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# TOWARDS A LARGE-$N_C$ ESTIMATE OF THE $O(p^6)$ CHIRAL LOW ENERGY CONSTANTS

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I discuss recent progress in large-$N_C$ inspired determinations of chiral low energy constants of order $p^6$. I illustrate the general method and then describe how it works in the case of the $\langle VAP \rangle$ three-point function.

Chiral Perturbation Theory [1] (CHPT) is the effective theory of QCD describing the interactions of pseudo-Goldstone modes $(\pi, K, \eta)$ associated with the spontaneous breaking of chiral symmetry. CHPT is a fundamental tool in low energy hadron phenomenology. State of the art calculations involve expansions to NNLO ($p^6$) in external momenta or quark masses [2]. At this level the effective CHPT lagrangian [3,4] involves more than 90 low-energy constants (LECs) in the even-intrinsic parity sector for three flavours. Therefore, in order to do phenomenology to NNLO in the low-energy expansion it is highly desirable to develop a non-perturbative framework to predict such LECs by matching to QCD. Among non-lattice QCD techniques, one common framework is provided by the $1/N_C$ expansion (for a review see Ref. [5]).

Within this scheme, for a given QCD Green function one constructs an hadronic interpolation between the known low and high momentum regimes, dictated respectively by chiral symmetry and the operator product expansion (OPE) for QCD. The procedure of matching between low and high energies is especially transparent for Green functions that are order parameters of chiral symmetry breaking, as a finite number of hadronic states suffices to provide the correct fall-off at high momentum. Working in the chiral limit and at leading order in the $1/N_C$ expansion, the matching is performed by saturating the operator product expansion (OPE) with a certain number of narrow resonance multiplets [6]. While this matching can be performed without reference to a chiral resonance lagrangian, we choose to work with one. Compatibility between the constraints from QCD at high energies and a suitable chiral resonance lagrangian, as already established at $\mathcal{O}(p^4)$ [7,8], would allow for a consistent lagrangian treatment of resonance contributions at least up to $O(p^6)$. The approach employed here [9,10] should be seen as an approximation to large-$N_C$ QCD. The modeling consists in the choice of the hadronic content of the theory and the set of QCD short-distance constraints to be satisfied. For the *Hadronic ansatz*, we include all lowest-lying resonance multiplets that can contribute to the given Green function (one multiplet per channel). This choice relies on the assumption that the low-lying hadronic spectrum determines the chiral LECs that govern the low-energy behaviour of the Green function. Concerning the *Short-distance constraints*, it is known that when working with a finite number of resonance multiplets, conflicts may arise between different types of asymptotic constraints [11]. We assign the highest priority to the constraints dictated by the OPE to leading order in inverse powers of large momenta. We then consider the constraints implied by the high-momentum behaviour of hadronic form factors (as predicted by the quark counting rules [12]) with on-shell Goldstone modes and photons.

We have constructed the most general chirally invariant resonance lagrangian which, upon integrating out the heavy fields, prduces contributions up to order $p^6$ in the resulting Goldstone chiral lagrangian. Apart from kinetic and mass terms, it has the structure:

$$\mathcal{L} \ = \mathcal{L}_{(2)}^{R_i} + \mathcal{L}_{(4)}^{R_i} \ + \ \mathcal{L}_{(2)}^{R_i R_j} \ + \ \mathcal{L}_{(0)}^{R_i R_j R_k} \ , \tag{1}$$

where $\mathcal{L}_{(n)}^{[...]}$ is a term of chiral order $p^n$ involving the number of resonances specified in [...]. The full list of resonance couplings and the formal procedure of integrating out heavy fields and obtaining the $p^6$ LECs will be presented elsewhere [10]. Here we show that the above chiral resonance lagrangian is fully consistent with a set of QCD constraints for the $\langle VAP \rangle$ Green function, and we go on to determine six of the LECs of $\mathcal{O}(p^6)$ from $V, A$ and $P$ resonance contributions [9].

