



Theoretical Cosmology

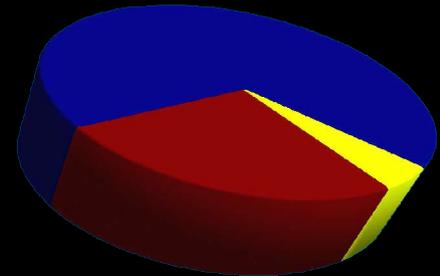
Sean Carroll, University of Chicago

<http://pancake.uchicago.edu/~carroll/>

Executive Summary:

We have plenty of good models,
not enough good theories.

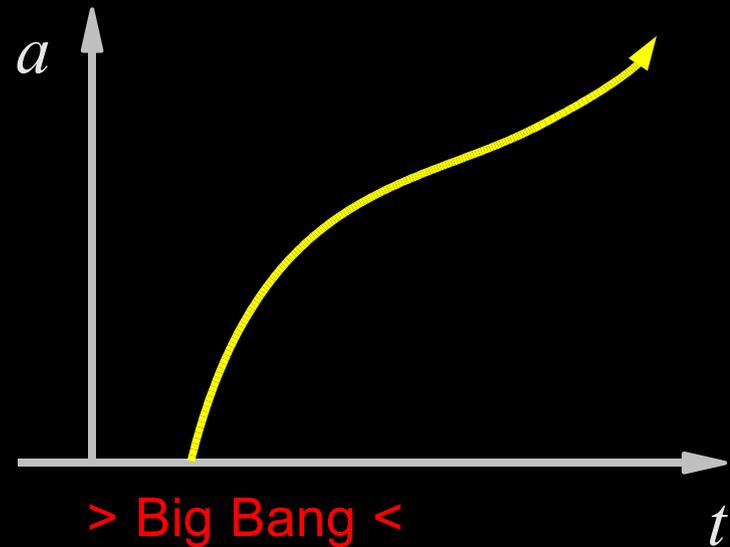
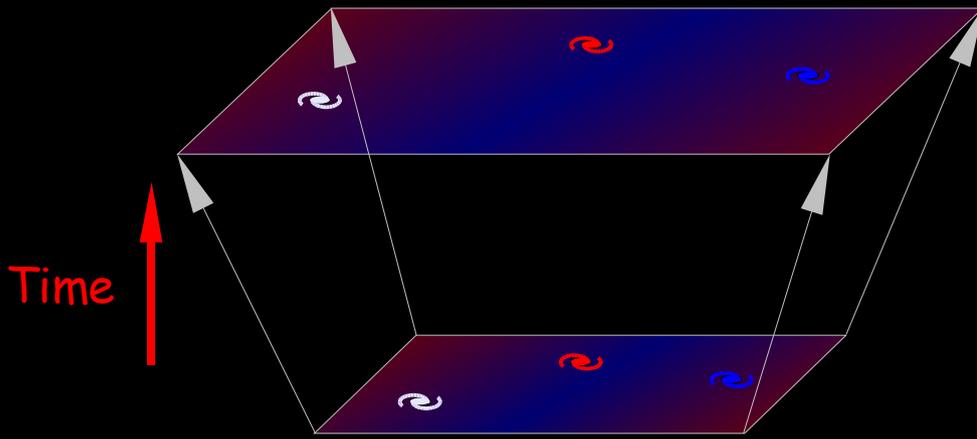
We know much, and
understand very little.



Homogeneous Isotropic Cosmology

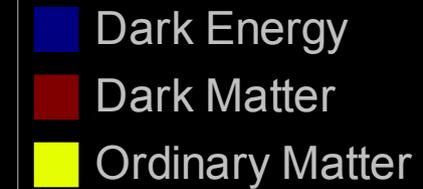
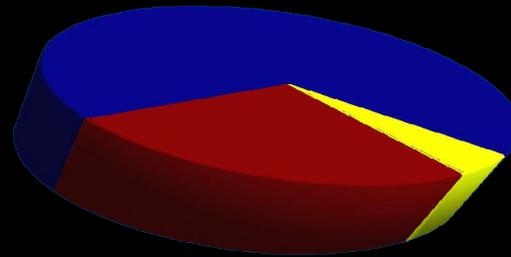
The scale factor a obeys the Friedmann equation:

$$H^2 = \frac{8\pi G}{3} \rho - \frac{\kappa}{a^2}$$



| | |
|----------------|---|
| expansion rate | $H_0 \sim 72 \text{ km/s/Mpc} \sim 10^{-33} \text{ eV}$ |
| age | $t_0 \sim 13.7 \text{ Gyr} \sim (10^{-33} \text{ eV})^{-1}$ |
| energy density | $\rho_0 \sim 10^{-8} \text{ ergs/cm}^3 \sim (10^{-3} \text{ eV})^4$ |
| curvature | $\kappa_0 \ll 8\pi G\rho_0$ (nearly flat) |

Inventory



Define the density parameter $\Omega_i = (3H^2/8\pi G)\rho_i$,
such that $\Omega_{\text{total}} = 1$ implies a spatially flat universe.

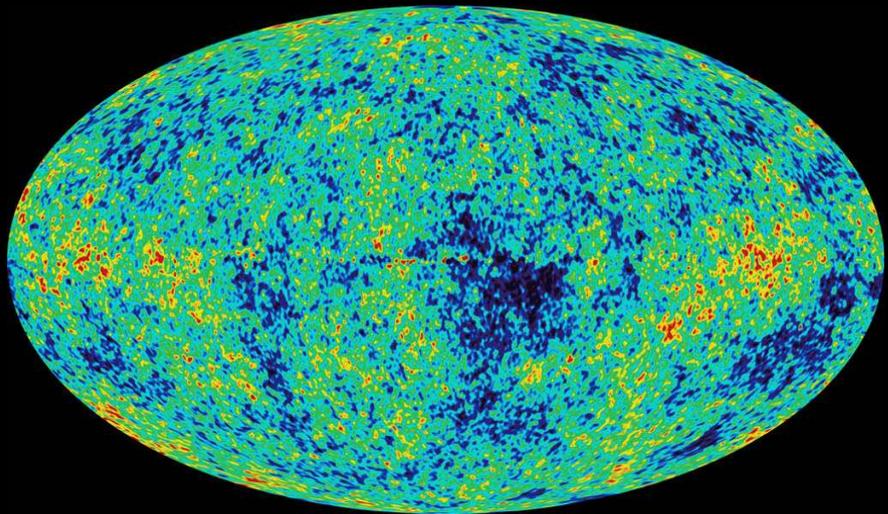
| | | |
|--------------|---------------------------|-------------------------------------|
| Matter: | $\Omega_M \sim 0.3$ | $\rho_M \propto a^{-3}$ |
| baryons: | $\Omega_b \sim 0.05$ | (from BBN and CMB) |
| dark matter: | $\Omega_{DM} \sim 0.25$ | (cold, collisionless) |
| neutrinos: | $\Omega_\nu \sim 10^{-3}$ | |
| Radiation: | $\Omega_R \sim 10^{-4}$ | $\rho_R \propto a^{-4}$ |
| Dark Energy: | $\Omega_{DE} \sim 0.7$ | $\rho_{DE} \approx \text{constant}$ |

Perturbations

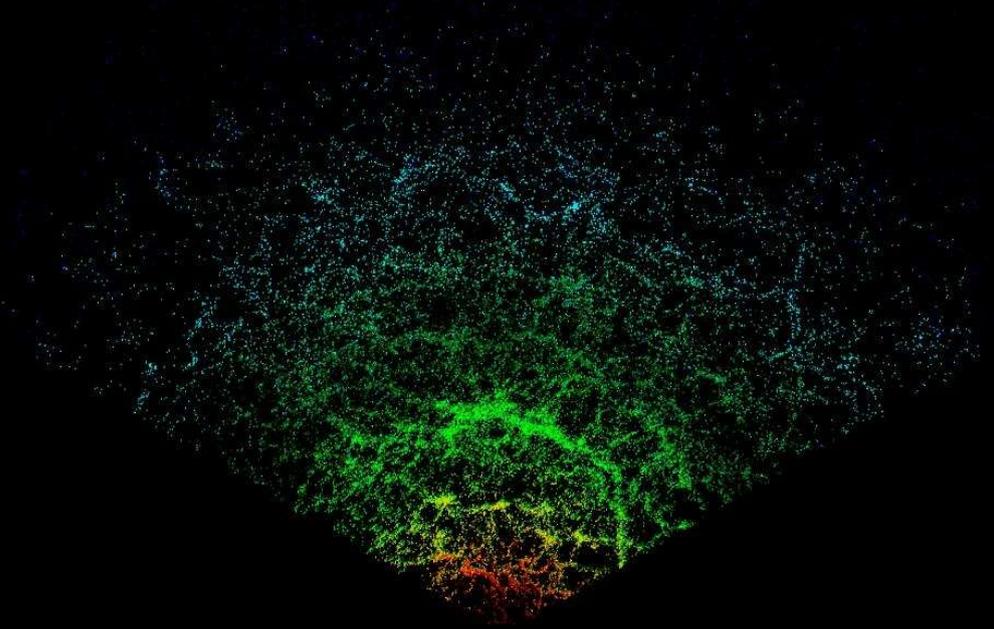
Primordial (super-Hubble) perturbations with $\delta\rho/\rho \sim 10^{-5}$

Consistent with: Gaussian (uncorrelated phases)
Adiabatic (species are correlated)
Scale-free ($|n-1| < 0.1$)

Evolution from early universe to recombination to today



[WMAP]



[SDSS]

Questions we still have

Evolution

How did galaxies and clusters form?

