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S-brane Solutions in String Theory

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Recent progress in the construction of time dependent solutions of supergravity theories corresponding to so called spacelike branes will be reviewed.

1. Introduction

Time dependent solutions in string theory and supergravities have been of great interest recently. Classical solutions of supergravity theories can be the starting point to analyze questions of stability under small perturbations and particle creation. The discovery of D-branes by Polchinski [1] fueled our understanding of nonperturbative string theory and the resolution of static singularities as exemplified in the conifold.

A particular class of such solutions are called S-branes (for 'spacelike' branes) [2]. S-branes describe a shell of radiation moving in from infinity and forming an unstable brane which subsequently decays. Since the brane only exists for a finite time it is localized in time and is therefore spacelike.

A concrete realization of this was found by Sen in terms of open string tachyon condensation in [3,4,5]. Recently various supergravity S-brane solutions were found in [6,7,8,9](For related earlier work see [10,11,12,13]).

2. Analytic continuation

Many S-brane solutions can be obtained by analytic continuation of known static solutions. As an example consider the metric for the four dimensional Schwarzschild black hole.

$$ds^2 = -(1-\frac{m}{r})dt^2 + \frac{1}{1-\frac{m}{r}}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2). \qquad (1)$$

A time dependent solution can be obtained from (1) by replacing

$$r \to i\tau, \quad t \to ix, \quad m \to im, \quad \theta \to i\theta, \qquad (2)$$

which gives

$$ds^2 = -\frac{1}{1-\frac{m}{\tau}}d\tau^2 + (1-\frac{m}{\tau})dx^2 + \tau^2(d\theta^2 + \sinh^2\theta d\phi^2). \qquad (3)$$

The analytic continuation turns the metric on the two sphere S_2 into the metric on H_2, the Poincaré plane. The Penrose diagram of this spacetime is the one of the Schwarzschild black hole which is rotated by 90^0 degrees.

Another example is given by the Reissner-Nordström solution of Einstein-Maxwell theory

$$\begin{aligned} ds^2 &= -(1-\frac{2m}{r}+\frac{q^2}{r^2})dt^2 + (1-\frac{2m}{r}+\frac{q^2}{r^2})^{-1}dr^2 + r^2(d\theta^2+\sin^2\theta d\phi^2), \\ F_{rt} &= \frac{q}{r^2}. \end{aligned} \tag{4}$$

The analytic continuation

$$r \to i\tau, \quad t \to ix, \quad m \to \pm im, \quad q \to q, \quad \theta \to i\theta \tag{5}$$

produces the time dependent metric

$$\begin{aligned} ds^2 &= -(1\mp\frac{2m}{\tau}-\frac{q^2}{\tau^2})^{-1}d\tau^2 + (1\mp\frac{2m}{\tau}-\frac{q^2}{\tau^2})dx^2 + \tau^2(d\theta^2+\sinh^2\theta d\phi^2), \\ F_{\tau x} &= \frac{q}{\tau^2}. \end{aligned} \tag{6}$$

For an extremal black hole, obeying $m = \pm q$, the analytic continuation (5) produces a complex and therefore unphysical solution. This is a generic feature. The S-brane solutions obtained by analytic continuation are always nonsupersymmetric, as expected from spacetimes without timelike or null Killing vectors.

A black hole in AdS_5 has a metric

$$ds^2 = -(1-\frac{m}{r^2}+r^2)dt^2 + \frac{1}{1-\frac{m}{r^2}+r^2}dr^2 + r^2(d\theta^2+\sin^2\theta d\Omega_2^2), \tag{7}$$

where $d\Omega_2^2$ is the metric on the unit two sphere. As before one can obtain a real time dependent solution from the above metric by performing

$$r \to i\tau, \quad t \to ix, \quad \theta \to i\theta, \tag{8}$$

which should be associated with an S-brane in AdS. The analytically continued metric is

$$ds^2 = (1+\frac{m}{\tau^2}-\tau^2)dx^2 - \frac{1}{1+\frac{m}{\tau^2}-\tau^2}d\tau^2 + \tau^2(d\theta^2+\sinh^2\theta d\Omega_2^2). \tag{9}$$

