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I=2 S-wave pion scattering phase shift with two flavor fullQCD

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We present a lattice calculation of the scattering phase shift for the $I = 2$ S-wave pion-pion system in the continuum limit with two-flavor dynamical effect. Calculations are made at three lattice spacings, using the finite volume method proposed by Lüscher in the center of mass system, and its extension to the laboratory system by Rummukainen and Gottlieb.

Keywords: Lattice QCD; $\pi\pi$ scattering.

1. Introduction

Study of scatterings of hadrons based on lattice QCD is an important step to understand strong interaction. Exploratory studies of the scattering phase shift for the $I = 2$ S-wave two-pion system [1,2,3] were carried out. These studies were made only at one lattice spacing employing the quenched approximation. In this work [4], we present a calculation including two flavors of dynamical quarks, so that we can get rid of errors from unitarity violation, at three different lattice spacings to carry out the continuum extrapolation.

Our study is based on the finite volume method of Lüscher formulated for the center of mass (CM) system [6], and its extension to the laboratory system by Rummukainen and Gottlieb [7]. While the inclusion of the laboratory system adds little extra computational cost, the use of the laboratory system allows more dense sampling of the results.

We use the full QCD configurations [5] previously generated with an RG-improved gauge action and a mean-field improved clover quark action at three gauge couplings $\beta = 1.80, 1.95$ and 2.10, corresponding to $a = 0.2150(22), 0.1555(17)$ and $0.1076(13)$ fm, on $12^3 \times 24$, $16^3 \times 32$ and $24^3 \times 48$ lattices. The physical volumes are about 2.5 fm in all lattice spacings. Four hopping parameters corresponding to $m_\pi \approx 1.1, 0.9, 0.75$ and 0.55 GeV are taken at each β for chiral extrapolation. The numbers of the configurations vary from 380 to 725.

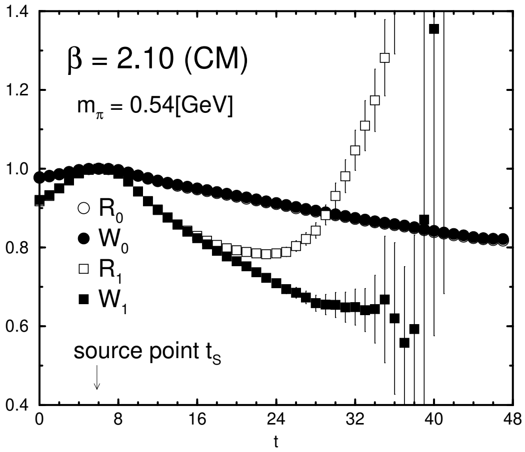

Fig. 1. Values before diagonalization $R_n(t)$ are compared to those after diagonalization $W_n(t)$. Note that $W_0(t)$ is completely overlayed on $R_0(t)$.

2. Methods

On finite volume of L^3 the interaction between two pions shifts the two-pion energy E. In the CM system, writing $E = 2\sqrt{m_\pi^2 + p^2}$ with $p^2 = (2\pi/L)^2\overline{n}$ ($\overline{n} \notin Z$), the phase shift $\delta(p)$ is given by the equation $\tan\delta(p) = \pi^{3/2}\sqrt{\overline{n}}/Z_{00}(1;\overline{n})$ where $\sqrt{4\pi}Z_{00}(i;\overline{n}) = \sum_{\vec{l}}(l^2 - \overline{n})^{-i}$, $\vec{l}$ being integer vectors.

In the laboratory system, where the total momentum of two pions $P = p_1 + p_2$ is non-zero, the two-pion energy E_L is also shifted by the two-pion interaction as in the CM system. The relative momentum of two-pion in the CM system is given by the two-pion energy and total momentum as $p^2 = (E_L^2 - P^2)/4 - m_\pi^2 = (2\pi/L)^2\overline{m}$. The phase shift in the CM system is now $\tan\delta(p) = \gamma\pi^{3/2}\sqrt{\overline{m}}/Z_{00}^P(1;\overline{m})$ where $\gamma = E_L/\sqrt{E_L^2 - P^2}$ and $\sqrt{4\pi}Z_{00}^P(i;\overline{m}) = \sum_{\vec{r}}(r^2 - \overline{m})^{-i}$. Here the summation is taken over the vectors $\vec{r}$ that are constructed as $\vec{r} = \hat{\gamma}^{-1}(\vec{l} + \vec{d}/2)$ where $\vec{d} = L\vec{P}/2\pi$.

The pion four-point function behaves as a sum of exponentials due to the presence of a number of states having the same quantum numbers. To resolve the energy eigenvalues, we construct in each system the pion four-point function matrix $C_{nm}(t) = \langle 0|\Omega_n(t)\Omega_m(t_S)|0\rangle$, where t_S is the source point and $\Omega_n(t) = \pi(p_1, t)\pi(p_2, t)$ is the two-pion operator with momenta p_i that depends on the state n. The energy eigenvalues $\lambda_n(t) = \exp[-E_n(t - t_0)]$ are obtained by diagonalizing the matrix $C^{-1/2}(t_0)C(t)C^{-1/2}(t_0)$ with t_0 a reference time [8]. A cut-off N is introduced in the number of the operators and energy states considered.