The $\langle VAP \rangle$ three-point function in momentum space is defined as

$$(\Pi_{VAP})_{\mu\nu}^{abc}(p,q) = \int d^4x \int d^4y \ e^{i(p\cdot x + q\cdot y)} \langle 0 | T\{V_\mu^a(x) A_\nu^b(y) P^c(0)\} | 0 \rangle \, , \tag{2}$$

with $SU(3)$ octet currents $V_\mu^a = \bar{\psi}\gamma_\mu T^a \psi, A_\mu^a = \bar{\psi}\gamma_\mu\gamma_5 T^a\psi, P^a = \bar{\psi} i\gamma_5 T^a \psi$. The general solution of the chiral Ward identities [13,14] implies that the dynamics of this correlator is encoded in two invariant functions $\mathcal{F}(p^2, q^2, (p+q)^2)$ and $\mathcal{G}(p^2, q^2, (p+q)^2)$. The behaviour of the invariant functions $\mathcal{F}$ and $\mathcal{G}$ at small momentum transfer is governed by the contributions from Goldstone boson intermediate states. The behavior at high energy (one or two large momenta) is dictated by the OPE. The leading contribution is proportional to the quark condensate and is known [13,14]. Our hadronic lagrangian leads to invariant functions of the form ($r = (p+q)$):

$$\mathcal{F}(p^2, q^2, r^2) = \frac{\langle \overline{\psi}\psi \rangle_0}{(p^2 - M_V^2)(q^2 - M_A^2)} \left[ a_0 + \frac{b_1 + b_2 p^2 + b_3 q^2}{r^2} + \frac{c_1 + c_2 p^2 + c_3 q^2}{r^2 - M_P^2} \right] \, ,$$

$$\mathcal{G}(p^2, q^2, r^2) = \frac{\langle \overline{\psi}\psi \rangle_0}{(p^2 - M_V^2) q^2} \left[ \frac{d_1 + d_2 q^2}{r^2 (q^2 - M_A^2)} + \frac{f}{(p+q)^2 - M_P^2} \right] \, . \tag{3}$$

The short-distance conditions allow us to express six of the parameters in terms of $b_3$: $a_0 = -1/2, b_2 = 1 + b_3, d_2 = 2b_3, c_2 = -1/2 - b_3, c_3 = c_2, f = 2c_2$. Additional information on the parameters in Eq. (3) can be obtained by putting one or two momenta in the $\langle VAP \rangle$ Green function (2) on the pion mass shell The fall-off of the pion form factors and the s.d. constraints from the $VV - AA$ correlator imply[13]

$b_1 = M_A^2 - M_V^2$ and $d_1 = 2M_A^2$. Moreover, within our resonance lagrangian $c_1$ is related to $c_{2,3}$. Finally, the remaining dimensionless parameter $b_3$ can be set to zero by requiring that the axial form factor governing the matrix element $\langle\gamma|A_\mu|\pi\rangle$ vanishes at high momentum transfer. The matching described above allows to fully determine six LECs of the $p^6$ CHPT lagrangian in terms of the pion decay constant ($F$) and resonance masses:

$$
\begin{aligned}
C_{78} &= \frac{F^2(3M_A^2+4M_V^2)}{8M_V^4M_A^2} - \frac{F^2}{16M_V^2M_P^2}\ , & C_{82} &= -\frac{F^2(4M_A^2+5M_V^2)}{32M_V^4M_A^2} - \frac{F^2}{32M_A^2M_P^2}\ , \\
C_{87} &= \frac{F^2(M_A^4+M_V^4+M_A^2M_V^2)}{8M_V^4M_A^4}\ , & C_{88} &= -\frac{F^2}{4M_V^4} + \frac{F^2}{8M_V^2M_P^2}\ , \\
C_{89} &= \frac{F^2(3M_A^2+2M_V^2)}{4M_V^4M_A^2}\ , & C_{90} &= \frac{F^2}{8M_V^2M_P^2}\ .
\end{aligned}
$$

Our large-$N_C$ determination of these LECs cannot reproduce their renormalization scale dependence (a next-to-leading-order effect in the $1/N_C$ counting). We have checked that in all cases the variation of the renormalized LECs between $\mu = M_K$ and $\mu = 1$ GeV remains within a range of 30 %. Although there is no reason a priori that this result will be valid for *all* LECs, our findings quantify the reliability of these specific resulrs for phenomenological applications.


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