HIGH ENERGY ASTROPHYSICS TESTS OF LORENTZ INVARIANCE VIOLATION

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Observations of the multi-TeV spectra of the Mkn 501 and other nearby BL Lac objects exhibit the high energy cutoffs predicted to be the result of intergalactic annihilation interactions, primarily with IR photons having a flux level as determined by various astronomical observations. After correcting for such intergalactic absorption, these spectra can be explained within the framework of synchrotron self-Compton emission models. Stecker and Glashow have shown that the existence of this annihilation via electron-positron pair production puts strong constraints on Lorentz invariance violation. Such constraints have important implications for some quantum gravity and large extra dimension models. A much smaller amount of Lorentz invariance violation has potential implications for understanding the spectra of ultrahigh energy cosmic rays.

Keywords: relativity; quantum gravity; cosmic rays.

1. Consequences of Breaking of Lorentz Invariance

It has been suggested that Lorentz invariance (LI) may be only an approximate symmetry of nature [1]. Although no true quantum theory of gravity exists, it was independently proposed that LI might be violated in such a theory with astrophysical consequences [2]. A simple formulation for breaking LI by a small first order perturbation in the electromagnetic Lagrangian which leads to a renormalizable treatment has been given by Coleman and Glashow [3]. Using this formalism, these authors point out that with LI violation (LIV), different particles can have maximum attainable velocities (MAVs) which can be different from c. Using the formalism of Ref. 3, we denote maximum attainable velocity (MAV) of a particle of type i (not necessarily equal to $c \equiv 1$) by c_i. We futher define the difference $c_i - c_j \equiv \delta_{ij}$ and specifically here $c_{e\gamma} \equiv \delta << 1$. These definitions will be used to discuss the physics implications of cosmic ray and cosmic γ-ray observations[4,5,6].

If $\delta < 0$, the decay of a photon into an electron-positron pair is kinematically allowed for photons with energies exceeding $E_{\max} = m_e \sqrt{2/|\delta|}$. This decay would take place rapidly, so that photons with energies exceeding $E_{\max}$ could not be observed either in the laboratory or as cosmic rays. Since photons have been observed with energies $E_\gamma \geq 50$ TeV from the Crab nebula [7], this implies that $E_{\max} \geq 50$ TeV, or that $|\delta| < 2 \times 10^{-16}$.

If, on the other hand, $\delta > 0$, electrons become superluminal if their energies exceed $E_{\max}/\sqrt{2}$. Electrons traveling faster than light will emit light at all frequencies by a process of 'vacuum Čerenkov radiation.' The electrons then would rapidly lose energy until they become subluminal. Because electrons have been seen in the cosmic radiation with energies up to ~ 2 TeV[8], it follows that $\delta < 3 \times 10^{-14}$. A smaller, but more indirect, upper limit on δ for the $\delta > 0$ case can be obtained from theoretical considerations of γ-ray emission from the Crab Nebula. Its emission above 0.1 GeV, extending into the TeV range, is thought to be Compton emission of the same relativistic electrons which produce its synchrotron radiation at lower energies[9]. The Compton component, extends to 50 TeV and thus implies the existence of electrons having energies at least this great in order to produce 50 TeV photons, even in the extreme Klein-Nishina limit. This indirect argument, based on the reasonable assumption that the 50 TeV γ-rays are from Compton interactions, leads to a smaller upper limit on δ, *viz.,* $\delta < 10^{-16}$. A further constraint on δ for $\delta > 0$ follows from a change in the threshold energy for the pair production process $\gamma + \gamma \rightarrow e^{+} + e^{-}$. This follows from the fact that the square of the four-momentum is changed to give the threshold condition

$$2\epsilon E_{\gamma}(1 - cos\theta) \;-\; 2E_{\gamma}^{2}\delta \;\geq\; 4m_{e}^{2},$$

