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WARD-TAKAHASHI IDENTITIES AND SPIN-0 COMPONENTS OF W AND Z FIELDS

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It is shown that three scalar fields are dynamically generated after spontaneous symmetry breaking. Their masses are dynamically determined at about $10^{14}GeV$. They are ghosts. Unitarity of the SM is broken at 10^{14}GeV.

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Comparing with QED and QCD, there are axial-vector and charged vector currents in the SM. These new currents have not been studied in 't Hooft's and Lee and Zinn-Justin's papers.

Before spontaneous symmetry breaking the Lagrangian of the SM is invariant under $SU(2)_L \times U(1)$ gauge transformation. At the lowest order $\partial_\mu \bar{t}\gamma_\mu\gamma_5 t = 0$, $\partial_\mu \bar{b}\gamma_\mu\gamma_5 b = 0$, $\partial_\mu(\bar{t}\gamma_\mu(1+\gamma_5)b) = 0$. The amplitudes of the vacuum polarization of W and Z fields are proportional to $q_\mu q_\nu - g_{\mu\nu}q^2$.

According to 't Hooft, Lee and Zinn-Justin, **the vacuum polarizations of Z and W fields by gauge bosons and Higgs don't have any effects on unitarity of the theory**. Therefore, we only study the vacuum polarizations by fermions after spontaneous symmetry breaking[1]. The change of the WT identities of fermion axial-vector and charged vector currents after spontaneous symmetry breaking can be shown clearly by the equations at the lowest order

$$\partial_\mu(\bar{t}\gamma_\mu\gamma_5 t) = 2im_t\bar{t}\gamma_5 t, \quad \partial_\mu(\bar{b}\gamma_\mu\gamma_5 b) = 2im_b\bar{b}\gamma_5 b,$$

$$\partial_\mu(\bar{t}\gamma_\mu(1+\gamma_5)b) = i(m_t - m_b)\bar{b}t + i(m_t + m_b)\bar{t}\gamma_5 b.$$

The amplitude of the vacuum polarization of Z fields by massive fermions at the lowest order is expressed as

$$\Pi^Z_{\mu\nu} = \frac{1}{2}F_{Z1}(z)(p_\mu p_\nu - p^2 g_{\mu\nu}) + F_{Z2}(z)p_\mu p_\nu + \frac{1}{2}\Delta m^2_Z g_{\mu\nu}, \tag{1}$$

$$F_{Z2} = -\frac{\bar{g}^2}{64\pi^2}\{N_C\sum_q f_2(z_q) + \sum_{l=e,\mu,\tau} f_2(z_l)\}, \tag{2}$$

where $z_i = \frac{p^2}{m_i^2}$,

$$f_2(z) = \frac{1}{z}\int_0^1 dxlog\{1 - x(1-x)z\}. \tag{3}$$

$F_{Z2}(z)p_\mu p_\nu$ is a new term. Only the axial-vector currents of massive fermion contribute to F_{Z2}. In the limits, $m_{q(l)} \to 0$, $F_{Z2}(z) = 0$. Therefore, the new term is resulted in the changed WT identities of the axial-vector currents of fermions of the SM after spontaneous symmetry breaking. Because of the new term there is no current conservation. It is interesting to notice that the symmetry breaking is soft. F_{Z2} is finite and no new divergence is generated. Weinberg's counting rule is still held. The term F_{Z2} indicates that Z field has nonzero divergence. $\partial_\mu Z^\mu$ can be defined as a spin-0 field ϕ_Z whose mass is labeled as m_{ϕ_Z} which can be determined. We rewrite F_{Z2} as

$$F_{Z2}(z) = \xi_Z + (p^2 - m_{\phi_Z}^2)G_{Z2}(p^2), \tag{4}$$

The new free Lagrangian of the Z-field is constructed as

$$\mathcal{L}_{Z0} = -\frac{1}{4}(\partial_\mu Z_\nu - \partial_\nu Z_\mu)^2 + \xi_Z(\partial_\mu Z^\mu)^2 + \frac{1}{2}m_Z^2 Z_\mu^2. \tag{5}$$

The term $(\partial_\mu Z^\mu)^2$ is dynamically generated and the coefficient ξ_Z is no longer a parameter of gauge condition and it can be determined. The new spin-0 field is defined $\phi_Z = \mp\frac{m_Z}{m_{\phi_Z}^2}\partial_\mu Z^\mu$. The mass of ϕ_z is defined as $m_{\phi_Z}^2 = -\frac{m_Z^2}{2\xi_Z}$, which satisfies

$$2m_{\phi_Z}^2 F_{Z2}|_{p^2=m_{\phi_Z}^2} + m_Z^2 = 0. \tag{6}$$

The value of $m_{\phi_Z}^2$ is determined by

$$3\sum_q \frac{m_q^2}{m_Z^2} z_q f_2(z_q) + \sum_l \frac{m_l^2}{m_Z^2} z_l f_2(z_l) = \frac{32\pi^2}{\bar{g}^2}. \tag{7}$$

