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PLANCK MASS PLASMA ANALOG OF STRING THEORY

F. WINTERBERG *

Department of Physics, University of Nevada, M/S 220,
Reno, Nevada 89557-0058, USA

In recent years there had been a growing interest in analog models of general relativity, with certain superfluid solutions simulating black hole solutions of Einstein's gravitational field equation. The quantization of a superfluid, composed of discrete particles (helium atoms), treated as a nonrelativistic many body problem does not lead to divergencies as the quantization of Einstein's field equations. Quantization of gravity is possible in string theory, but only if one introduces the daring hypothesis of higher dimensions. But if the gravitational field is made up of discrete elements as superfluid helium is made up of helium atoms, then gravity can be quantized without difficulty in three space and one time dimension. Such a hypothesis, of course, implies that Lorentz invariance is a dynamic symmetry caused by real rod and clock deformations, as it was assumed in the pre-Einstein theory of relativity by Lorentz and Poincaré, which required the existence of an aether. Making the hypothesis that this aether is a kind of superfluid plasma made up of positive and negative Planck mass particles interacting with the Planck force over a Planck length, one obtains an analog of the standard model, including gravity, which can be quantized as a nonrelativistic many body problem. In this model nonrelativistic vortex rings in three space dimensions and one time dimension simulate the relativistic theory of closed strings in ten space-time dimensions. But because in the vortex lattice, one obtains a large dimensionless number conceivably advancing our understanding of the finestructure constant.

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1. Introduction

Because of the impossibility to carry out laboratory experiments with black holes, there has been in recent years a growing interest to simulate black holes in the laboratory with condensed matter physics analogues of general relativity, in particular analogues offered by the physics of superfluid helium. The connection between these two very different areas of physics is that the propagation of phonons can be described by an effective metric. These models, though, require that Lorentz invariance must be understood as a dynamic symmetry, as in the pre-Einstein theory of relativity by Lorentz and Poincaré. It assumed the existence of an aether, which in the condensed matter analogue is the superfluid.

In my talk I will try to go a crucial step beyond these models, making the case that they may tell us something more, namely a possible novel interpretation of

*University of Nevada, Reno

general relativity and ultimately of string theory, with Einstein's nonlinear gravitational field equation as an effective field theory. Since the analogue is based on the nonrelativistic many body problem of a superfluid, the quantization cannot lead to any divergencies. This raises the question, if this can give an analogue of string theory, where the divergencies can be avoided as well. The connection with string theory is made is one identifies the quantized vortices of a superfluid with strings, albeit not in nine spaces plus one time dimension, but in the three space plus one time dimension of ordinary physics.

Generating a vortex tangle in superfluid helium, one should then be able to simulate solutions of general relativity in the laboratory. This experimental simulation, however, fails for supersymmetry where there is no condensed matter physics analogue, but one can ask what property the analogue would have to posses to simulate supersymmetry. It is here where the hypothesis of negative masses enters.

2. Quantized Vortex Analog Models

To reach a deeper insight for the description of gravity by a condensed matter physics analogue, we start from the microscopic theory of a bosonic quantum fluid like superfluid helium. Assuming that in between the atoms of the fluid there are short range repulsive forces with a delta function potential [5,6]

$$V(r) = g\delta(r) \tag{1}$$

Where g is a coupling constant, the operator field equation of the quantum fluid is given by

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + g\psi^\dagger\psi\psi \tag{2}$$

With the Hartree approximation

$$< \psi^\dagger\psi\psi > \rightarrow |\varphi|^2\varphi \tag{3}$$

the nonlinear operator field equation (2), becomes the nonlinear Schrödinger equation

$$i\hbar\frac{\partial\varphi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\varphi + g|\varphi|^2\varphi \tag{4}$$

which by the Madelung transformation is brought into the hydrodynamic form

$$\begin{aligned}\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v}\cdot\nabla)\boldsymbol{v} &= -\frac{1}{m}\nabla(gn - \frac{\hbar^2}{2m}\frac{\nabla^2\sqrt{n}}{\sqrt{n}}) \\ \frac{\partial n}{\partial t} + \nabla\cdot(n\boldsymbol{v}) &= 0\end{aligned} \tag{5}$$

where

$$Q = -\frac{\hbar^2}{2m}\frac{\nabla^2\sqrt{n}}{\sqrt{n}} \tag{6}$$

is called the quantum potential.