What is the distribution of the dark matter?

What is the chemical evolution of the universe?

How did supermassive black holes form?

Can we disentangle lensing effects from tensor modes in the CMB?

Was Friedmann right?

Composition

What kind of particle is the dark matter?

Can we detect/produce dark matter astrophysically or in the lab?

What the hell is the dark energy?

Does dark energy evolve?

What is the origin of ultra-high-energy cosmic rays?

What is the origin of the matter/antimatter asymmetry?

Origins

Did the universe inflate?

What is the origin of the cosmological perturbations?

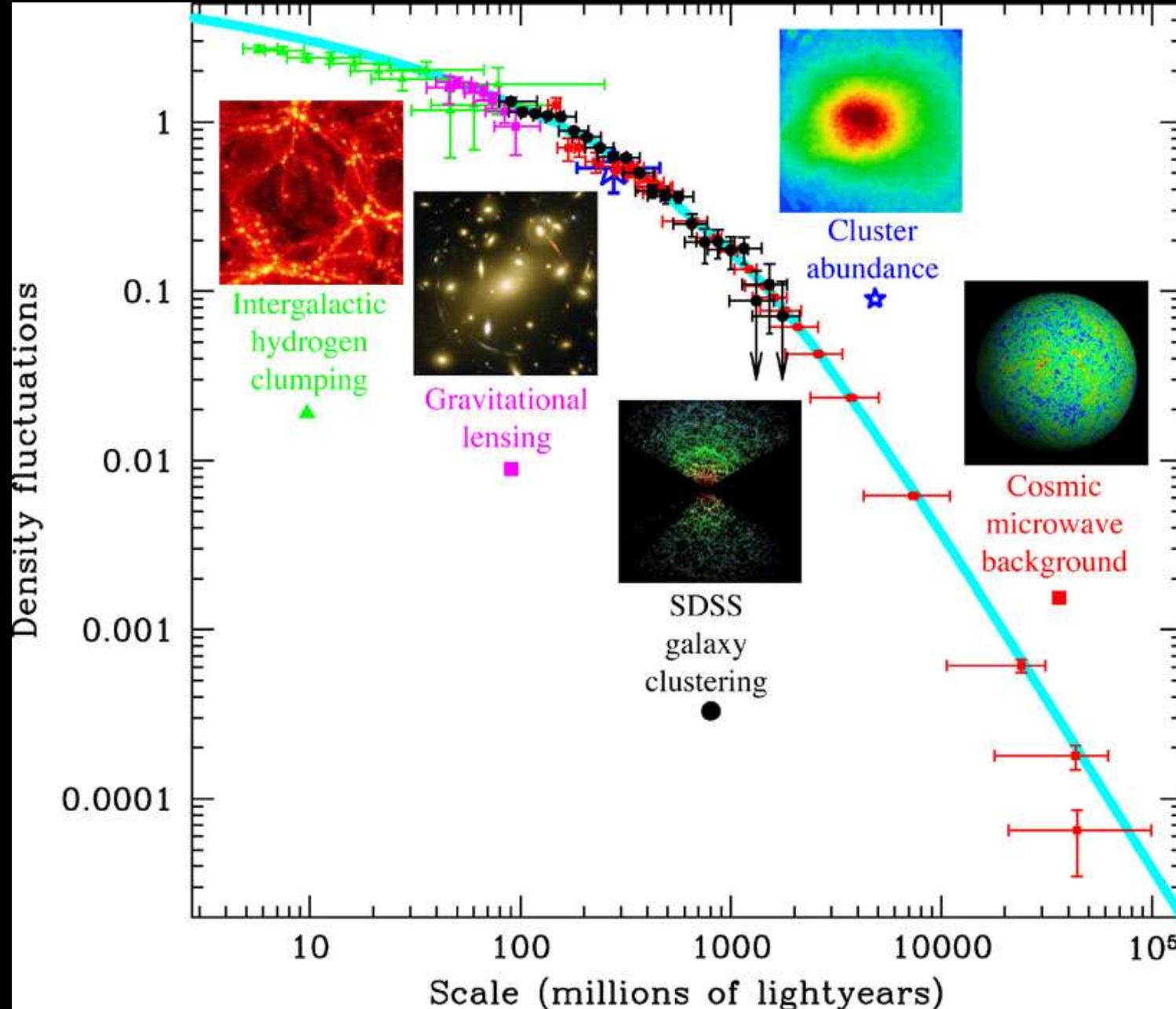
Is there a gravitational-wave background?

What is the role of extra dimensions, if any?

Are there multiple universes with different conditions?

What happened before the Big Bang?

Evolution Questions



We're pretty good at power-spectrum issues, especially in the linear regime.

Less good at the nonlinear universe: galaxies and clusters.

But we're getting better! Only hope is numerical simulation; codes are become faster and including more astrophysics (beyond dark-matter-only).

Questions still linger about the distribution of dark matter, especially in central regions of clusters and galaxies. Roughly, there is not enough DM there (or it's not concentrated enough). Do we need better data, better simulations, or new physics?

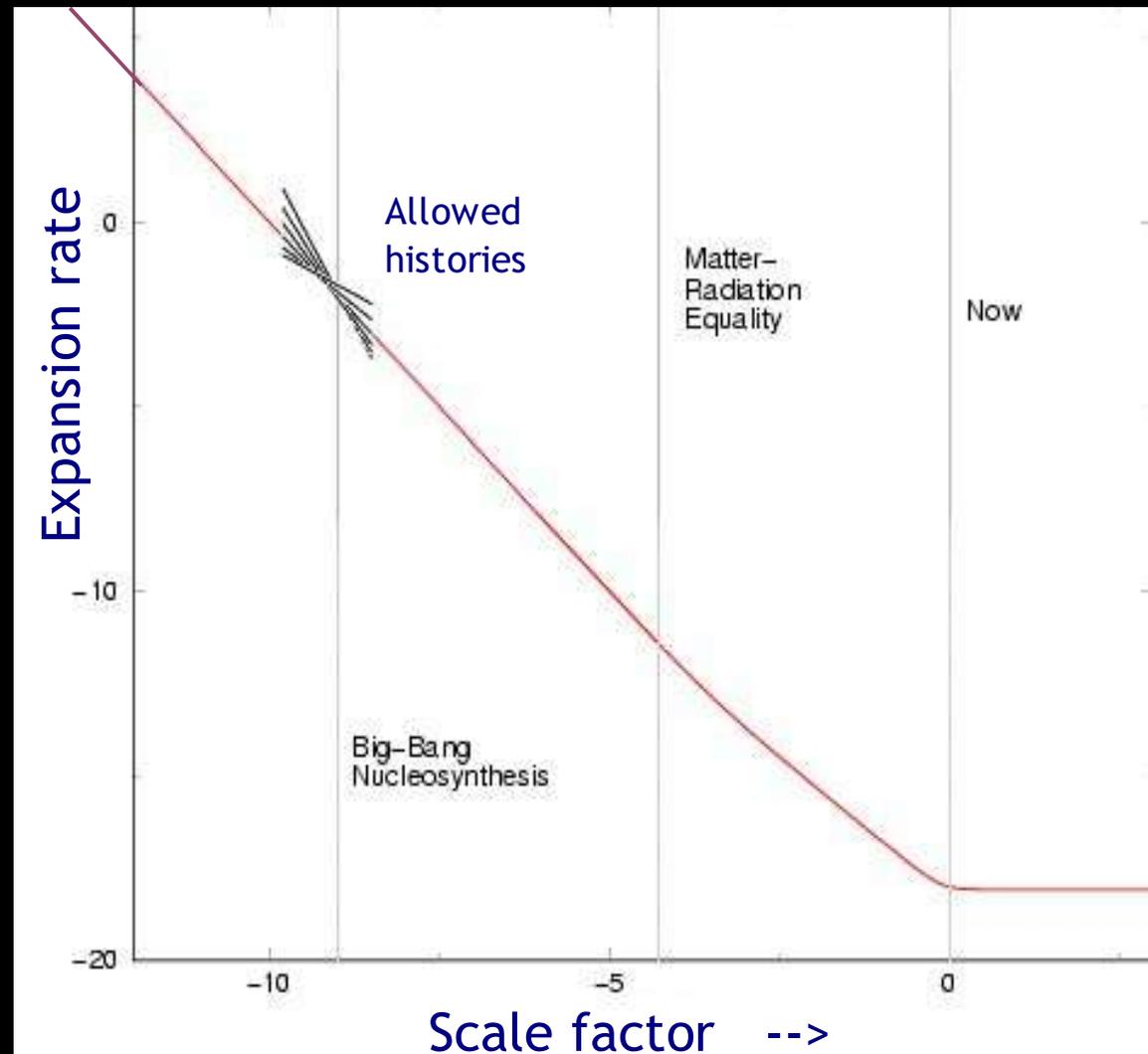


Was Friedmann Right?

