International Journal of Modern Physics A

THRESHOLD RESUMMING WITH TIME-LIKE PRESCRIPTION

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Received (Day Month Year)
Revised (Day Month Year)

As it is well known, large threshold logarithmic contributions can be resummed in the form factors within a fixed order approximation. Such form factors, however, are divergent at large x, as a consequence of the QCD coupling hitting the Landau pole in the resumming procedure. A possible solution is presented here, and it requires using an effective coupling which takes into account additional –usually neglected– absorptive physical effects; as a consequence, the theory becomes completely predictive with regular form factors at large x. These results are pertinent to a wide class of phenomenological applications, and in particular are of interest for processes at the B meson mass scale.

Keywords: QCD; threshold logarithmic resumming; Landau pole.

In hadronic decays or interactions, the threshold limit is where the observable final state obtains its maximum energy. In this limit, the whole energy of the partons starting the process is devoted to producing the observable final state. There is no energy left over for real gluon emission, which is therefore inhibited. Physically speaking, the decay "looses" real gluons with respect to virtual ones, remaining unbalanced and acquiring soft and collinear singularities. As a consequence, the perturbative calculation of the differential rate is plagued by singular corrections, of infrared nature, that can grow numerically quite large. Meaningful results are obtained only once these singular corrections are resummed.

The threshold form factor resumming soft and collinear singularities has the following expression in the Mellin N-space [1,2,3]:

$$f_N(\alpha_S) = \exp \int_0^1 dz \frac{z^{N-1}-1}{1-z} \Big\{ \int_{Q^2(1-z)^2}^{Q^2(1-z)} \frac{dk_t^2}{k_t^2} \left[A_1\, \alpha_S(k_t^2) + A_2\, \alpha_S(k_t^2)^2 \, + \, \cdots \right] \, + \\ + \, B_1\, \alpha_S(Q^2(1-z)) \, + \, D_1\, \alpha_S(Q^2(1-z)^2) + \cdots \Big\}, \quad (1)$$

where Q is the hard scale of the process and A_1, $A_2, \cdots B_1, \cdots D_1, \cdots$ are the first coefficients of the expansion in α_S of the functions $A(\alpha_S)$, $B(\alpha_S)$ and $D(\alpha_S)$. These functions describe the emission of soft and collinear partons and their values depend, in general, on the process.

Let us observe that in eq. (1) the running coupling $\alpha_S\left(k_t^2\right)$ is integrated over all gluon transverse momenta k_t, from the hard scale Q down to zero. Therefore, there are regions of integration where the coupling constant becomes large; by hitting the Landau pole, the coupling constant actually diverges. As a net result, the form factors diverge at large x (that is, in the Mellin space, at large N). A prescription [2,3,4] has to be assigned to give a meaning to the formal expression (1).

An alternative solution is possible by reconsidering the role of $\alpha_S\left(k_t^2\right)$ in eq. (1). The presence of $\alpha_S(k_t^2)$, instead than the tree-level, momentum-independent QCD coupling, has the aim to include in the resumming the so-called inclusive-gluon-decay quantities[5,6]. Formally, at one loop, one start from the sum over all the cuts of dressed gluon propagators:

$$\frac{1}{2\pi i}\int_0^{k_t^2} ds\ \mathrm{Disc}_s\ \frac{1}{s\ \beta_0 \log\frac{-s}{\Lambda^2}} = \int_{-\pi}^{+\pi}\frac{d\varphi}{2\pi}\frac{1}{\beta_0\left[\log\frac{k_t^2}{\Lambda^2}+i\varphi\right]}. \tag{2}$$

The discontinuity is defined as: $\mathrm{Disc_s F(s) = F(s+i\epsilon) - F(s-i\epsilon)}$, with ϵ being a positive infinitesimal number; the second passage follows by closing the integration contour with a circle of radius k_t^2 and a circle of infinitesimal radius. The QCD coupling evaluated at the gluon transverse momentum squared, $\alpha_S\left(k_t^2\right)$, in the standard resummation formula (1), is obtained by neglecting the imaginary term $i\varphi$ in eq. (2).

The previous approximation does not take into account the decay of the jets which are originated from the gluons emitted by the hard parton. The radiated gluons have a positive virtuality $k^2 > 0$, as the development of jets is intrinsically a time-like process. By taking into account absorptive effects, i.e. by leaving the imaginary term $i\varphi$ in eq. (2), one aims at including the physical effects of such gluon cascades [7].

The net result is that $\alpha_S(k_t^2)$ does not occur anymore in the resummation formula, and it is replaced by an effective coupling $\tilde{\alpha}_S(k_t^2)$. At one loop [8]:

$$f_N(\alpha_S) = \exp\int_0^1 dz\frac{z^{N-1}-1}{1-z}\Big\{\int_{Q^2(1-z)^2}^{Q^2(1-z)}\frac{dk_t^2}{k_t^2}A_1\,\tilde{\alpha}_S(k_t^2)\ + \\ +\ B_1\,\tilde{\alpha}_S(Q^2(1-z))\ +\ D_1\,\tilde{\alpha}_S(Q^2(1-z)^2)\Big\}, \tag{3}$$

where

$$\tilde{\alpha}_S(k_t^2) = \frac{1}{\beta_0}\left[\frac{1}{2}-\frac{1}{\pi}\arctan\frac{\log\frac{k_t^2}{\Lambda^2}}{\pi}\right]. \tag{4}$$

Analogous reasonings can be carried on at two loops [8]. The main point is that the effective coupling never becomes large, staying within the perturbative domain; therefore, the improved resummation formula, eq. (3), is not plagued by Landau pole singularities. The interesting consequences are that the form factor is regularized and the model, not involving any new free parameter, is strictly predictive.

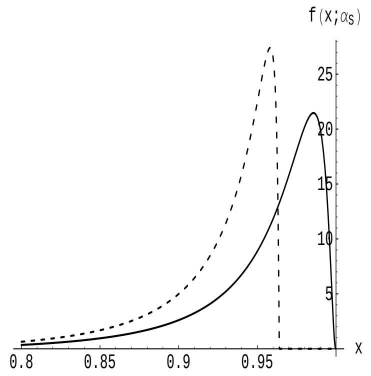

Fig. 1. Form factor in x-space $f(x;\alpha_S)$ for the *beauty* case. Solid line: effective coupling; dashed line: standard coupling.

In fig. 1 [8], the form factor calculated within the time-like prescription is compared with the standard one (solid and dashed lines, respectively) for the decay of a beauty quark, at $Q = m_B$. In the Mellin space, the momentum can be plotted up to $N = 20$, since above these values the standard form factor becomes singular, while the improved form factor is regular over the whole range. At the same way, in the x space, the standard form factor presents a singularity at $x_{crit} \sim 1 - \Lambda/Q$, so in fig. 1 the dashed line plot is truncated above this point.

The phenomenological relevance of this model is more evident at moderately large scales, such as the beauty scale. At larger scales, f.i. in top decays, there is practically no difference with the standard prescription[8]. In the beauty case, however, we expect large non-perturbative corrections. In general, the physical spectrum is obtained by convoluting the perturbative form factor with a phenomenological function, aimed at describing non-perturbative effects. In the standard approach, the convolution is usually implemented with numerical methods, employing prescriptions[4] which circumvent the mathematical problems introduced by the singularities. Within the present model, based on the inclusion of existing physical absorptive effects, we are in the position to implement analytical convolutions, without any additional prescription.

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