We work with the CM and two laboratory systems, L1 and L2. The sums of the two pion momenta in L1 and L2 are $(1,0,0)$ and $(1,1,0)$ in $2\pi/L$ unit, respectively. We choose the operator cut-off $N = 3$ in the CM system and 4 in the laboratory systems. In each system, 3×3 or 4×4 four-point function matrices are diagonalized and the phase shift is determined from the energies of the ground ($n = 0$)

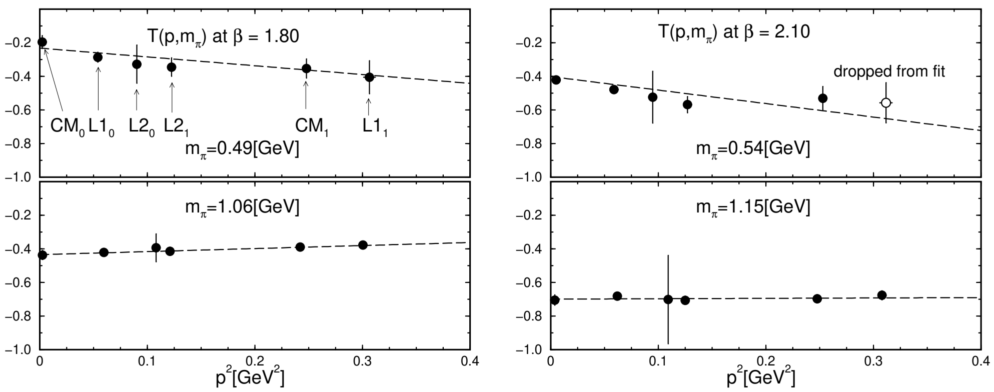

Fig. 2. Scattering amplitudes $T(p, m_\pi) = E/(2p) \cdot \tan\delta(p)$ for the heaviest and lightest pion masses at $\beta = 1.80$ (left panels) and 2.10 (right panels). The dashed lines show the global polynomial fit.

and the first excited ($n = 1$) states. The other states are used to investigate the contamination of the higher states.

Figure 1 illustrates the effect of diagonalization in the CM system. We compare the ratio $R_n(t) = C_{nn}(t)/G_n^2(t)$, where $G_n(t) = \langle 0|\pi(p_n, t)\pi(-p_n, t_S)|0\rangle$ with $p_n^2 = (2\pi/L)^2 n$, before the diagonalization (denoted by open symbols) to $W_n(t) = \lambda_n(t)/G_n^2(t)$ after the diagonalization (closed symbols) normalized at $t = t_S$. For the ground state, $W_0(t) \simeq R_0(t)$ so that the diagonalization is unnecessary. For the first excited state, however, the ratio shows an exponential fall-off only after diagonalization: $R_1(t)$ blows up at large t. We apply an exponential fit to $W_1(t)$. The energy eigenvalues for the $n = 0$ and 1 states change very little even if we adopt the cut-off $N = 2$ instead of 3 and 4.

3. Result for phase shift

We define the 'scattering amplitude' as $T(p, m_\pi) = E/(2p) \cdot \tan\delta(p)$ to extrapolate the phase shift to the physical pion mass $m_\pi = 0.14$ GeV and the continuum limit. This gives the scattering length a_0 in the zero-momentum limit: $\lim_{p\to 0} T(p, m_\pi) = a_0 m_\pi$.

Figure 2 shows the scattering amplitude at $\beta = 1.80$ and 2.10 for the lightest and heaviest pion masses. The CM_n refers to the amplitude obtained from the n-th state in the CM system, and the $\mathrm{L1}_n$ and $\mathrm{L2}_n$ to those in the laboratory systems.

We carry out a global fit to the amplitude at each β with a polynomial $T(p, m_\pi) = A_{10}m_\pi^2 + A_{20}m_\pi^4 + A_{30}m_\pi^6 + A_{01}p^2 + A_{11}m_\pi^2 p^2 + A_{21}m_\pi^4 p^2$. Open symbol in Fig. 2 is excluded from the fit, since the $W_n(t)$ does not show a good exponential behavior. The fit results are also shown in the figure. We find a reasonable fit to the data for the both CM and laboratory systems.

The continuum limit is taken by extrapolating the fits linear in a to the scattering amplitude at the physical pion mass for fixed momentum, $T(p, m_\pi) = A + aB$.

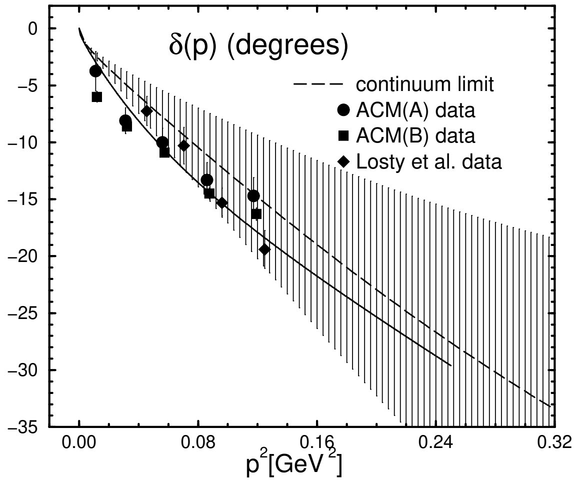

Fig. 3. Phase shifts in the continuum limit (indicated by the dashed curve). The band of bars shows our estimate of errors. The solid curve is parametrized by experimental inputs [9] and the symbols are the experimental data [10,11].

The phase shift in the continuum limit is presented in Fig.3 with the dashed line, associated by a band of error bars. This is compared to the experimental data. The scattering length obtained by taking the zero-momentum limit of $T(p, m_\pi)$ in the continuum, $a_0 m_\pi = -0.0484(49)$, may be compared with the prediction of chiral perturbation theory: $a_0 m_\pi = -0.0444(10)$ [9].

To check the reliability of the result we need to investigate the volume dependence, and the chiral behavior and the dependence of the lattice spacing more precisely. It is more important to apply the calculation to other scatterings and decays of hadrons.

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