From the two equations (5) and neglecting the quantum potential one obtains for small amplitudes the scalar phonon wave equation

$$-\frac{\partial^2 \boldsymbol{v}}{\partial t^2} + \frac{g n_0}{m}\ grad\ div\ \boldsymbol{v} = 0 \tag{7}$$

In this equation n_0 is the atomic number density of the undisturbed quantum fluid, with the wave velocity given by

$$a = \sqrt{\frac{g n_0}{m}} \tag{8}$$

A second fundamental solution of (5) is a potential vortex for which $curl\ \boldsymbol{v} = 0$, and where

$$\begin{aligned} v_\varphi &= a(\frac{r_0}{r}), \qquad r > r_0 \\ v_\varphi &= 0, \qquad\qquad r < r_0 \end{aligned} \tag{9}$$

with $v_\varphi = a$ at the vortex core radius $r = r_0$. With the quantization condition for line vortices

$$m r v_\varphi = m r_0 a = \hbar \tag{10}$$

one can write for (9)

$$\begin{aligned} v_\varphi &= \frac{\hbar}{m}\frac{1}{r}, \qquad r > r_0 \\ v_\varphi &= 0, \qquad\quad r < r_0 \end{aligned} \tag{11}$$

Then, with the identity $(\boldsymbol{v} \cdot \nabla)\boldsymbol{v} = grad\ (\frac{v^2}{2}) - \boldsymbol{v} \times curl\ \boldsymbol{v}$ and with $curl\ \boldsymbol{v} = 0$, one obtains from the first equation of (5), and as before neglecting the quantum potential,

$$\frac{v^2}{2} + (\frac{g}{m})n = const. \tag{12}$$

or since for $r \to \infty$, $v \to 0$ and $n = n_0$

$$\frac{v^2}{2} = \frac{g}{m}(n_0 - n) \tag{13}$$

Setting $v^2 = v_\varphi^2$ and making use of (9) one obtains

$$n/n_0 = 1 - (1/2)(r_0/r)^2 \tag{14}$$

The quantum potential at the vortex core radius $r = r_0 = \hbar/ma$ is of the order

$$|Q| \sim \frac{\hbar^2}{2 m r_0^2} = \frac{1}{2} m a^2 \tag{15}$$

It describes the zero point quantum fluctuations near the vortex core, which have the energy density

$$\varepsilon \sim \frac{|Q|}{r_0^3} = \frac{m a^3}{2 r_0^3} \tag{16}$$

These fluctuations set up a virtual phonon field surrounding the line vortex, which thereby becomes the source of a an attractive force field with a field strength f, where $f^2 \sim \varepsilon$. We compare it with the square of the field strength for a Newtonian gravitational force field of mass m at a distance $r = r_0$

$$f^2 = \frac{Gm^2}{r_0^4} \tag{17}$$

Equating (16) and (17), assuming that $a = c$, and hence $r_0 = \hbar/mc$, we find that

$$Gm^2 = \hbar c \tag{18}$$

This means that $m = m_p$, where

$$m_p = \sqrt{\frac{\hbar c}{G}} \tag{19}$$

is the Planck mass, and where because of $r_0 = \hbar/m_p c$, r_0 is equal to the Planck length

$$r_p = \sqrt{\frac{\hbar G}{c^3}}$$

This result explains the phenomenon of "charge," in this case the gravitational charge, to have its cause in the zero point fluctuations of Planck mass particles bound in vortex filaments with the vortex core radius equal the Planck length.

With $a = c$ one then obtains from (8) that

$$g = \frac{m_p c^2}{n_0} \tag{20}$$

or since in the condensed state $n_0 = 1/r_p^3$, that $g = m_p c^2 r_p^3 = \hbar c r_p^2$ whereby (2) becomes

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m_p} \nabla^2 \psi + \hbar c r_p^2 \psi^\dagger \psi \psi \tag{21}$$

We, therefore, arrive at the important conclusion that for the superfluid to have phonons with luminal velocity, and to have vortex filaments coupled to these phonons with a force equal to the force of Newton's law, the fluid must consist of Planck mass particles.

With the phonon velocity equal the velocity of light, Lorentz invariance follows as a dynamic symmetry. And with the gravitational coupling strength explained by the field of virtual phonons set up by the zero point fluctuations of the Planck mass particles bound in quantized vortex filaments, a close relationship between Lorentz invariance, quantum mechanics and gravity is demonstrated.