$$H^2 = \frac{8\pi G}{3} \rho ?$$

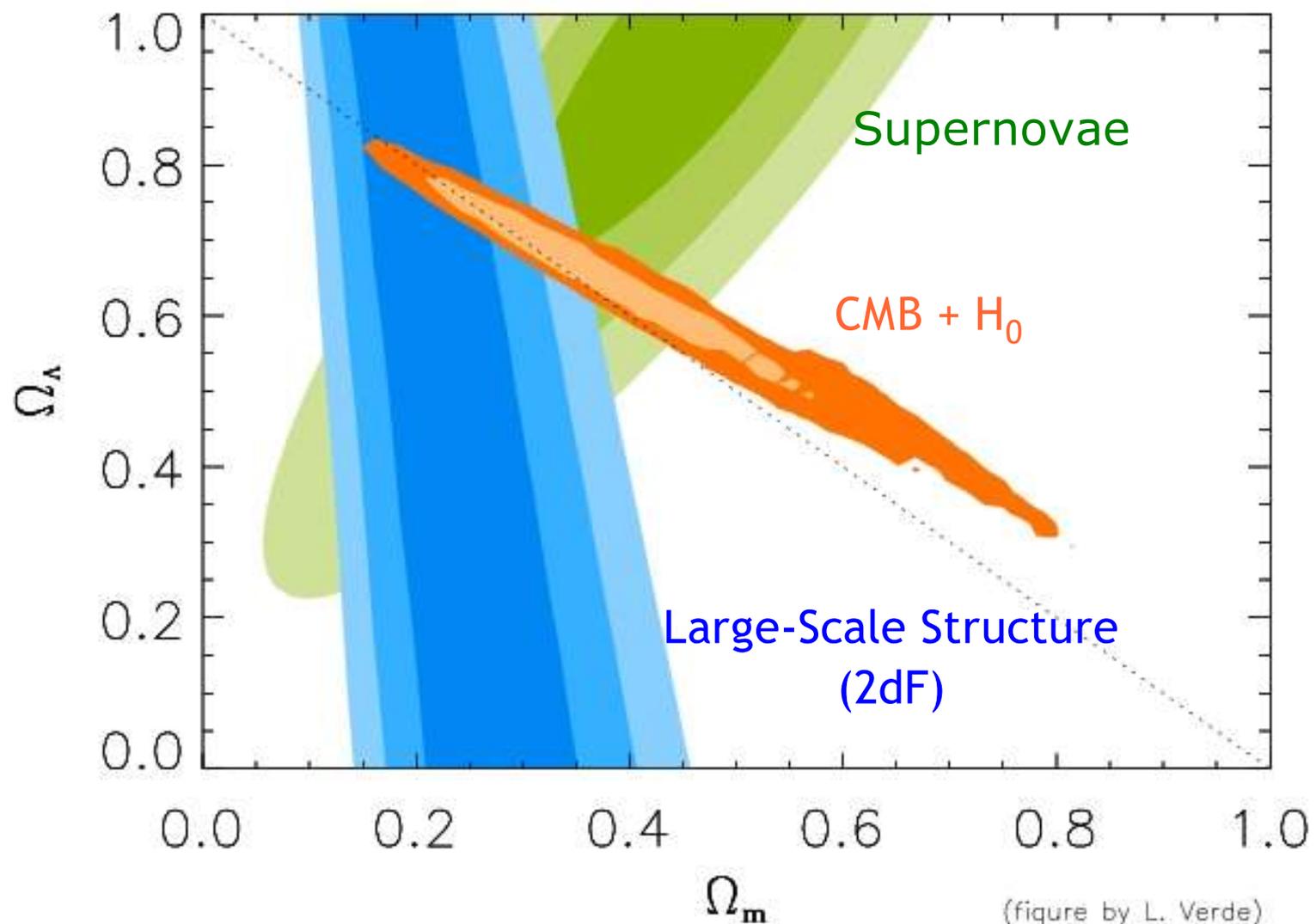
Evidence for conventional expansion history:

- **Big-Bang Nucleosynthesis** ($z \sim 10^9$) is the most model-independent test; unconventional expansion possible, but constrained.
- **CMB anisotropies** ($z \sim 10^3$), e.g. location of acoustic peaks, are consistent with conventional expansion.
- **Structure growth** harder to quantify, but consistent with $a = t^{2/3}$ (MD) until quite recently.



Composition Questions

5% Ordinary Matter
25% Dark Matter
70% Dark Energy



Dark Matter: well-motivated candidates

- **Weakly Interacting Massive Particles (WIMPs)**
 - in equilibrium early; freeze-out after becoming nonrelativistic (cold)
 - must be neutral, color singlets
 - A linear collider would be helpful here!
- **Axions**
 - light pseudoscalars predicted by Peccei-Quinn solution to the strong-CP problem
 - produced out of equilibrium, by vacuum misalignment or topological-defect radiation
 - colliders no good, need dedicated experiments
- **anything else**

The Early Universe

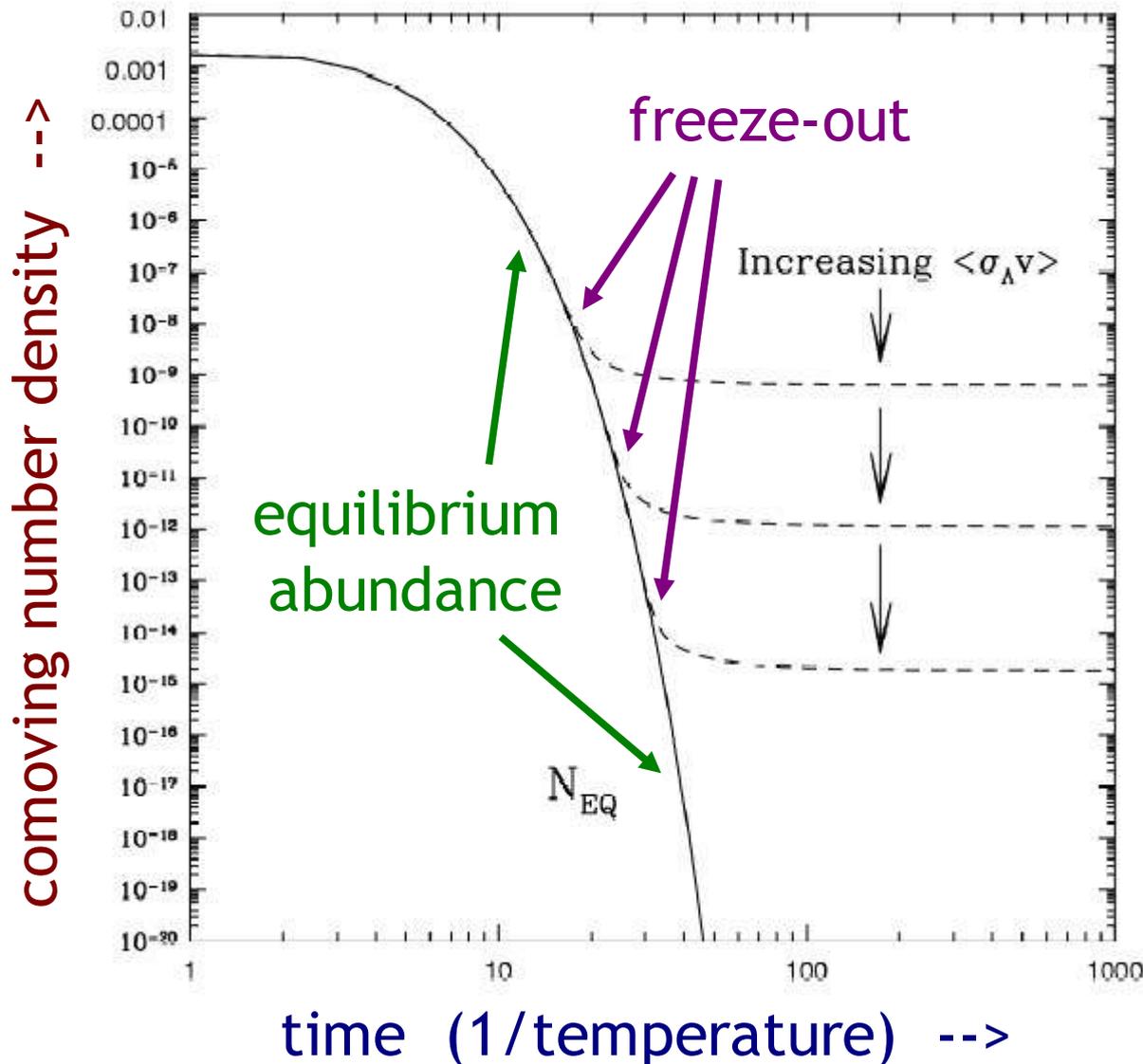
- It was hot, dense, nearly homogeneous.
- Expanding, but slowly (in a sense). At an energy density $\rho = E^4$, the expansion rate is

