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HOLOGRAPHIC MESONS: ADDING FLAVOR TO THE ADS/CFT DUALITY

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We study mesons in an $\mathcal{N} = 2$ super Yang-Mills theory with fundamental matter from its dual string theory on $AdS_5 \times S^5$ with a D7-brane probe. For quarks with a finite mass m_q, the meson spectrum is discrete and exhibits a mass gap of order $m_q/\sqrt{g_{YM}^2 N}$. The spectrum of mesons with large spin J is obtained from semiclassical rotating open strings attached to the D7-brane. It displays Regge behaviour for $J \ll \sqrt{g_{YM}^2 N}$ whereas for $J \gg \sqrt{g_{YM}^2 N}$ it corresponds to that of two non-relativistic quarks bound by a Coulomb potential. We calculate $\langle \mathrm{Tr} F^2 \rangle$ around an isolated quark, to gain insight into the Regge behaviour of the spinning mesons.

Keywords: AdS/CFT correspondence; Gauge theory; Mesons.

1. Introduction

The AdS/CFT correspondence[1] provides a remarkable equivalence between ten-dimensional string theory and four-dimensional gauge theory. The best understood framework is the 'duality' between type IIB string theory on $AdS_5 \times S^5$ and $\mathcal{N} = 4$ $U(N)$ super Yang-Mills (SYM) theory. While the quantization of superstrings in the $AdS_5 \times S^5$ background remains an open technical problem, one can make great progress in a regime where calculations with the low-energy supergravity fields are reliable. On the gauge theory side, this weakly coupled string regime corresponds

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precisely to a strong coupling 't Hooft limit with $N \to \infty$, $g_{YM} \to 0$ and $\lambda = g_{YM}^2 N$ finite and large. Hence this equivalence provides a new tool with which to study strongly coupled (four-dimensional) gauge theories.

Much of the excitement about this duality stems from the hope of gaining new insights into QCD.[2] However, there is an immediate stumbling block to studying QCD-like theories. Namely, all matter fields in the above gauge theory (and its simple generalizations) are in the adjoint representation of the gauge group. Ref. [3] provided a simple solution by introducing k D7-branes into $AdS_5 \times S^5$ which corresponds to adding k flavours of *dynamical* quarks to the gauge theory. More precisely, the new gauge theory is an $\mathcal{N} = 2$ SYM theory coupling the original $\mathcal{N} = 4$ multiplet to k hypermultiplets transforming in the fundamental representation of $U(N)$.

This $\mathcal{N} = 2$ theory with dynamical quarks possesses a rich spectrum of mesons, that is, quark-antiquark bound states, which was described in Ref. [4]. The AdS duals of these mesons are open strings attached to the D7-brane. Hence to compute the meson spectrum, one should in principle quantize these open strings. While that is not possible at the moment, we have calculated with two complementary approximations. In particular, for mesons with large (four-dimensional) spin, which are dual to highly-excited rotating strings, we found the spectrum numerically using a semiclassical approximation. This spectrum shows Regge-like behaviour for small J. This paper gives a short summary of the calculations of the spectra appearing in Ref. [4]. We also describe further calculations which attempt to understand the Regge behavior, noted above, from the point of view of the dual field theory.[5]

2. The model

The model of interest can be understood by starting with a stack of N D3-branes and k parallel D7-branes in Minkowski space, which we represent by the array

$$\begin{array}{ll} \text{D3:} & 1\,2\,3\,\_\,\_\,\_\,\_\,\_\,\_ \\ \text{D7:} & 1\,2\,3\,4\,5\,6\,7\,\_\,\_\,. \end{array} \tag{1}$$

The massless modes of open strings with both ends on the N D3-branes give rise to an $\mathcal{N} = 4$ vector multiplet consisting of the vector bosons, four Weyl fermions and six scalars, all in the adjoint of $U(N)$. $\mathcal{N} = 4$ SYM is a conformal theory with R-symmetry group $SO(6)$, under which the fermions and the scalars transform in the $\mathbf{4}$ and the $\mathbf{6}$ representations, respectively. If $g_s N \sim g_{YM}^2 N \gg 1$ the D3-branes may be replaced, in the appropriate decoupling limit,[1] by an $AdS_5 \times S^5$ spacetime, which corresponds to the throat geometry generated by the D3-branes. The four-dimensional conformal symmetry group, $SO(2,4)$, and the R-symmetry group of the gauge theory correspond to the isometry groups of AdS_5 and S^5, respectively.