For $z > 4$

$$f_2(z) = -\frac{2}{z} - \frac{1}{z}(1-\frac{4}{z})^{\frac{1}{2}} log\frac{1-(1-\frac{4}{z})^{\frac{1}{2}}}{1+(1-\frac{4}{z})^{\frac{1}{2}}} \tag{8}$$

is a very good approximation. Top quark dominates. The equation has a solution at very large value of z

$$\frac{2(4\pi)^2}{\bar{g}^2} + \frac{6m_t^2}{m_Z^2} = 3\frac{m_t^2}{m_Z^2} log\frac{m_{\phi_Z}^2}{m_t^2}. \tag{9}$$

The mass of the ϕ_Z is determined to be

$$m_{\phi_Z} = m_t e^{\frac{m_z^2}{m_t^2}\frac{16\pi^2}{3\bar{g}^2}+1} = m_t e^{28.4} = 3.78 \times 10^{14} GeV, \tag{10}$$

and $\xi_Z = -1.18 \times 10^{-25}$. The Z-filed is decomposed as $Z_\mu = Z'_\mu \pm \frac{1}{m_Z}\partial_\mu \phi_Z$.

The propagator of Z boson is found

$$\Delta_{\mu\nu} = \frac{1}{p^2 - m_Z^2}\{-g_{\mu\nu} + (1 + \frac{1}{2\xi_Z})\frac{p_\mu p_\nu}{p^2 - m_{\phi_Z}^2}\}, \tag{11}$$

It can be separated into two parts

$$\Delta_{\mu\nu} = \frac{1}{p^2 - m_Z^2}\{-g_{\mu\nu} + \frac{p_\mu p_\nu}{m_Z^2}\} - \frac{1}{m_Z^2}\frac{p_\mu p_\nu}{p^2 - m_{\phi_Z}^2}. \tag{12}$$

The minus sign indicates that the Fock space has indefinite metric and there is problem of negative probability when ϕ_Z is on mass shell. The ϕ_Z is a ghost field. Therefore, unitarity of the SM is broken at $E = m_{\phi_Z}$. This ghost field is dynamically generated by fermion loops. The ghost boson is resulted by the masses of the fermions. Therefore, there is no way it can be canceled within the SM.

Similarly, the expression of the vacuum polarization of W-fields at one loop of fermions is calculated

$$\Pi^W_{\mu\nu} = F_{W1}(p^2)(p_\mu p_\nu - p^2 g_{\mu\nu}) + 2F_{W2}(p^2)p_\mu p_\nu + \Delta m_W^2 g_{\mu\nu}, \tag{13}$$

Both the axial-vector and charged vector currents contribute to F_{W2} and it is finite, which leads to $(\partial_\mu W^{\pm\mu})^2 \neq 0$ and charged scalar components are revealed, whose mass is dynamically generated

$$m_{\phi_W} = m_t e^{\frac{16\pi^2}{3g^2}\frac{m_W^2}{m_t^2}} = m_t e^{27} = 9.31 \times 10^{13} GeV \tag{14}$$

The propagator of W-field is derived

$$\Delta^W_{\mu\nu} = \frac{1}{p^2 - m_W^2}\{-g_{\mu\nu} + (1 + \frac{1}{2\xi_W})\frac{p_\mu p_\nu}{p^2 - m_{\phi_W}^2}\}, \tag{15}$$

$\xi_W = -3.73 \times 10^{-25}$. It can be separated into two parts

$$\Delta^W_{\mu\nu} = \frac{1}{p^2 - m_W^2}\{-g_{\mu\nu} + \frac{p_\mu p_\nu}{m_W^2}\} - \frac{1}{m_W^2}\frac{p_\mu p_\nu}{p^2 - m_{\phi_W}^2}. \tag{16}$$

The minus sign in front of the propagator of $\phi^\pm_W$ fields indicates that there is problem of indefinite metric and negative probability when $\phi^\pm_W$ are on mass shell. Therefore, $\phi^\pm_W$ are ghost fields. Unitarity of the SM is broken at $E > m_{\phi_W}$.

Three scalar fields are dynamically generated after spontaneous symmetry breaking. They are ghosts whose masses are at 10^{14} Gev. Unitarity of the SM is broken at this energy level.

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References

1. Bing An Li, International J. of Modern Physics **A17**,3607(2002).