$$H = \left(\frac{E}{E_{Pl}} \right) E$$

- Thus, nearly in equilibrium.
- But reactions eventually freeze out (decouple); e.g. photons decouple at recombination.
- For a number density n and cross-section σ , a reaction rate $\Gamma = n\langle\sigma v\rangle$ freezes out when

$$\Gamma < H.$$

Cold relics: comoving equilibrium abundance plummets while non-relativistic, then stabilizes after freeze-out.
 (To do it right, solve Boltzmann equation.)



Predicted mass density is almost independent of m , but depends sensitively on annihilation cross-section $\langle\sigma v\rangle$.

For σ at the weak scale, we naturally get $\Omega_{\text{wimp}} \sim 1$.

This compelling story can easily be upset by including additional particles.

[Griest and Seckel]

"**Coannihilation.**" Imagine there is a particle χ_2 , slightly heavier than the DM particle χ_1 , with the same quantum number but a larger $\langle\sigma v\rangle$. Then χ_1 can annihilate more quickly by first converting into χ_2 .

"**Forbidden annihilation.**" Imagine that χ_1 can annihilate into heavier particles that don't decay back into χ_1 , but enhance $\langle\sigma v\rangle$ for χ_1 . Because freeze-out occurs at finite temperature, this channel becomes allowed.

For masses within 10%, abundances can change by $\mathcal{O}(1)$:
we need to understand an entire network of reactions.

(Not to mention angular-momentum dependence, resonances, etc.)

Actual models for WIMP dark matter:

- **Supersymmetry.**

In MSSM with R -parity, the LSP is a perfect DM candidate if it is neutral and a color singlet. Some linear combination of bino, photino, higgsino.

- **Universal extra dimensions.**

Forget branes, imagine Kaluza-Klein extra dimensions with size $\sim (\text{TeV})^{-1}$.

Then "KK parity" is a conserved quantity, and the lightest KK mode (photon, maybe ν) can be dark matter.

Both of these models feature the nearly-degenerate particle spectra that deform relic abundance calculations through coannihilation and forbidden annihilations. (E.g., a neutrino LSP can coannihilate with squarks or staus, or have a forbidden annihilation channel into Higgs bosons.)

Moral of the story:

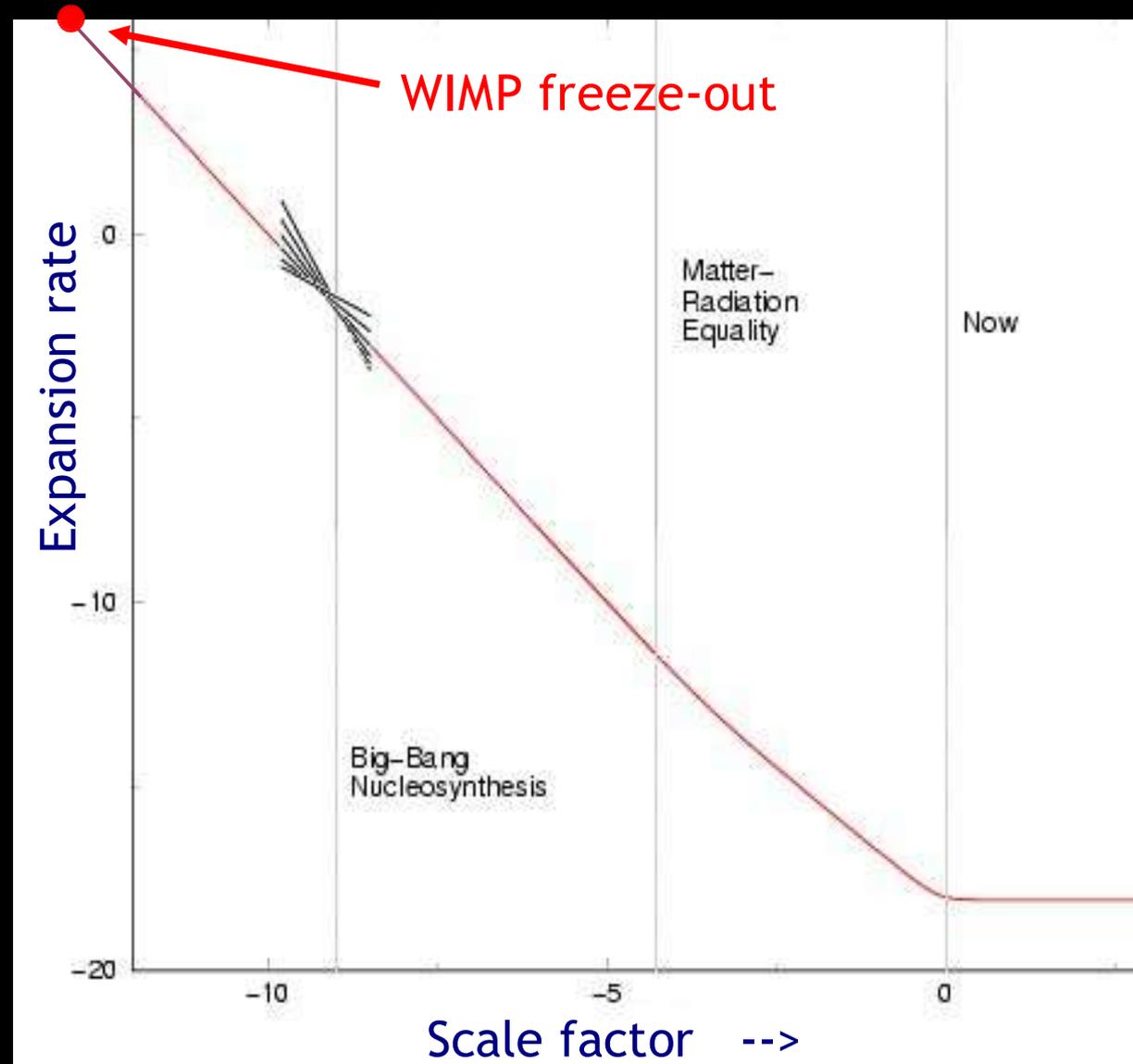
Understanding the dark matter abundance to an order of magnitude may require measuring model parameters at percent-level precision.

That is why cosmologists need a linear collider.

Bonus: probing the expansion rate (and thus the Friedmann equation) at $T \sim 10$ GeV.

Best current test of Friedmann eq. in the early universe: Big Bang Nucleosynthesis, at 1 MeV - 50 keV.

So we can push the known history of the universe back by a factor of 10,000.



