, hep-th/0409207; F. Denef and M. R. Douglas, *JHEP* **0405**, 072 (2004).
10. L. A. Pando Zayas and A. A. Tseytlin, *JHEP* **0011**, 028 (2000).
11. A. Strominger, S. T. Yau and E. Zaslow, *Nucl. Phys. B* **479**, 243 (1996).
12. K. Dasgupta, K. Oh and R. Tatar, *Nucl. Phys. B* **610**, 331 (2001); *JHEP* **0208**, 026 (2002); *JHEP* **0201**, 031 2002; *Advances in Theor. Math. Phys.*, **6**, 141 (2003).
13. I. Bena, R. Roiban and R. Tatar, *Nucl. Phys. B* **679**, 168 (2004), R. Roiban, R. Tatar and J. Walcher, *Nucl. Phys.B* **665**, 211 (2003).
14. K. Landsteiner, C. I. Lazaroiu and R. Tatar, *Fortsch. Phys.* **52**, 590 (2004); *JHEP* **0311**, 057 (2003); *JHEP* **0402**, 044 (2004); *JHEP* **0311**, 044 (2003).
15. M. Atiyah, J. M. Maldacena and C. Vafa, *J. Math. Phys.* **42**, 3209 (2001); A. Brandhuber, J. Gomis, S. S. Gubser and S. Gukov, *Nucl. Phys. B* **611**, 179 (2001).
16. S. Gurrieri, J. Louis, A. Micu and D. Waldram, *Nucl. Phys. B* **654**, 61 (2003).
17. S. Alexander, K. Becker, M. Becker, K. Dasgupta, A. Knauf and R. Tatar, hep-th/0408192, to appear in *Nucl. Phys. B.*
18. K. Becker, M. Becker, P. S. Green, K. Dasgupta and E. Sharpe, Nucl. Phys. B **678**, 19 (2004); K. Becker and K. Dasgupta, JHEP **0211**, 006 (2002).
19. M. Serone and M. Trapletti, JHEP **0401**, 012 (2004); S. Gurrieri, A. Lukas and A. Micu, hep-th/0408121.