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COLLINEAR AND SOFT RESUMMATION IN THE LARGE-X LIMIT

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I discuss general unified formulas for resumming collinear and soft contributions to QCD hard scattering cross sections at large x. Expansions of the resummed cross sections to next-to-next-to-next-to-leading order are also shown along with applications of the formalism.

Keywords: collinear gluons; soft gluons; resummation.

1. Introduction

The calculation of cross sections in perturbative QCD involves the convolution of hard-scattering factors, which are perturbatively calculable, with parton distribution functions. Near threshold for production of any given system of particles the restricted phase space for real gluon emission results in large logarithms. These soft and collinear corrections[1] appear in the form of logarithmic plus distributions, $\mathcal{D}_l(x) \equiv [\ln^l(1-x)/(1-x)]_+$, where $x \to 1$ at threshold. There are also purely collinear corrections of the form $\ln^l(1-x)$.

If we define moments of the cross section by $\hat{\sigma}(N) = \int dx\, x^{N-1}\hat{\sigma}(x)$ then the corrections take the form $[\ln^{2n-1}(1-x)/(1-x)]_+ \to \ln^{2n} N$ and $\ln^n(1-x) \to \ln^n N/N$. We can formally resum these logarithms in moment space to all orders in α_s by factorizing soft and collinear gluons from the hard scattering.[2,3]

Inverting back to momentum space and expanding the resummed cross section at fixed order, we find at NLO soft and collinear corrections involving $\mathcal{D}_1(x)$ and $\mathcal{D}_0(x)$ terms. At NNLO, we have $\mathcal{D}_3(x)$, $\mathcal{D}_2(x)$, $\mathcal{D}_1(x)$, and $\mathcal{D}_0(x)$ terms.

A unified approach and a master formula for calculating these distributions at NNLO for any process in hadron-hadron and lepton-hadron colliders, for both total and differential cross sections, was presented in Ref. 4. Results are now available for many processes, including top pair[5] and FCNC single-top[6] production, (charged) Higgs[7] production, W-boson[8] and direct photon[9,10] production, and jet production.[11] Regarding the purely collinear corrections, at NLO we have $\ln(1-x)$ and constant terms, while at NNLO we have $\ln^3(1-x)$, $\ln^2(1-x)$, $\ln(1-x)$, and constant terms.

2. Soft and collinear resummation: A unified approach

The resummed cross section in moment space takes the form[4]

$$\hat{\sigma}^{res}(N) = \exp\left[\sum_i E_i(N) + E_i^{coll}(N)\right] \, \exp\left[\sum_j E'_j(N) + {E'}_j^{coll}(N)\right]$$
$$\times \, \exp\left[\sum_i 2\int_{\mu_F}^{\sqrt{s}} \frac{d\mu'}{\mu'}\left(\frac{\alpha_s(\mu'^2)}{\pi}\gamma_i^{(1)} + {\gamma'}_{i/i}(\mu'^2)\right)\right] \times \exp\left[2\, d_{\alpha_s}\int_{\mu_R}^{\sqrt{s}} \frac{d\mu'}{\mu'}\beta(\mu'^2)\right]$$
$$\times \mathrm{Tr}\left\{H(\mu_R^2)\exp\left[\int_{\sqrt{s}}^{\sqrt{s}/\tilde{N}} \frac{d\mu'}{\mu'}\Gamma_S^\dagger(\mu'^2)\right] S(s/\tilde{N}^2)\exp\left[\int_{\sqrt{s}}^{\sqrt{s}/\tilde{N}} \frac{d\mu'}{\mu'}\Gamma_S(\mu'^2)\right]\right\}, \quad (1)$$

where i and j denote incoming and outgoing massless partons, respectively. In the $\overline{\mathrm{MS}}$ scheme, which we will use here,

$$E_i(N) = -\int_0^1 dz \frac{z^{N-1}-1}{1-z}\left\{\int_{(1-z)^2 s}^{\mu_F^2} \frac{d\mu'^2}{\mu'^2} A_i\left(\alpha_s(\mu'^2)\right) + \nu_i\left[\alpha_s((1-z)^2 s)\right]\right\} \quad (2)$$

with $A_i(\alpha_s) = C_i\left[\alpha_s/\pi + (\alpha_s/\pi)^2 K/2\right] + A_i^{(3)} + \cdots$, $\nu_i = (\alpha_s/\pi)C_i + (\alpha_s/\pi)^2\nu_i^{(2)} + \cdots$ and, for any massless final-state partons at lowest order,

$$E'_j(N) = \int_0^1 dz \frac{z^{N-1}-1}{1-z}\left\{\int_{(1-z)^2}^{1-z} \frac{d\lambda}{\lambda} A_j(\lambda s) - B'_j((1-z)s) - \nu_j((1-z)^2 s)\right\} \quad (3)$$

where $B'_j = (\alpha_s/\pi){B'}_j^{(1)} + (\alpha_s/\pi)^2 {B'}_j^{(2)} + \cdots$ with ${B'}_q^{(1)} = 3C_F/4$ and ${B'}_g^{(1)} = \beta_0/4$.