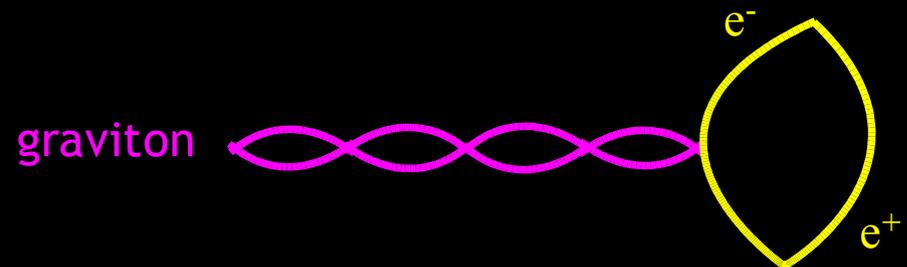
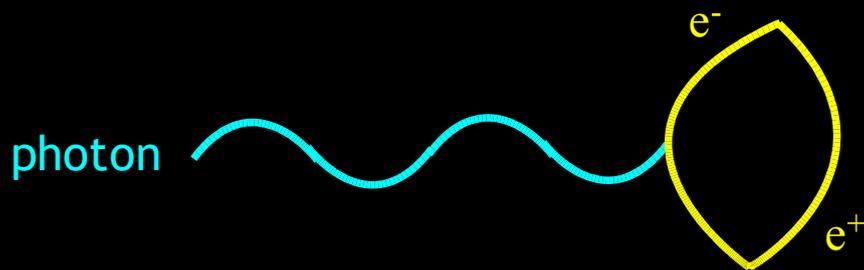
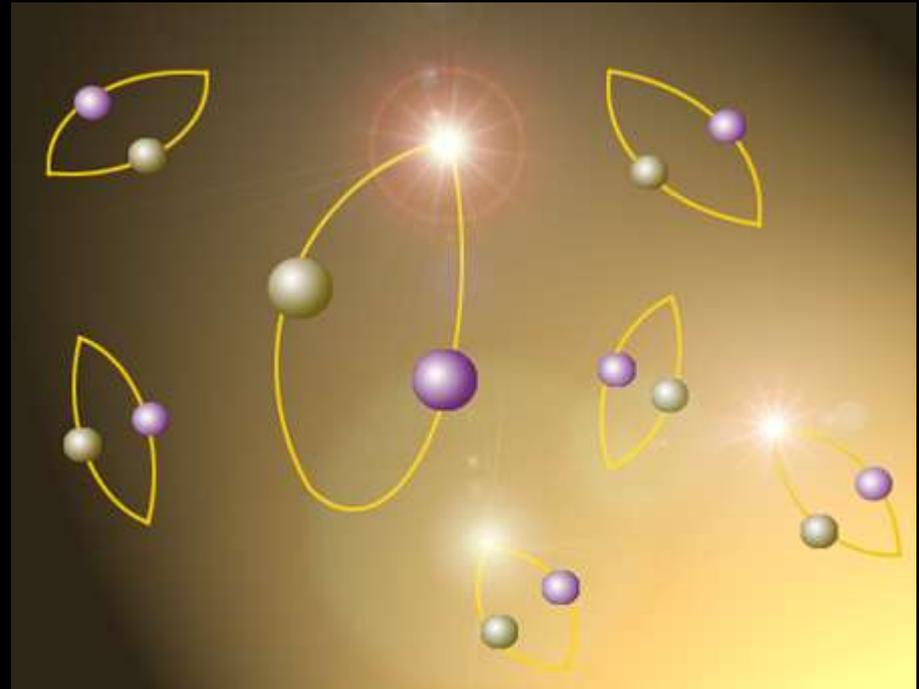
Dark Energy: well-motivated candidates

Dark Energy: ill-motivated candidates

- **Vacuum energy, a/k/a cosmological constant**
 - a strictly constant energy density inherent in empty spacetime
- **Dynamical dark energy**
 - evolution characterized by equation-of-state parameter $w = p/\rho$
- **Modified gravity**
 - Friedmann was wrong, but only at late times

Issue 1:
Why is the vacuum energy so small?

We know that virtual particles couple to photons (e.g. Lamb shift); why not to gravity?



Natural expectation: $\rho_{\text{vac}} = E_{\text{Pl}}^4 = 10^{120} \rho_{\text{vac}}^{(\text{obs})}$.

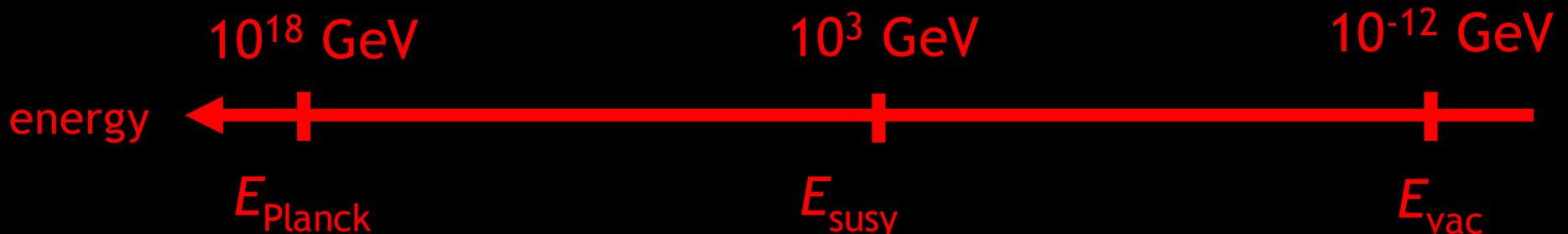
Supersymmetry can squelch the vacuum energy by canceling bosonic and fermionic vacuum energies; unfortunately, in the real world it must be broken at $E_{SUSY} \sim 10^3$ GeV. Typically we would then expect

$$E_{vac} = E_{SUSY} ,$$

which is off by 10^{15} . But if instead we were able to predict

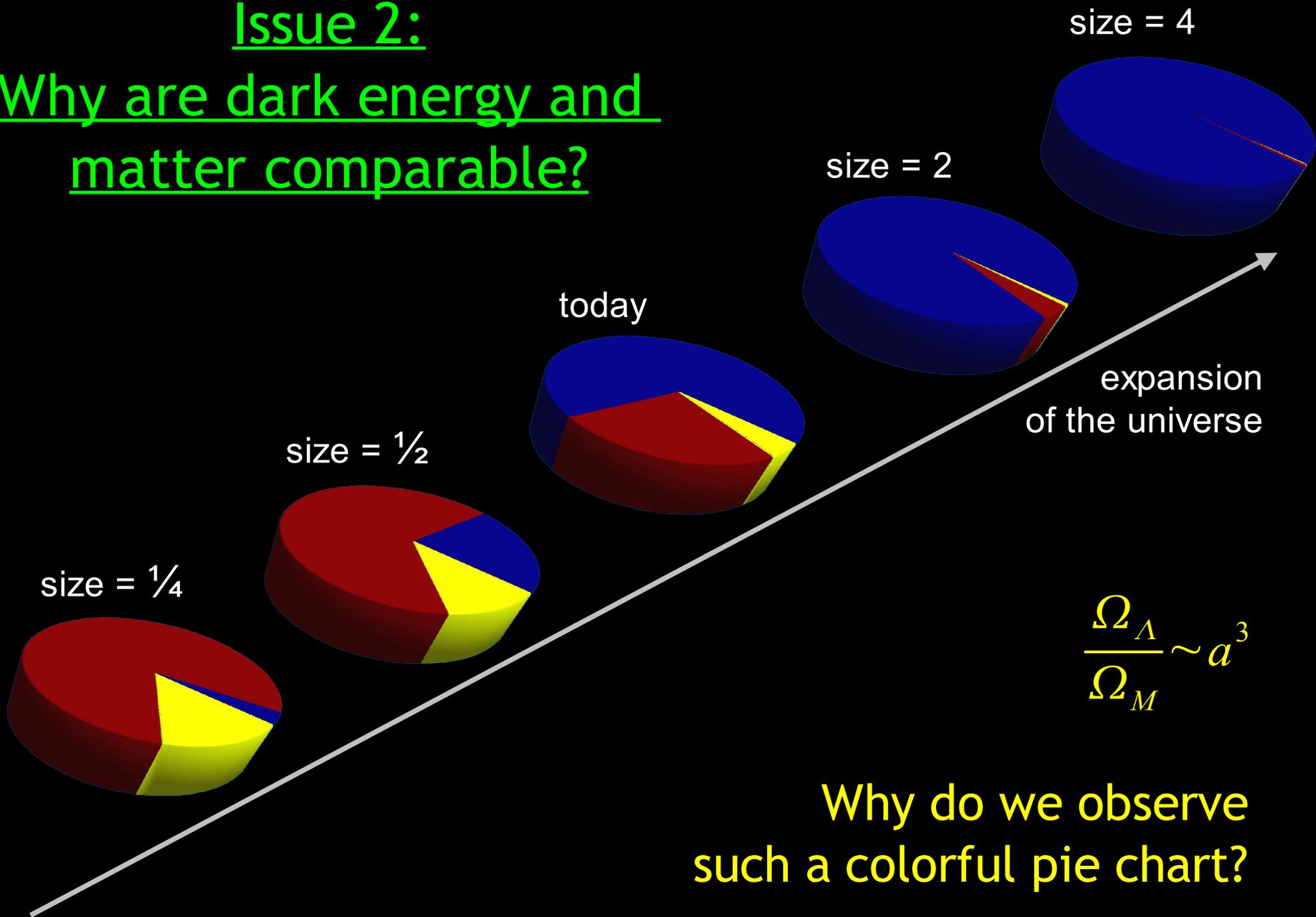
$$E_{vac} = \left(\frac{E_{SUSY}}{E_{Planck}} \right) E_{SUSY} ,$$

it would agree with experiment. (All we need is a theory that predicts this relation.)