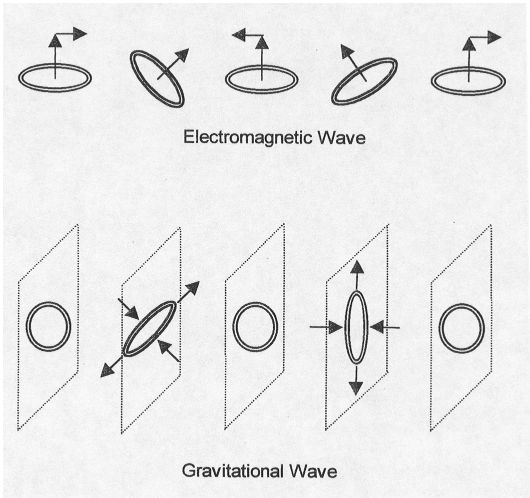

Fig. 1. Deformation of the vortex lattice for an electromagnetic and a gravitational wave

3. The Vortex Sponge Analog of Maxwell's and Einstein's Field Equations

In the general theory of relativity, gravity is described by a second rank symmetric tensor. A field with this property can be modeled by a tangle of a large number of vortex filaments, or equivalently by a lattice of vortex rings. As Kelvin had shown [7], such a lattice can propagate transverse waves mimicking electromagnetic waves, but in addition it can also propagate transverse wave mimicking a gravitational waves [4]. The deformation of the vortex lattice for electromagnetic and gravitational waves is shown in Fig.1.

In this model gravitational waves are described as highly nonlinear vortex waves. There the general theory of relativity is only an approximation for energies small compared to the Planck energy. For smaller energies these waves can be derived from a nonlinear field theory in flat space-time, which as Gupta has shown is equivalent to Einstein's theory in a curved space-time for a special gauge [8]. For wavelengths not small compared to the Planck length, the hydrodynamic nonlinearities of large amplitude vortex waves cannot be modeled by a curved space-time. In the vortex sponge analogue, the gravitational wave has two nonlinearities, first, as in Einstein's theory where the energy momentum tensor of the gravitational field is a source of this field, and second, for small wavelengths where the microscopic structure of the vortex wave must be taken into account. The latter type of nonlinearity also occurs for the electromagnetic waves.

4. Extension of the Model to Include Fermionic Matter

Because the model given by (21) cannot explain Dirac spinors, and thus cannot describe ordinary matter composed of fermions, it cannot be complete. Holding firm to Planck's conjecture that all of physics should be reduced to equations containing as free parameters only the Planck length, mass and time, the only freedom left is to introduce besides positive, also negative Planck mass particles. The introduction of negative masses is not possible in a relativistic theory where the particle number operator does not commute with the Hamilton operator, but it is possible in an exactly nonrelativistic theory.

Going beyond the simple model described by equation (21) I had suggested as the fundamental equation [4]

$$i\hbar \frac{\partial \psi_\pm}{\partial t} = \mp \frac{\hbar^2}{2m_p} \nabla^2 \psi_\pm \pm 2\hbar c r_p^2 (\psi_\pm^\dagger \psi_\pm - \psi_\mp^\dagger \psi_\mp) \psi_\pm \tag{22}$$

It assumes that the vacuum is occupied by an equal number of positive and negative Planck mass particles, with repulsive short-range forces between particles of equal sign, and attractive forces between those of opposite sign. Since during the interaction between particles of opposite sign, the momentum, not the energy, fluctuates, momentum conservation is violated. With $\Delta q \sim r_p$ and $\Delta p \sim m_p c$, Heisenberg's relation $\Delta p \Delta q \sim \hbar$ is thereby recovered at the most fundamental microscopic level. Furthermore, as Schrödinger [9] had shown in his interpretation of the Dirac equation as a "Zitterbewegung," caused by in the negative energy states of the Dirac equation, negative masses are hidden in the Dirac equation. But if this is so, then the Dirac equation should, and can in fact, be derived from the assumption of the existence of negative masses, as it was shown by Hönl and Bopp [10,11].

In the Hartree-Fock approximation (22) the interaction between equal sign Planck mass particles becomes twice as large as the interaction between Planck mass particles of opposite sign. And in making thereafter the Madelung transformation, one arrives at two coupled equations describing two slightly interacting superfluids, with wave-like and vortex-like solutions for both of them.

