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DESCRIBING RECENTLY DISCOVERED NARROW STATES AS QUARKONIA USING A POTENTIAL MODEL*

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We examine to what extent several recently discovered narrow resonances can be interpreted as conventional $c\bar{c}$ bound states describable using a potential model. In doing so, we use a semirelativistic approach, which includes both the v^2/c^2 and QCD one-loop corrections to the short distance potential and a long range linear potential together with its scalar and vector v^2/c^2 spin-dependent terms.

1. Introduction

With the recent experimental results for several expected states (η'_C and h_C) in the charmonium spectrum, and the discovery of a state ($X(3872)$), which could be a $^3D_2$ charmonium level, it seems an appropriate time to revisit the potential model interpretation of the $c\bar{c}$ spectrum. For such models, the challenges seem to be:

- Are potential models capable of describing the spin splitting in a quantitatively satisfactory way?
- Including the additional data, is it possible to determine the Lorentz properties of the phenomenological confining potential?
- How well are the leptonic and radiative decays predicted?
- Under what circumstances can the state at 3872 MeV be interpreted as a charmonium level?

Here, we will attempt to answer these questions using a potential model which includes the v^2/c^2 and all one-loop corrections to the short distance potential supplemented with a linear phenomenological confining potential and its v^2/c^2 corrections.

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2. Potential Models

Potential models range from non-relativistic forms such as the Cornell model [1]

$$\mathcal{H}_C = \frac{p^2}{2m} + Ar - \frac{4\alpha_S}{3r}\,, \tag{1}$$

recently used to identify new charmonium states observable in B decays [2], to those including all v^2/c^2 spin-dependent corrections, [3,4] $\mathcal{H} = \mathcal{H}_C + V_{HF} + V_{LS} + V_{TEN}$, which can be used to analyze the spectrum and decays of charmonium [5]. One-loop QCD corrections can then be added to complete the non-relativistic treatment.[6]

In our analysis, we utilize a semi-relativistic Hamiltonian of the form

$$\mathcal{H} = 2\sqrt{p^2+m^2} + Ar - \frac{4\alpha_S}{3r}F(\mu,r) + V_S + V_L = \mathcal{H}_0 + V_S + V_L\,. \tag{2}$$

Explicit forms of the short distance potential V_S, the long distance potential V_L and $F(\mu,r)$, the one-loop QCD correction to $4\alpha_S/3r$, can be found in Ref. 7. The $c\bar{c}$ mass spectrum and the corresponding wave functions were obtained using a variational approach. The wave functions were expanded as

$$\psi_\ell^m(\vec{r}) = \sum_{n=0}^{N} C_n \left(\frac{r}{R}\right)^n e^{-r/R}Y_\ell^m(\Omega)\,, \tag{3}$$

and the C_n's were determined by minimizing $E = \langle\psi\,|\mathcal{H}\,|\psi\rangle/\langle\psi\,|\psi\rangle$. This procedure results in a linear eigenvalue equation for the C_n's and the energies. The wave functions corresponding to different eigenvalues are orthogonal and the $j^{\rm th}$ eigenvalue λ_j is an upper bound on the exact energy E_j. For $N = 10$, the lowest four eigenvalues are stable to a part in 10^6.

3. Results and Conclusions

The energies and wave functions were obtained by treating $V_S{+}V_L$ as a perturbation to $\mathcal{H}_0$ and by treating $\mathcal{H}$ nonperturbatively. We fit the spectrum to ten well established [a] $c\bar{c}$ states by adjusting the parameters A, α_S, m, μ and the vector fraction f_V to minimize χ^2. Our nonperturbative fit gives: $A = 0.175\,\mathrm{GeV}^2, \alpha_S = 0.361, m = 1.49\,\mathrm{GeV}, \mu = 1.07\,\mathrm{GeV}$ and $f_V = 0.18$. Interestingly, the perturbative fit yields similar results for A, α_S and m, but prefers $\mu = 2.32\,\mathrm{GeV}$ and $f_V = 0$. The results for the levels are given in Table 1 and the predicted E_1 transitions widths are given in Table 2.

The semi-relativistic model provides a quantitatively good description of the charmonium spectrum. Of the states included in the fit, only the ${}^3D_1(3770)$ is poorly described. The E_1 widths agree reasonably well with experiment[8]. However, based on the model considered here, the $X(3872)$ cannot be explained solely in terms of a charmonium ${}^3D_2$ state described by a potential. Spin effects alone can only separate the ${}^3D_2$ from the ${}^3D_1$ by 40 MeV or so, which suggests that the inclusion of open channel effects is essential if this identification is to be established.

[a] $1^1S_0, 1^3S_1, 1^3P_J, 2^1S_0, 2^3S_1, 1^3D_1, 3^3S_1$ and 2^3D_1.

Table 1. Perturbative and nonperturbative results for the $c\bar{c}$ spectrum.

	Pert	Non-pert	Expt		Pert	Non-pert	Expt
η_c	2985	2981	2979.7 ± 1.5	η'_c	3599	3624	(3637.7 ± 4.4)
J/ψ	3096.9	3096.9	3096.87 ± 0.04	ψ'	3686	3686	3686.0 ± 0.1
χ_0	3418.4	3415.8	3415.1 ± 0.8	χ'_0	3849	3872	
χ_1	3510.2	3510.4	3510.51 ± 0.12	χ'_1	3946	3951	
χ_2	3556.5	3556.3	3556.18 ± 0.17	χ'_2	3999	3996	
h_c	3527	3524	(3526.21 ± 0.25)	h'_c	3966	3966	
1^3D_1	3809	3790	3770 ± 2.5	2^3D_1	4174	4157	4160 ± 20
1^3D_2	3827	3826	3872 ± 1.0	2^3D_2	4198	4201	
1^3D_3	3831	3845		2^3D_3	4209	4223	
1^1D_2	3824	3825	3836 ± 13.0	2^1D_2	4199	4202	

Table 2. E_1 transition widths.

$\Gamma_\gamma(E1)$ (keV)	TH	EX	$\Gamma_\gamma(E1)$ (keV)	TH	EX
$\chi_0 \to \gamma J/\psi$	169	119 ± 17	$1^3D_2(3826) \to \gamma\chi_1$	314	
$\chi_1 \to \gamma J/\psi$	357	288 ± 51	$1^3D_2(3826) \to \gamma\chi_2$	76.3	
$\chi_2 \to \gamma J/\psi$	468	426 ± 48	$1^3D_2(3872) \to \gamma\chi_1$	459	
$h_c \to \gamma\eta_c$	670		$1^3D_2(3872) \to \gamma\chi_2$	119	
$\psi' \to \gamma\chi_0$	22	24.2 ± 2.5	$\psi(3770) \to \gamma\chi_0$	291	320 ± 100
$\psi' \to \gamma\chi_1$	33	23.6 ± 2.7	$\psi(3770) \to \gamma\chi_1$	125	280 ± 100
$\psi' \to \gamma\chi_2$	29	24.2 ± 2.5	$\psi(3770) \to \gamma\chi_2$	5.6	≤ 330
$\eta'_c \to \gamma h_c$	22				

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