The addition of a D7-brane to the system, as in (1), breaks the supersymmetry to $\mathcal{N} = 2$. The light modes coming from strings with one end on the D3-branes and the other one on the D7-brane give rise to an $\mathcal{N} = 2$ hypermultiplet in the fundamental representation, whose field content is two complex scalar fields ϕ^m

and two Weyl fermions $\psi_\pm$, of opposite chirality. If the D3-branes and the D7-brane overlap then the original $SO(6)$ symmetry is broken to $SO(4) \times SO(2) \sim SU(2)_R \times SU(2)_L \times U(1)_R$, where $SO(4)$ and $SO(2)$ rotate the 4567- and the 89-directions in (1), respectively. In this case the hypermultiplet is massless and the R-symmetry of the theory is $SU(2)_R \times U(1)_R$. If the D7-brane is separated from the D3-branes in the 89-plane then the hypermultiplet acquires a mass, given by $m_q = L/2\pi\alpha'$, where L is the distance between the D3- and the D7-branes in the 89-plane. It is known from field theory that the R-symmetry is then only $SU(2)_R$, in agreement with the geometric interpretation: separating the D7-brane breaks the $U(1)_R$ that acts on the 89-plane.

In the dual gravity description, if $N \gg k$ then the back-reaction of the D7-branes on the spacetime geometry may be neglected and one is left with k D7-brane probes in $AdS_5 \times S^5$ that preserve eight supersymmetries. If we locate the D7-brane at a distance L in the 89-plane, the induced worldvolume metric is

$$ds^2 = \frac{r^2}{R^2} ds^2(\mathbb{E}^{(1,3)}) + \frac{R^2}{r^2} \frac{dr^2}{1 - \frac{L^2}{r^2}} + R^2 \left(1 - \frac{L^2}{r^2}\right) d\Omega_3^2 \tag{2}$$

where Ω_3 are spherical coordinates in the 4567-space and as usual $R^2 = \sqrt{g_{YM}^2 N}\alpha'$ (and $g_{YM}^2 = 4\pi g_s$). This metric becomes $AdS_5 \times S^3$ asymptotically, *i.e.*, for $r \gg L$, reflecting the fact that in the gauge theory conformal invariance is explicitly broken by the mass $m_q \propto L$ of the hypermultiplet, but is restored asymptotically at energies $E \gg m_q$. Note also that the radius of the three-sphere is not constant; in particular, it *smoothly* shrinks to zero at $r = L$, at which point the D7-brane 'terminates' from the viewpoint of the projection on AdS_5.

3. Meson spectrum

The spectrum of scalar and vector mesons with arbitrary R-charge can be obtained from the fluctuations of the scalar and vector fields in the D7-brane worldvolume theory. In Ref. [4], the result was given in closed analytical form. Here we simply observe that the spectrum is discrete and determined by a single scale:

$$m_0 = \frac{L}{R^2} = \frac{2\pi}{\sqrt{g_{YM}^2 N}} m_q \ . \tag{3}$$

In particular, the mass of the lightest meson is $2\sqrt{2}m_0$. This means that in the present regime with $\sqrt{g_{YM}^2 N} \gg 1$, the meson masses are much smaller than the quark mass. In perturbation theory one would find that $M \simeq 2m_q - E_b$, with a binding energy $E_b \sim (g_{YM}^2 N)^2$. At large 't Hooft coupling, however, we obtain that the binding energy almost cancels the rest energy of the quarks. This is clear from the AdS_5 picture of meson 'formation', in which two strings of opposite orientation stretching from the D7-brane to the horizon (the quark-antiquark pair) join together to form an open string with both ends on the D7-brane (the meson); this resulting string is much shorter than the initial ones, and hence corresponds to a configuration with much lower energy.