where ϵ is the energy of the low energy photon and θ is the angle between the two photons. The second term on the left-hand-side comes from the fact that $c_{\gamma} = \partial E_{\gamma}/\partial p_{\gamma}$. It follows that the condition for a significant increase in the energy threshold for pair production is $E_{\gamma}\delta/2 \geq m_{e}^{2}/E_{\gamma}$, or equivalently, $\delta \geq 2m_{e}^{2}/E_{\gamma}^{2}$. The γ-ray spectrum of the active galaxy Mkn 501 while flaring extended to $E_{\gamma} \geq 24$ TeV [10] and exhibited the high energy absorption expected from γ-ray annihilation by extragalactic pair-production interactions with extragalactic infrared photons[11], [12]. This has led Stecker and Glashow [4] to point out that the Mkn 501 spectrum presents evidence for pair-production with no indication of Lorentz invariance violation (LIV) up to a photon energy of ~ 20 TeV and to thereby place a quantitative constraint on LIV given by $\delta < 2m_{e}^{2}/E_{\gamma}^{2} \simeq 10^{-15}$. This constraint on positive δ is more secure than the smaller, but indirect, limit given above.

2. Constraints on Quantum Gravity and Extra Dimension Models

LIV has been proposed consequence of quantum gravity physics at the Planck scale $M_{Planck} = \sqrt{\hbar c/G} \simeq 1.22 \times 10^{19}$ GeV, [13], [14]. In models involving large extra dimensions, the energy scale at which gravity becomes strong can occur at a scale, $M_{QG} << M_{Planck}$, even approaching a TeV [15]. In the most commonly considered case, the usual relativistic dispersion relations between energy and momentum of the photon and the electron are modified[2], [14] by a term of order p^{3}/M_{QG}.[a]

[a]We note that there are variants of quantum gravity and large extra dimension models which do not violate Lorentz invariance and for which the constraints considered here do not apply. There are also variants for which there are no cubic terms in momentum, but rather much smaller quartic terms of order $\sim p^{4}/M_{QG}^{2}$.

Generalizing the LIV parameter δ to an energy dependent form

$$\delta \equiv \frac{\partial E_e}{\partial p_e} - \frac{\partial E_\gamma}{\partial p_\gamma} \simeq \frac{E_\gamma}{M_{QG}} - \frac{m_e^2}{2E_e^2} - \frac{E_e}{M_{QG}}, \quad (1)$$

the threshold condition from pair production implies $M_{QG} \geq E_\gamma^3/8m_e^2$. Since pair production occurs for energies of at least 20 TeV, we find a constraint on the quantum gravity scale[5] $M_{QG} \geq 0.3 M_{Planck}$. This constraint contradicts the predictions of some proposed quantum gravity models involving large extra dimensions and smaller effective Planck masses. In a variant model of Ref. 21, the photon dispersion relation is changed, but not that of the electrons. In this case, we find the even stronger constraint $M_{QG} \geq 0.6 M_{Planck}$.

Within the context of a more general cubic modification of the dispersion relations, Jacobson, *et al.*[17] obtained an indirect limit on M_{QG} from the apparent cutoff in the synchrotron component of the in the Crab Nebula γ-ray emission at ~ 0.1 GeV. However, their very strong constraint, $M_{QG}{>}1.2 \times 10^7 M_{Planck}$, is qualified by considerations of electron helicity and photon polarization[18] and is thus not as general as the constraint from photon-photon pair-production. Also, for the model suggested in 21, this constraint does not hold.

3. LIV and the Ultrahigh Energy Cosmic Ray Spectrum

Coleman and Glashow [3] have shown that for interactions of protons with CBR photons of energy ϵ and temperature $T_{CBR} = 2.73K$, pion production is kinematically forbidden and thus *photomeson interactions are turned off* if

$$\delta_{p\pi} > 5 \times 10^{-24} (\epsilon / T_{CBR})^2.$$

Thus, given even a very small amount of LIV, photomeson and pair-production interactions of UHECR with the CBR can be turned off. Such a violation of Lorentz invariance might be produced by Planck scale effects[19], [20]. If Lorentz invariance violation is the explanation for the missing GZK effect, indicated in the *AGASA* data, but not the *HiRes* data (see Fig. 1)[6]), one can also look for the absence of a "pileup" spectral feature and for the absence of photomeson neutrinos, but these may be more difficult to detect.