Issue 2:

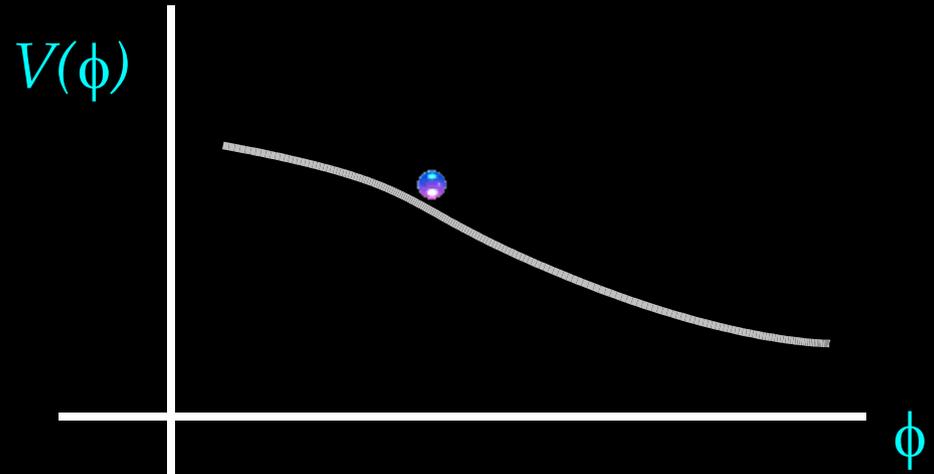
Why are dark energy and matter comparable?



Is the dark energy a slowly-varying dynamical component?

e.g. a slowly-rolling scalar field: "quintessence"

$$\rho_\phi = \underbrace{\frac{1}{2} \dot{\phi}^2}_{\text{kinetic energy}} + \underbrace{\frac{1}{2} (\nabla \phi)^2}_{\text{gradient energy}} + \underbrace{V(\phi)}_{\text{potential energy}}$$

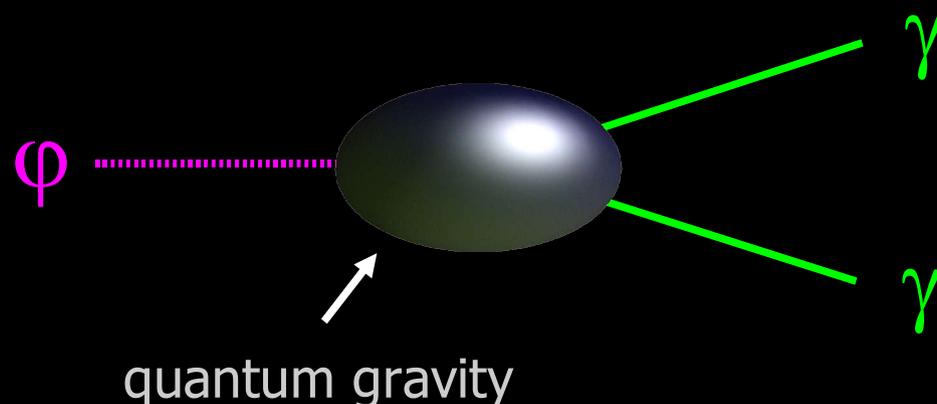


[Wetterich; Peebles & Ratra;
Caldwell, Dave & Steinhardt; etc.]

- This is an observationally interesting possibility, and at least holds the possibility of a dynamical explanation of the coincidence scandal.
- But it is inevitably finely-tuned: requires a scalar-field mass of $m_\phi < 10^{-33}$ eV, and very small couplings to matter.

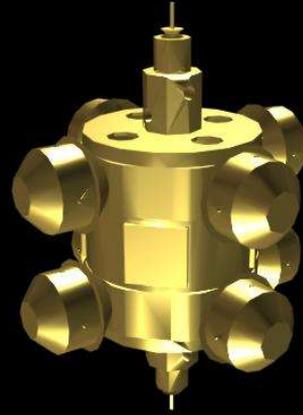
Don't forget the possibility of direct detection of dark energy.

Dynamical dark energy has no right to be completely "dark"; even if it only directly couples to gravity, there will be indirect couplings to all standard-model fields.

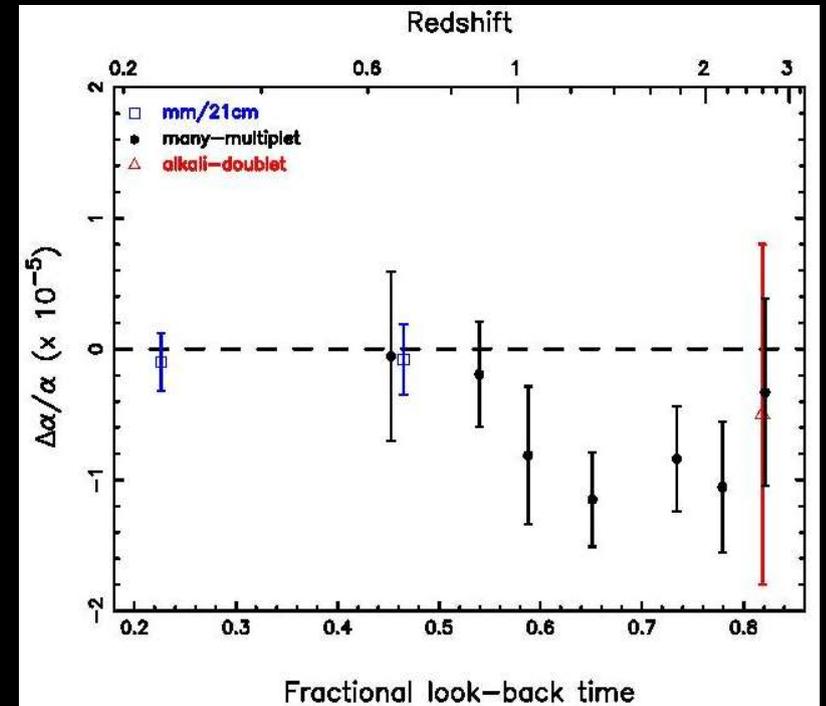


The fact is, if the dark energy is a light scalar field **we should have detected it already**, unless some mechanism suppresses its couplings.

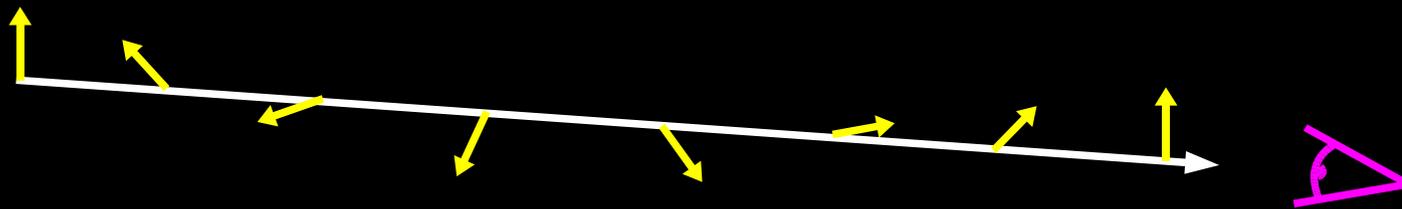
Direct dark energy detection search strategies:



- 5th forces.
- Time-dependent "constants of nature" (e.g., α).
- Cosmological birefringence (rotation of polarized light).



[Webb et al.]



Current limits: couplings must be suppressed by $\sim 10^5 M_p$.