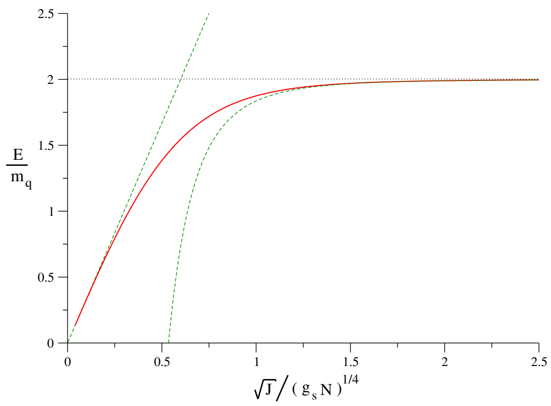

Fig. 1. Meson spectrum $E(J)$ obtained numerically. The horizontal line represents the rest mass of the quark-antiquark pair. The other dashed, straight line is a plot of the Regge behaviour exhibited by the spectrum for $J \ll \sqrt{g_{YM}^2 N}$. The curved dashed line, which fits well for $J \gg \sqrt{g_{YM}^2 N}$, is a plot of $E(J)$ as calculated two quarks bound by a Coulomb potential.

We now turn to the spectrum of mesons with four-dimensional spin $J \gg 1$. In principle, one has to compute the corresponding spectrum of open strings attached to the D7-brane in the $AdS_5 \times S^5$ background, which remains an open problem. However, for $J \gg 1$, the spectrum can be obtained by treating the open strings semiclassically. The spectrum in Figure 1 was found by numerically solving the open string equations of motion with appropriate boundary conditions.

This spectrum can be determined analytically in two particularly interesting regimes. For $J \gg \sqrt{g_{YM}^2 N}$ the two quarks are far apart and the string is longer than the AdS radius. Here the result is $M \simeq m_q(2-\kappa^4/4J^2)$ where κ is a particular constant. This behavior can be understood in terms of the quark-antiquark potential that is computed by studying a static open string attached to the D7-brane, in the spirit of Ref. [6]. Then one understands the m_q/J^2 term as the binding energy of non-relativistic quarks bound by a Coulomb potential $V(r) = -\kappa^2/r$.

For $1 \ll J \ll \sqrt{g_{YM}^2 N}$ the distance between the two quarks is small in comparison with the scale set by the mass of the lightest meson, $O(1/m_0)$, and the string is much shorter than the AdS_5 radius. Hence in this regime, the strings behave as though they are in flat space and the spectrum exhibits Regge behaviour with $M \simeq \sqrt{J/\alpha'_{\rm eff}}$. The effective tension $1/\alpha'_{\rm eff} \sim m_q^2/\sqrt{g_{YM}^2 N}$ is the string tension redshifted to the end of the D7-brane where the strings lie. While the Regge behavior is natural from this AdS point of view, it must be seen as an unusual strong-coupling effect from that of the dual field theory, since the latter is not confining. However, in the next section, we provide some calculations to elaborate on this result from the field theory point of view.

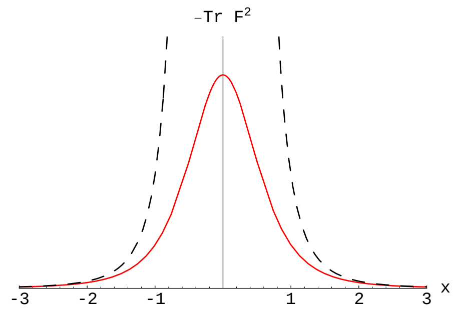

Fig. 2. $\langle \mathrm{Tr} F^2 \rangle$ versus x for an isolated quark with a finite mass (in units with $m_0 = 1$). The dashed line represents the same for a quark with infinite mass.