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References

1. H. Sato and T. Tati, *Prog. Theor. Phys.* **47**, 1788 (1972).
2. G. Amelino-Camelia *et al.*, *Nature* **393**, 763 (1998).
3. S. Coleman and S.L. Glashow, *Phys. Rev.* **D59**, 116008 (1999).
4. F.W. Stecker and S.L. Glashow, *Astropart. Phys.* **16**, 97 (2001).

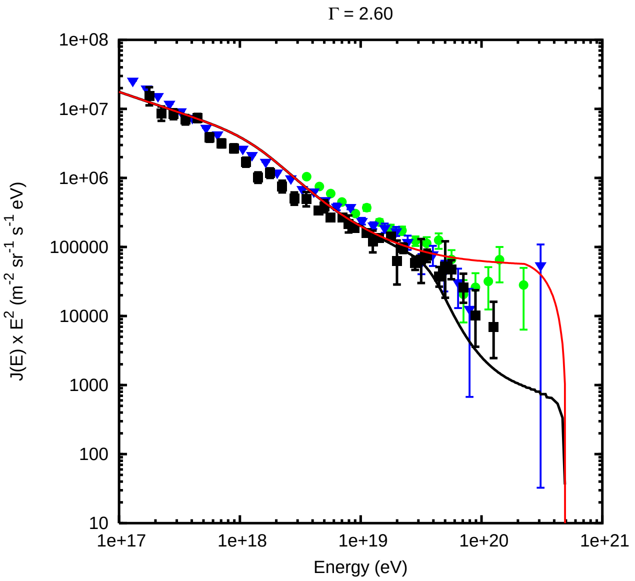

Fig. 1. Predicted spectra for an $E^{-2.6}$ source spectrum with redshift evolution and E_{max} = 500 EeV, shown with pair-production losses included and photomeson losses both included (black curve) and turned off (lighter (red) curve). The curves are shown with ultrahigh energy cosmic ray spectral data from *Fly's Eye* (triangles), *AGASA* (circles), and *HiRes* monocular data (squares)[6].

5. F.W. Stecker *Astropart. Phys.* **20**, 85 (2003).
6. F.W. Stecker, *Nucl. Phys B*, in press, e-print astro-ph/0407311; Stecker and Scully, in prep.
7. T. Tanimori *et al.*, *Astrophys. J.* **492**, L33 (1998).
8. J. Nishimura *et al.*, *Astrophys. J.* **238**, 394 (1980).
9. O.C. de Jager and A. K. Harding, *Astrophys. J.* **396**, 161 (1992).
10. F. Aharonian *et al.*, *Astron. and Astrophys.* **366**, 62 (2001).
11. O.C. de Jager and F.W. Stecker, *Astrophys. J.* **566**, 738 (2002).
12. A. Konopelko *et al. Astrophys. J.* **597**, 851 (2003).
13. L.J. Garay, *Intl. J. Mod. Phys. A* **10**, 165 (1995).
14. J. Alfaro *et al.*, *Phys. Rev. D* **65**, 103509 (2002).
15. J.Ellis *et al*, *Phys. Rev.D* **63**, 124025 (2001).
16. C. Myers and M. Pospelov *Phys. Rev. Letters* **90**, 211601 (2003).
17. T. Jacobson *et al.*, *Nature* **424**, 1019 (2003).
18. T. Jacobson *et al.*, *Phys. Rev. Letters* **93**, 021101 (2004).
19. R. Aloisio *et al.*, *Phys. Rev. D* **62**, 053010 (2000).
20. J. Alfaro and G. Palma, *Phys. Rev. D* **67**, 083003 (2003).
21. J. Ellis, *et al.*, *Astropart. Phys.* **20**, 669 (2004)