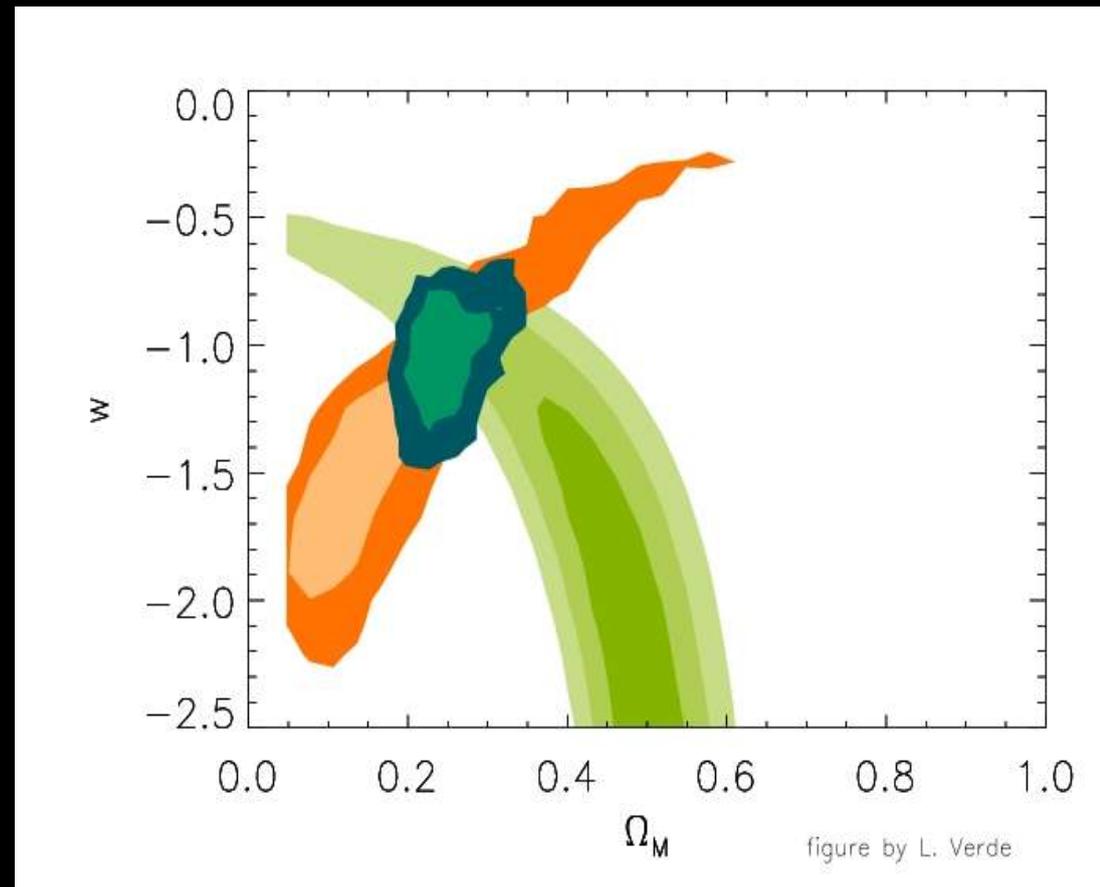
Characterize dynamical dark energy using an effective equation of state relating pressure to energy density:

$$p = w\rho \quad \longrightarrow \quad \rho \propto a^{-3(1+w)}$$

Matter: $w = 0$

Vacuum: $w = -1$.

We can place limits on the w - Ω_M plane (assuming a flat universe, $\Omega_{\text{tot}} = 1$) using supernovae, CMB, H_0 , BBN, and large-scale structure.



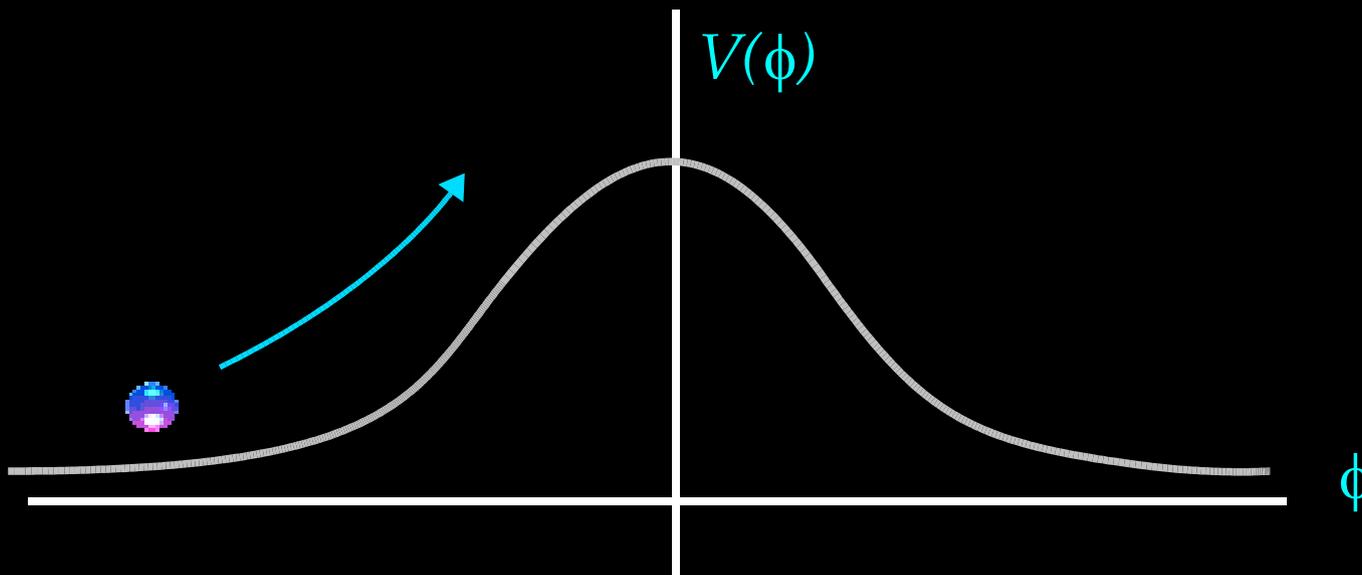
Should we consider $w < -1$?

[Caldwell]

If $w=p/\rho$ is less than -1, it means that the dark energy density is increasing with time - seemingly crazy.

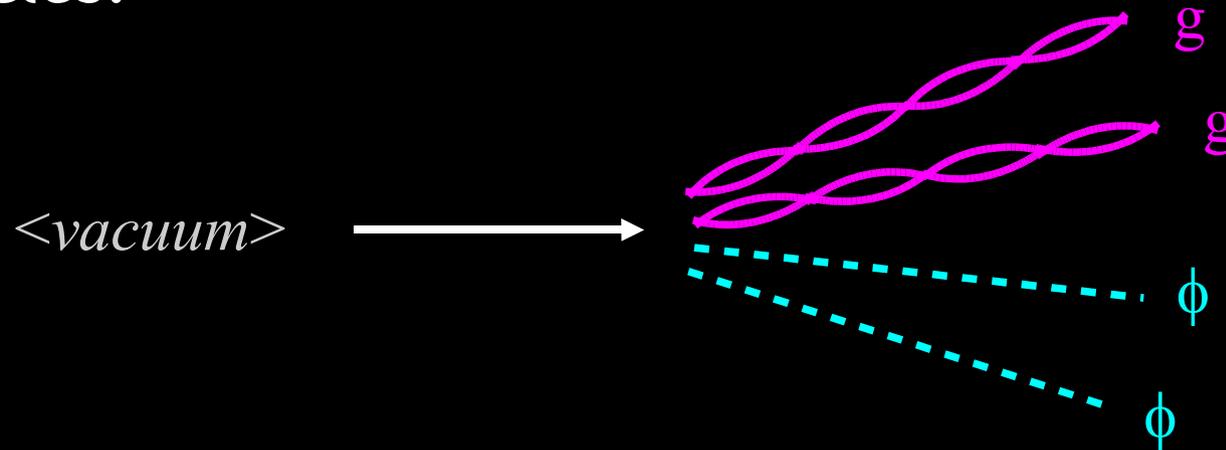
But we can make a field theory with $w < -1$: a **negative-kinetic-energy** scalar field, with

$$\rho_\phi = -\frac{1}{2}\dot{\phi}^2 - \frac{1}{2}(\nabla\phi)^2 + V(\phi)$$



Problem: **the vacuum is unstable to decay.**

If a scalar field has negative kinetic energy, its particle excitations have negative energy. So empty space can decay into positive-energy gravitons and negative-energy ϕ particles.