The collinear exponents for incoming and outgoing massless partons are, respectively, $E_i^{coll}(N) = \int_0^1 dz\, z^{N-1}\left\{\int_{(1-z)^2 s}^{\mu_F^2}(d\mu'^2/\mu'^2)A_i(\alpha_s(\mu'^2)) + \cdots\right\}$ and ${E'}_j^{coll}(N) = -\int_0^1 dz\, z^{N-1}\left\{\int_{(1-z)^2}^{1-z}(d\lambda/\lambda)A_j(\alpha_s(\lambda s)) + \cdots\right\}$.

The γ's are parton anomalous dimensions, H are hard scattering matrices, and S are soft matrices that describe noncollinear soft-gluon emission whose evolution is given by the soft anomalous dimension matrices Γ_S.[2,3,4]

If we expand the resummed cross section, Eq. (1), to fixed order, and then invert to momentum space, we get master formulas for the NLO, NNLO, and higher-order corrections. At NLO the master formula for soft and collinear corrections is[4]

$$\hat{\sigma}^{(1)} = \sigma^B \frac{\alpha_s}{\pi}\left\{c_3\mathcal{D}_1(x) + c_2\mathcal{D}_0(x)\right\} + \frac{\alpha_s^{d_{\alpha_s}+1}}{\pi} A^c \mathcal{D}_0(x) \quad (4)$$

where σ^B is the Born term, α_s is at scale μ_R, $c_3 = \sum_i 2C_i - \sum_j C_j$, with $C_F = (N_c^2-1)/(2N_c)$ for quarks and $C_A = N_c$ for gluons,

$$c_2 = -\sum_i\left[C_i + C_i\ln\left(\frac{\mu_F^2}{s}\right)\right] - \sum_j\left[{B'}_j^{(1)} + C_j + C_j\ln\left(\frac{M^2}{s}\right)\right], \quad (5)$$

with M a hard scale, and $A^c = \mathrm{tr}(H^{(0)}\Gamma_S^{(1)\,\dagger}S^{(0)} + H^{(0)}S^{(0)}\Gamma_S^{(1)})$. We can also calculate the purely collinear terms from the expansion. As an example, for the Drell-Yan process, $q\bar{q} \to V$, whose cross section is known to NNLO,[12] we find

$$\hat{\sigma}^{(1)} = \sigma^B \frac{\alpha_s}{\pi} \left\{ 4C_F \left[\frac{\ln(1-x)}{1-x}\right]_+ - 2C_F \ln\left(\frac{\mu_F^2}{Q^2}\right) \left[\frac{1}{1-x}\right]_+ - 4C_F \ln(1-x) \right\}. \quad (6)$$

At NNLO the master formula for the soft and collinear corrections is[4]

$$\hat{\sigma}^{(2)} = \sigma^B \frac{\alpha_s^2(\mu_R^2)}{\pi^2} \frac{1}{2} c_3^2 \, \mathcal{D}_3(x) + \sigma^B \frac{\alpha_s^2(\mu_R^2)}{\pi^2} \left\{ \frac{3}{2} c_3 c_2 - \frac{\beta_0}{4} c_3 + \sum_j C_j \, \frac{\beta_0}{8} \right\} \mathcal{D}_2(x)$$
$$+ \frac{\alpha_s^{d_{\alpha_s}+2}(\mu_R^2)}{\pi^2} \frac{3}{2} \, c_3 \, A^c \, \mathcal{D}_2(x) + \cdots \quad (7)$$

where here we show only the leading and next-to-leading logarithms. We note that two-loop calculations[13] are needed to get all soft logarithms at NNLO.

We can continue this procedure to higher orders. At next-to-next-to-next-to-leading order (NNNLO) the master formula is

$$\hat{\sigma}^{(3)} = \sigma^B \frac{\alpha_s^3(\mu_R^2)}{\pi^3} \frac{1}{8} \, c_3^3 \, \mathcal{D}_5(x)$$
$$+ \sigma^B \frac{\alpha_s^3(\mu_R^2)}{\pi^3} \left\{ \frac{5}{8} \, c_3^2 \, c_2 - \frac{5}{2} \, c_3 \, X_3 \right\} \mathcal{D}_4(x) + \frac{\alpha_s^{d_{\alpha_s}+3}(\mu_R^2)}{\pi^3} \frac{5}{8} \, c_3^2 \, A^c \, \mathcal{D}_4(x) + \cdots \quad (8)$$

where $X_3 = (\beta_0/12)c_3 - \sum_j C_j \beta_0/24$, and again we only show explicitly the leading and next-to-leading logarithms.

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