International Journal of Modern Physics A

LARGE EXTRA DIMENSIONS AND THE MINIMAL SCALE CONSTRAINTS THROUGH HIGH PRECISION EXPERIMENTS

U. HARBACH [1,2,*], S. HOSSENFELDER [3], B. KOCH [1,2], M. BLEICHER [1] & H. STÖCKER [1]

[1] *Institut für Theoretische Physik, Johann Wolfgang Goethe-Universität, Robert-Mayer-Str. 8-10 60054 Frankfurt, Germany*

[2] *Frankfurt Institute for Advanced Studies, Johann Wolfgang Goethe-Universität, Robert-Mayer-Str. 8-10 60054 Frankfurt, Germany*

[3] *Department of Physics, University of Arizona, 1118 East 4th Street Tucson, AZ 85721, USA*

Received (Day Month Year)
Revised (Day Month Year)

We give a brief introduction to the recent idea of Large eXtra Dimensions (LXDs). We give a motivation for the existence of a minimal length scale and show how to incorporate it into quantum theory. We derive first order corrections to high precision measurements such as transition frequencies in He^+ and the muon gyromagnetic moment. The results clearly show that effects due to a minimal length have to be considered in models with LXDs.

Keywords: Large eXtra Dimensions; minimal length.

In String Theory, gravitation is interpreted as an interaction via closed strings, whereas Standard Model particles are represented by open strings. Since extra spatial dimensions are required in String Theory but not detected in collider experiments, it is assumed that the Standard model particles are bound to our $3 + 1$-dimensional submanifold via Dirichlet boundary conditions and thus cannot propagate into the extra dimensions. The closed strings are able to penetrate all dimensions. This implies that the extra dimensions have to be compactified, since - at least for large distances - we observe the usual gravitational potential that follows from Gauss' law in 3 spatial dimensions. Since we expect a smooth behavior of the potential at the compactification radius, the boundary condition yields the relation between the observed Planck mass and the fundamental mass scale M_f [3,2]:

$$m_{Pl}^2 = R^d M_f^{d+2}$$

Here, m_{Pl} represents the Planck mass and R is the compactification radius. This

*email: harbach@th.physik.uni-frankfurt.de.

relation clearly shows that the hugeness of the Planck scale (which equals the weakness of gravity via $G = 1/m_{Pl}^2$) is merely a consequence of the number and size of the extra dimensions. The fundamental mass scale can be significantly lower.

String theory also suggests the existence of a minimal length scale. In perturbative string theory [7,1], the feature of a fundamental minimal length scale arises from the fact that strings cannot probe distances smaller than the string scale. If the energy of a string reaches the fundamental energy scale, excitations of the string can occur and increase the extension [10]. Due to this, there is a fundamental uncertainty in position measurement. In the present model with LXDs, this fact grows important for collider physics at high energies or for high precision measurements at low energies due to the lowered fundamental scale M_f, which results in a new fundamental length scale $L_f = 1/M_f$.

In order to implement the notion of a minimal length L_f, we suppose that one increases the momentum p of a particle arbitrarily, but that the wave number k has an upper bound. The physical interpretation of this is that particles cannot possess arbitrarily small Compton wavelengths $\lambda = 2\pi/k$ so that arbitrarily small scales cannot be resolved anymore. To incorporate this behaviour, we assume a relation $k = k(p)$ between p and k which is an uneven function (because of parity) and which asymptotically approaches $1/L_f$ [9].

This altered de Broglie-relation leads to modified commutation relations and operators. With a low momentum approximation, one can derive first order deviations due to a minimal length scale:

If we look at the precession of a dipole in a magnetic field without minimal length and compare its precession frequency to that of the spin 1/2 particle under investigation, the so-called Landé-g-factor occurs. Without minimal length the frequency from quantum mechanics is two times the classical one. A modification from minimal length is expected. In our model, this modification results from the new relation between energy and frequency.

Starting from the modified Dirac equation, the expression $\tilde{g}$ for the gyromagnetic moment for $k \to 0$ is[8]:

$$\tilde{g} = g \cdot \left(1 - \alpha \frac{m^2}{M_f^2}\right) \quad .$$

The theoretical and experimental results[5,4] indicate that modifications to the standard model calculation have to be smaller than 10^{-8}. This leads to the following constraint on the fundamental scale of the theory:

$$M_f \gtrsim 500 \text{ TeV} \quad .$$

As a second example, we discuss the energy levels of the Helium atom. Especially those of the $S1 - S2$-transition are very well known experimentally. To simplify matters, we used the semiclassical calculation according to N. Bohr, which leads to the same solution as the Schrödinger equation. As our result, we get the modified energy levels of the Coulomb central potential to first order:

$$E_n \approx -\frac{e^4 m}{2n^2}\left(1 - \frac{e^4 m^2}{M_f^2 n^2}\right) \quad .$$

For $M_f \approx 1.1\,\mathrm{TeV}$ (the present limit)[6] the predicted relative modification of the 1S-2S transition in He^+ is 3.8×10^{-17}.

We have discussed the model of Large eXtra Dimensions and the notion of a minimal length. We have seen that a combination of both scenarios leads to a new possibility of constraining such models, especially in the high precision sector where virtual graviton calculations have not turned out to be very promising. In particular, we have shown our recent results on the muon gyromagnetic moment and the He^+-ion. We have compared these results with the experimental data and have given limits on the fundamental scale M_f. Furthermore, the results clearly indicate that effects of a minimal length scale have to be considered in models with LXDs.

Acknowledgments

The authors thank S. Hofmann and J. Ruppert for fruitful discussions. This work was supported by the Graduiertenkolleg (Land Hessen), DFG, GSI and BMBF. U.H. and B.K. thank the Frankfurt International Graduate School of Science (FIGSS) for financial support through a PhD fellowship. S.H. is supported by the German Academic Exchange Service (DAAD) and NSF PHY/0301998.

References

1. D. Amati, M. Ciafaloni, and G. Veneziano. Can space-time be probed below the string size? *Phys. Lett.*, B216:41, 1989.
2. I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, and G. R. Dvali. New dimensions at a millimeter to a fermi and superstrings at a tev. *Phys. Lett.*, B436:257–263, 1998.
3. N. Arkani-Hamed, S. Dimopoulos, and G. R. Dvali. The hierarchy problem and new dimensions at a millimeter. *Phys. Lett.*, B429:263–272, 1998.
4. G. W. Bennett et al. Measurement of the positive muon anomalous magnetic moment to 0.7-ppm. *Phys. Rev. Lett.*, 89:101804, 2002.
5. M. Davier, S. Eidelman, A. Hocker, and Z. Zhang. Updated estimate of the muon magnetic moment using revised results from e+ e- annihilation. *Eur. Phys. J.*, C31:503–510, 2003.
6. S. Eidelman et al. Review of particle physics. *Phys. Lett.*, B592:1, 2004.
7. D. J. Gross and P. F. Mende. String theory beyond the planck scale. *Nucl. Phys.*, B303:407, 1988.
8. U. Harbach, S. Hossenfelder, M. Bleicher, and H. Stocker. Probing the minimal length scale by precision tests of the muon g-2. *Phys. Lett.*, B584:109–113, 2004.
9. S. Hossenfelder et al. Collider signatures in the planck regime. *Phys. Lett.*, B575:85–99, 2003.
10. E. Witten. Reflections on the fate of space-time. *Phys. Today*, 49:24, 1996.