There are coordinate singularities at $\tau_\pm^2 = \frac{1}{2}(1\pm\sqrt{1+4m})$. For $0 > m > -1/4$ there are two horizons $\tau_\pm$, whereas for $m > 0$ there is only one horizon at τ_+. For $\tau > \tau_+$ the metric becomes static and one gets

$$ds^2 = -(-1-\frac{m}{\tau^2}+\tau^2)dx^2 + \frac{1}{-1-\frac{m}{\tau^2}+\tau^2}d\tau^2 + \tau^2 dH_3^2. \tag{10}$$

This metric is the 'topological' black hole solution [14,15,16], i.e. a black hole in AdS with a hyperbolic horizon. In contrast to the asymptotically flat case, in AdS the analytic continuation (8) of a black hole with a spherical horizon does not produce a new time dependent background but instead produces the inside of a black hole with a hyperbolic horizon.

3. Gauged Supergravities

In this section we review new S-brane solutions of theories of d-dimensional gravity coupled to gauge and scalar fields. These solutions where found in collaboration with W. Sabra [17]. The Lagrangian we will consider is given by

$$e^{-1}L_d = R - \frac{1}{2}M_{ij}\partial_\mu\phi^i\partial^\mu\phi^j - \frac{1}{4}G_{IJ}F^I_{\mu\nu}F^{\mu\nu\,J} - l^2 P. \tag{11}$$

with gauge kinetic term given by

$$G_{IJ} = \frac{1}{(X_I)^2}\delta_{IJ}. \tag{12}$$

Where the potential for the scalar fields is given by

$$P = -\frac{(d-3)^2}{8}\left((\sum_{I=1}^N X_I)^2 - 2\sum_{I=1}^N X_I^2\right). \tag{13}$$

The N scalars X_I are subject to the constraint

$$\prod_{I=1}^N X_I = 1. \tag{14}$$

The constrained scalars X_I are parameterized in terms of $(N-1)$ independent dilatonic scalars $\vec{\varphi}$ as follows

$$X_I = e^{-\frac{1}{2}\vec{b}_I\cdot\vec{\varphi}}, \tag{15}$$

where the $\vec{b}_I$ are the weight vectors of the fundamental representation of $SL(N,R)$, satisfying

$$\vec{b}_I\cdot\vec{b}_J = 8\delta_{IJ} - \frac{8}{N}, \qquad \sum_I \vec{b}_I = 0, \qquad (\vec{u}\cdot\vec{b}_I)\,\vec{b}_I = 8\vec{u}. \tag{16}$$

The vector $\vec{u}$ is an arbitrary N-vector. The above relations allow $\vec{\varphi}$ to be determined in terms of X_I

$$\vec{\varphi} = -\frac{1}{4}\sum_I \vec{b}_I \log X_I. \tag{17}$$

We take the same ansatz for the metric as in the previous section

$$ds^2 = fe^{-2(d-3)V}dx^2 + e^{2V}\left(-\frac{dt^2}{f} + t^2 ds^2_{d-2,k}\right). \tag{18}$$

It was shown in [17] that the Einstein equations of motion admit solutions given by

$$\begin{aligned}
e^{2V} &= \prod (H_I)^{\frac{1}{2(d-2)}}, f = -k + \frac{\mu}{t^{d-3}} - l^2t^2e^{2(d-2)V},\\
X_I &= \frac{1}{H_I}\prod_J (H_J)^{\frac{1}{4}\left(\frac{d-3}{d-2}\right)} = \frac{1}{H_I}e^{(d-3)V},\\
F^I_{xt} &= \frac{(d-3)}{H_I^2 t^{d-2}}\sqrt{(kq_I^2 + \mu q_I)},\\
H_I &= 1 + \frac{q_I}{t^{d-3}}.
\end{aligned} \tag{19}$$

provided we have the following relation

$$\vec{b}_I.\vec{b}_J = 8\delta_{IJ} - 2\frac{d-3}{d-2}. \tag{20}$$

It follows from the first equation in (16) that $N = 4(d-2)/(d-3)$. Since N and d have to be integers. Such a relation is only possible for dimensions $d = 4, 5, 7$, where $N = 8, 6, 5$ respectively.

These solutions have very similar properties than the examples discussed in the previous section. An extremal limit for the S-branes is unphysical since the gauge fields become complex. Hence these solutions are nonsupersymmtric.

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