4. Holographic Quarks and Mesons

The utility of the AdS/CFT correspondence extends beyond the calculation of various derived quantities, *e.g.*, mass spectra. The AdS/CFT dictionary allows us to produce a very direct and physical picture of various configurations. For example, one is able to map out the 'glue clouds' associated with quark or meson states. In particular, the supergravity dilaton is dual to the operator $\mathrm{Tr}F^2$ and so the expectation value of the latter can be determined by evaluating the asymptotic dilaton sourced by a particular (macroscopic) string configuration.

This approach was thoroughly developed in Ref. [7]. In particular, they determined the glue cloud surrounding an isolated static quark which is represented by a fundamental string stretching from the horizon to the boundary of AdS_5:

$$\frac{1}{4g_{YM}^2}\langle \mathrm{Tr}F^2\rangle = -\frac{15R^2}{32\pi^2\alpha'}\int_0^\infty dz' \frac{z'^2}{(x^2+z'^2)^{7/2}} = -\frac{\sqrt{g_{YM}^2 N}}{16\pi^2 x^4} \tag{4}$$

where x is the radial distance from the quark. This result provides a qualitative argument for a Coulomb potential matching the results of Ref. [6]: From eq. (4), the effective charge is proportional to $\sqrt[4]{g_{YM}^2 N}$. Hence the force on another such charge should be $\sim \sqrt{g_{YM}^2 N}/x^2$. This then implies a potential $\sim \sqrt{g_{YM}^2 N}/x$.

In the current context, it is interesting to extend the above calculation to determine the glue cloud around a dynamical quark. In this case, the infinite string is replaced by a fundamental string stretching from the horizon of AdS_5 to the lower limit of the D7-brane, $r = L$. The analogous calculation gives

$$\begin{aligned}\frac{1}{4g_{YM}^2}\langle \mathrm{Tr}F^2\rangle &= -\frac{15R^2}{32\pi^2\alpha'}\int_{R^2/L}^\infty dz' \frac{z'^2}{(x^2+z'^2)^{7/2}} \\ &= -\frac{\sqrt{g_{YM}^2 N}}{16\pi^2 x^4}\left[1-\frac{1+\frac{5}{2}(m_0\, x)^2}{(1+(m_0\, x)^2)^{5/2}}\right] .\end{aligned} \tag{5}$$

The only change in the initial integral is that the lower limit of the integration is finite because the string only extends out to the D7-brane. Figure 2 plots the result and compares it to that for an infinitely massive quark (4). For large x (*i.e.*, $m_0 x \gg 1$), the asymptotic field of the finite mass quark matches that of the static quark. From the figure, we see that with a finite quark mass, the field strength only reaches a finite value at the origin. For $m_0 x \ll 1$, the behavior of the expectation value (5) is given by

$$\frac{1}{4g_{YM}^2}\langle \mathrm{Tr}F^2\rangle = \frac{15\sqrt{g_{YM}^2 N}}{128\pi^2} m_0^4 \left[1 - \frac{7}{3}\,(m_0\,x)^2 + \dots\right]\,. \tag{6}$$

Extending the argument after eq. (4), one would say the field strength is constant near the quark and hence in this region a charge $\sqrt[4]{g_{YM}^2 N}$ feels a linear potential $\sim \sqrt{g_{YM}^2 N} m_0^2\, x \sim m_q^2/\sqrt{g_{YM}^2 N}\, x$. Remarkably this simple reasoning yields a linear potential with a slope of precisely the same parametric form as the effective string tension determined by the Regge behavior of the spinning mesons.

While the "Abelian" argument above should be taken with a grain of salt, the expectation value (5) is indicative of the physics underlying the meson spectrum in Figure 1. For small J (*i.e.*, $1 \ll J \ll \sqrt{g_{YM}^2 N}$), the constituent quarks are probing the near-core region of their respective glue clouds. There the asymptotic Coulomb potential has been weakened and so the mesons are not bound as strongly. The dilution of the core potential can be attributed to an effective spreading of the quark charge over a region $\sim 1/m_0$ by quantum fluctuations of the non-Abelian glue. We hope to expand on this reasoning to produce a more compelling argument for a short-range linear potential.[5]

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