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Measurement of the Electric Dipole Moment and Magnetic Moment Anomaly of the Muon

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The experimental precision of the anomalous magnetic moment of the muon has been improved to 0.5 part-per-million by the Brookhaven E821 experiment, similar to the theoretical uncertainty. In the same experiment, a new limit on the electric dipole moment of 2.8×10^{-19} e·cm (95% CL) was set. The experiment and plans to improve them are described.

Keywords: muons; anomalous magnetic moment; electric dipole moment.

1. THEORETICAL BACKGROUND

The electric and magnetic dipole moments of elementary particles are excellent tools to study the boundaries of the applicability of the Standard Model of Particle Physics (SM). Since its conception, the SM has been extremely in predicting and explaining every experimental result. Only in the last few years, neutrino oscillations are unveiling the limitations of the SM.

Within the SM, the leptonic magnetic and electric dipole moments can be calculated with great precision. Both dipole moments are aligned along the spin, with their respective proportionality constants defined as $\vec{\mu} = ge\hbar/2mc\,\vec{S}$ and $\vec{d} = \eta e\hbar/4mc\,\vec{S}$. According to Dirac's theory, the gyromagnetic ratio g is 2 for pointlike spin-1/2 particles. Radiative corrections give rise to the anomalous magnetic moment $a = (g-2)/2 \simeq \alpha/2\pi \simeq 1/860$. Corrections only involving photons and leptons are calculable using QED, and dominate the anomaly. The next largest contribution involves virtual hadrons, requiring QCD. Since low-energy QCD is non-perturbative, precise first-principles calculations are impossible. Fortunately, one can fall back on experimental data. By means of the dispersion relation the cross section for e^+e^--annihilation into hadrons can be related to the hadronic contribution to the anomalous magnetic moment. Only the so-called hadronic light-by-light contribution has to be modeled. The third contribution involves the weak interaction W and Z bosons. Recent predictions are[1]: $a_\mu^{QED} = (116\,584\,719 \pm 1.8)\times 10^{-11}$; $a_\mu^{had} = (6\,935 \pm 60)\times 10^{-11}$; $a_\mu^{weak} = (152 \pm 3)\times 10^{-11}$; $a_\mu^{SM} = a_\mu^{QED} + a_\mu^{had} + a_\mu^{weak}$ $= (116\,591\,806 \pm 60)\times 10^{-11}$. The sensitivity for phenomena outside of the scope of the SM is typically proportional to $(m_l/M)^2$, with m_l the lepton mass and M

a characteristic new physics mass scale, which explains the interest in the muon's magnetic moment.

A non-zero value for the electric dipole moment requires the violation of both parity (P) and time-reversal (T) symmetries. In the SM, the only source for P-odd, T-odd phenomena is CP-violation in the weak interaction. This follows from the assumption that the combined CPT symmetry is conserved, so that CP and T-violation are equivalent. Consequently, the predicted EDMs are exceedingly small and far below any foreseeable experimental sensitivity. The predicted muon EDM is of the order of $10^{-38}\,\mathrm{e{\cdot}cm^2}$. Various extensions of the SM predict values far in excess of this value. The observation of a non-zero EDM would be an unambiguous sign of physics beyond the SM.

2. EXPERIMENT

The interaction between the magnetic and electric dipole moments with an electromagnetic field causes a torque on the spin. At relativistic speeds, the spin precession, relative to the momentum of the particle, is described by the BMT-equation:

$$\frac{d\vec{s}}{dt} = \frac{e}{mc}\,\vec{s} \times \left[a\,\vec{B} + \left(a - \frac{1}{\gamma^2 - 1} \right) \vec{v} \times \vec{E} + \frac{\eta}{2} \left(\vec{v} \times \vec{B} + \vec{E} \right) \right], \qquad (1)$$

where it was assumed that $\vec{v}\cdot\vec{B} = \vec{v}\cdot\vec{E} = 0$. The factor in brackets can be interpreted as the spin precession frequency vector $\vec{\omega}$. The observation of the change of the spin orientation with time thus allows to gain insight in a and η, once $\vec{B}$, $\vec{E}$ and $\vec{v}$ are known.

2.1. *The Magnetic Moment Anomaly*

The magnetic moment anomaly is measured in the E821 experiment at Brookhaven National Laboratory (BNL) by storing polarized muons in a magnetic field, which is homogeneous at one part in a million. At the 'magic' momentum, $\gamma = \sqrt{(a+1)/a} \simeq 29$, the effect of the focusing electric field is zero. The spin precession rate combined with the magnetic field strength allows to extract the magnetic moment anomaly, provided that the electric dipole moment is negligible.

The experiment has reached a relative precision of about 0.7 parts-per-million for both the positive and negatively charged muon, $a_{\mu^+} = 11\,659\,203(8) \times 10^{-10}$ and $a_{\mu^-} = 11\,659\,214(9) \times 10^{-10}$, respectively[3,4]. These measurements are consistent, as predicted by the CPT theorem. The combined value is $a_\mu^{\mathrm{exp}} = 11\,659\,208(6) \times 10^{-10}$. The difference between experiment and theory is $(27 \pm 8) \times 10^{-10}$, *i.e.* 3.2 standard deviations away from zero. Several, none of them statistically definitive, explanations are possible for this difference, ranging from experimental or theory mistakes, to a statistical fluke or the manifestation of 'new physics'. A continued effort, both theoretically and experimentally will be needed to decide among these possibilities.

2.2. *The Electric Dipole Moment*

The difference between the observed and predicted magnetic moment anomaly could be interpreted as an EDM. For $\eta \neq 0$, $a' = \sqrt{a^2 + (\eta/2\,v)^2} \simeq a(1 + \eta^2/8a^2)$. The observed difference would correspond to an EDM of $(2.5 \pm 0.4) \times 10^{-19}\,$e·cm, *i.e* $\|d_\mu\| < 3.1 \times 10^{-19}\,$e·cm (95% C.L.). A model independent measurement was obtained from the radial component of the spin precession[5]. This yielded $\|d_\mu\| < 2.8 \times 10^{-19}$ e·cm (95% C.L.), some 19 orders of magnitude above the SM value.

In the E821 experiment, the EDM sensitivity is suppressed by $1/\omega_a$, the inverse of the anomalous spin precession rate. At momenta away from the magic one, a combination of a vertical *magnetic* field and a radial *electric* field can be used to freeze this precession[6]. For $E_r \simeq aBc\beta\gamma^2$, the first two terms between brackets in eq. (1) add to zero. In this case, only the radially oriented EDM precession is left, improving the sensitivity by five orders of magnitude.

3. The Future

The experimental value for $(g-2)_\mu$ is still statistics limited. A proposal to reduce the uncertainty to 0.2 ppm has been submitted to BNL and was regarded positively[8]. A letter-of-intend was submitted to J-PARC for an experiment to reach 0.06–0.1 ppm uncertainty.[7].

The present muon EDM limit can be improved by several orders of magnitude using the proposed new technique. The limit in a dedicated experiment will be statistics limited, even with a beam that is several orders of magnitude more intense than are presently available. The proposed muon facility at J-PARC is forseen to provide the highest possible intensity and a letter-of-intent for a possible experiment was submitted[9].

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