5. The Dimension of the Vortex Lattice Cell

In a two dimensional lattice of line vortices, realized in the von Karman vortex street, for a lattice to be stable requires that [12]

$$\frac{r_0}{l} = 3 \times 10^{-3} \tag{23}$$

where r_0 is the vortex core radius and l the distance in between two adjacent line vortices. With $R = l/2$ as the radius of the vortex lattice cell, one has

$$\frac{R}{r_0} \simeq 147 \tag{24}$$

To my knowledge no comparable stability calculation has been made for three-dimensional lattice of vortex rings, but one can there estimate a value for R/r_0 by the fact that the fluid velocity of a vortex ring at the distance R is larger by the factor $\log(\frac{8R}{r_0})$. Therefore, setting $r_0 = r_p$, one obtains a value for $\frac{R}{r_p}$, by solving for the equation

$$\frac{R}{r_p} \simeq 147 \log(\frac{8R}{r_p}) \tag{25}$$

$\frac{R}{r_p}$ with the result that [4]

$$\frac{R}{r_p} \simeq 1360 \tag{26}$$

6. Ring Vortex Resonance Energy

In the model described by equation (22), a ring vortex of radius R and core radius r_p has a resonance frequency against elliptic deformations of the ring given by

$$\omega_v \simeq \frac{c r_p}{R^2} \tag{27}$$

If quantized, this resonance leads to two energies, one for the vortex formed by the positive, and a second one by the negative mass component of Planck mass particle medium. Hence, for the energy

$$\hbar\omega_v = \pm m_p c^2 (\frac{r_p}{R})^2 \tag{28}$$

one has the two quasiparticle mass components

$$m_\pm = \pm m_p (\frac{r_p}{R})^2 \tag{29}$$

With the value $\frac{R}{r_p} = 1360$ one finds that $m_\pm \simeq \pm 5 \times 10^{12}$ GeV.

The existence of negative masses leads to the generation of positive masses by the positive gravitational field energy of a positive mass interacting with a negative mass. For a positive-negative mass dipole this energy is

$$E_{in} = \frac{G|m_\pm|^2}{r} \tag{30}$$

where the gravitational force comes from the attractive virtual phonon field set up by the zero point fluctuations of Planck mass particles bound in the quantized vortex filaments. With quantum mechanics requiring that

$$|m_\pm| r c = \hbar \tag{31}$$

and by setting $E_{in} = mc^2$, r can be eliminated from (30) and (31), with the result that

$$m = \frac{G|m_\pm|^3}{\hbar c} = \frac{|m_\pm|^3}{m_p^2} \tag{32}$$

or

$$\frac{m}{m_p} = (\frac{|m_\pm|}{m_p})^3 = (\frac{r_p}{R})^6 \tag{33}$$

For $\frac{R}{r_p} = 1360$ one has $\frac{m}{m_p} \simeq 2 \times 10^{-19}$ where m turns out to be about equal the typical mass of a baryon like the mass of the proton.

Inserting (33) into the expression for the strong nuclear force obtained by Wilczek [13].

$$\frac{m}{m_p} = e^{\frac{-k}{\alpha}} \tag{34}$$

where α is the finestructure constant at the grand unification scale, and where $k = \frac{11}{2\pi}$ is calculated from the antiscreening of the strong force, one finds that

$$\frac{1}{\alpha} = \frac{2\pi}{11} \log(\frac{m_p}{m}) = \frac{12\pi}{11} \log(\frac{R}{r_p}) \tag{35}$$

For $\frac{R}{r_p} = 1360$, this becomes

$$\frac{1}{\alpha} \simeq 24.8 \tag{36}$$

in surprisingly in good agreement with the empirical value $1/\alpha = 25$ given by Wilczek [13].

Our model, therefore, suggests that the large nondimensional numbers in elementary particle physics, like the finestructure constant, have their origin in the large nondimensional numbers of classical fluid dynamics.

7. Conclusion

In pursing the analogies between condensed matter physics, in particular superfluid condensed matter physics, many old problems, like quantum gravity (still at the forefront of fundamental physics research), appear in a new light and offer surprisingly different solutions. Furthermore, the analogies between general relativity and the vortex wave solutions of a superfluid make the study of these solutions accessible to experiments. However, an unsurpassable barrier for such analogies is supersymmetry, in the model related to the hypothetical existence of hidden negative masses. Most important though, is that these models offer a completely “finitistic,” that is free of all divergences, unified field theory of elementary particles in the three space and one time dimension of all physics laboratories.

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