[Carroll, Hoffman
& Trodden]

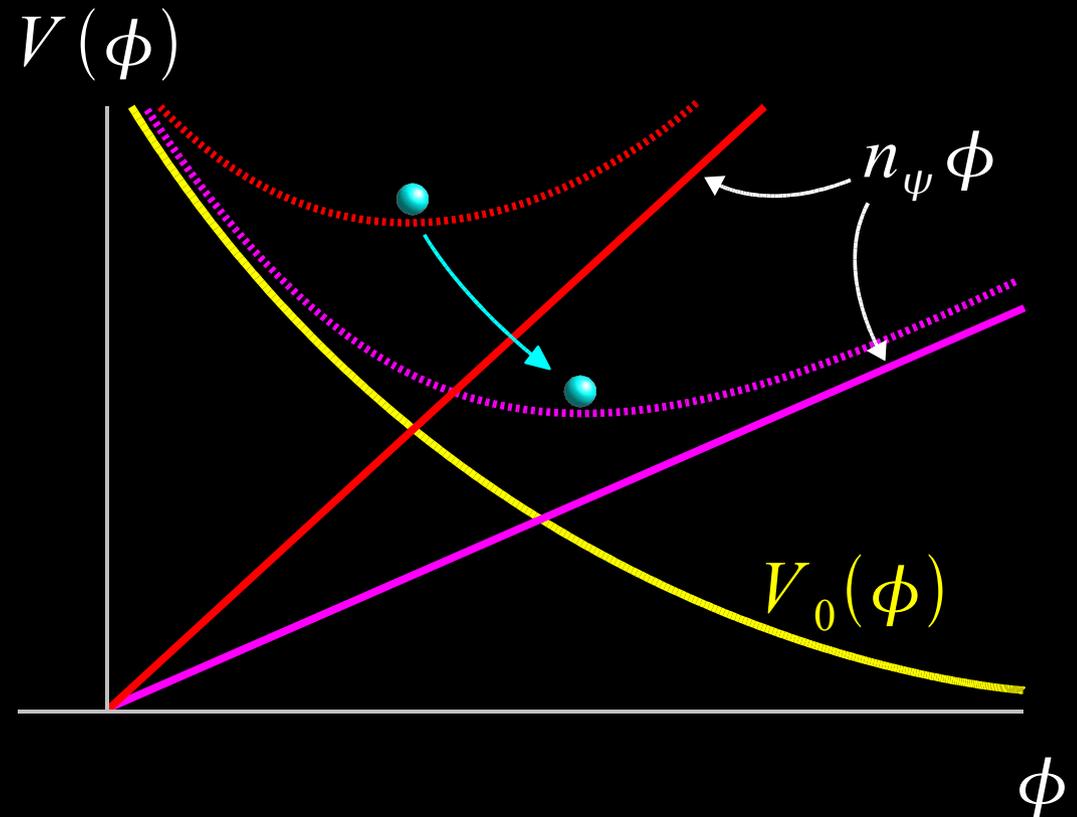
Can be avoided if we put a cutoff on the theory:
momenta less than 10^{-3} eV.

Theorists need to be careful, but observers should
keep an open mind. **Nobody ever measures w , really.**
We only measure the behavior of the scale factor.

Keep in mind: the "dark sector" could be complicated.

Variable-mass particles
("vamps") are dark-matter candidates coupled to a dark-energy field, so their masses increase with time.

[Garcia-Bellido et al.; Anderson & Carroll;
Amendola et al; Farrar & Peebles; Hoffman;
Gubser& Peebles]



Alternatively: neutrinos could couple directly to the dark-energy field, perhaps affecting experiments such as LSND.

[Fardon, Kaplan, Nelson & Weiner]

Moral: Models start simply, but reality could be complicated.

Is General Relativity breaking down?

We could try to modify the Friedmann equation to introduce a scale that only becomes important at late times. Phenomenological approaches:

- 1) modified energy-density dependence

$$H^2 = \frac{8\pi G}{3} \rho \left[1 + \left(\frac{\rho_x}{\rho} \right)^\alpha \right]$$

[Freese & Lewis]

- 2) modified Hubble-parameter dependence

$$H^2 \left[1 + \left(\frac{H_x}{H} \right)^\beta \right] = \frac{8\pi G}{3} \rho$$

[Dvali & Turner]

It's hard to distinguish between these and dark energy.

What about an actual theory? Here is a simple toy model that departs from GR at long distances:

Replace the Einstein-Hilbert action

$$S = \frac{1}{16\pi G} \int R d^4x$$

(R is the curvature scalar) with the modified form

$$S = \frac{1}{16\pi G} \int \left(R - \frac{m^4}{R} \right) d^4x$$

This implies deviations from ordinary GR at **low curvatures**.

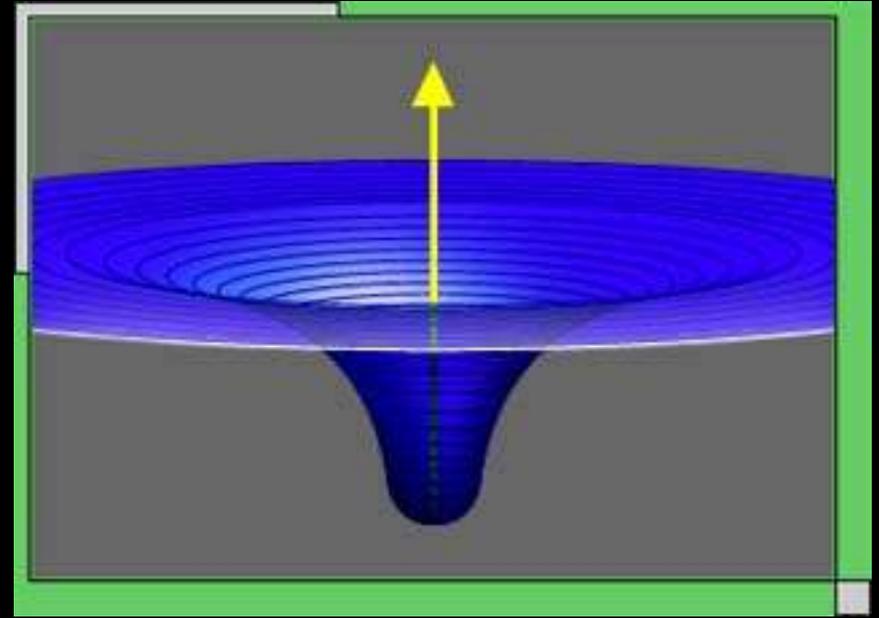
Problem: runs afoul of solar-system tests of gravity.

Can it be fixed?

[Carroll, Duvvuri, Trodden & Turner; Chiba; see also Dvali, Gabadadze & Porrati]

Origins Questions

Inflation is the guiding principle behind much thought about the very early universe. From a tiny starting patch, accelerated expansion creates a smooth, flat universe that expands into our own.



Explains: homogeneity, isotropy, flatness, absence of monopoles, nearly scale-free primordial fluctuations

Predictions:

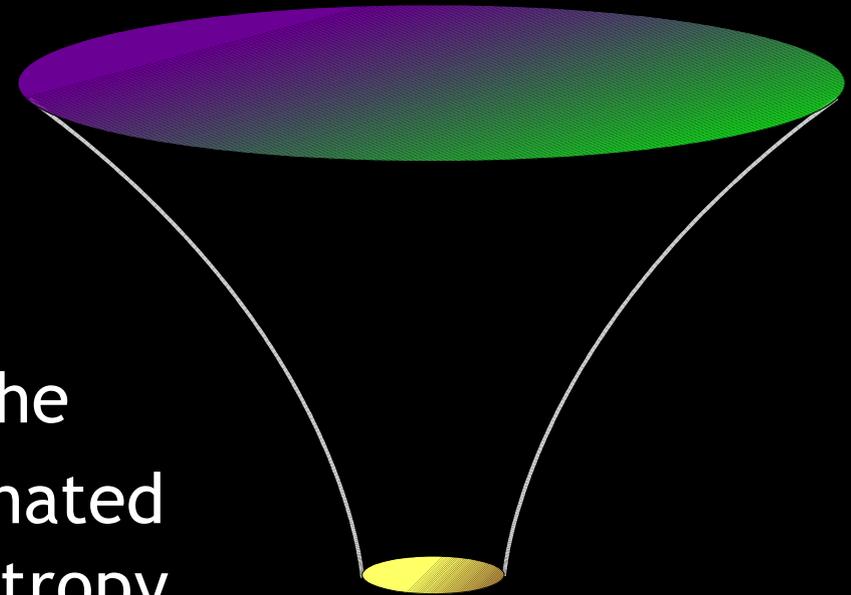
- fluctuations should not be precisely scale-free
- tensor gravity-wave fluctuations should exist along with scalar fluctuations; potentially observable in CMB B-mode polarization

A deep conceptual issue about inflation: Does it really provide more “natural” initial conditions?

Basic issue: Entropy of our current universe is about $S_{\text{today}} \sim 10^{100}$, and the entropy of the early radiation-dominated universe was $S_{\text{rad}} \sim 10^{88}$. But the entropy of a tiny inflationary patch is only $S_{\text{infl}} \sim 10^{10}$.

So: if we are going to “randomly fluctuate” into some state, shouldn't it be a high-entropy state, not a low-entropy one?

Moral: we really do need to understand the pre-inflationary universe, i.e. have a theory of initial conditions.

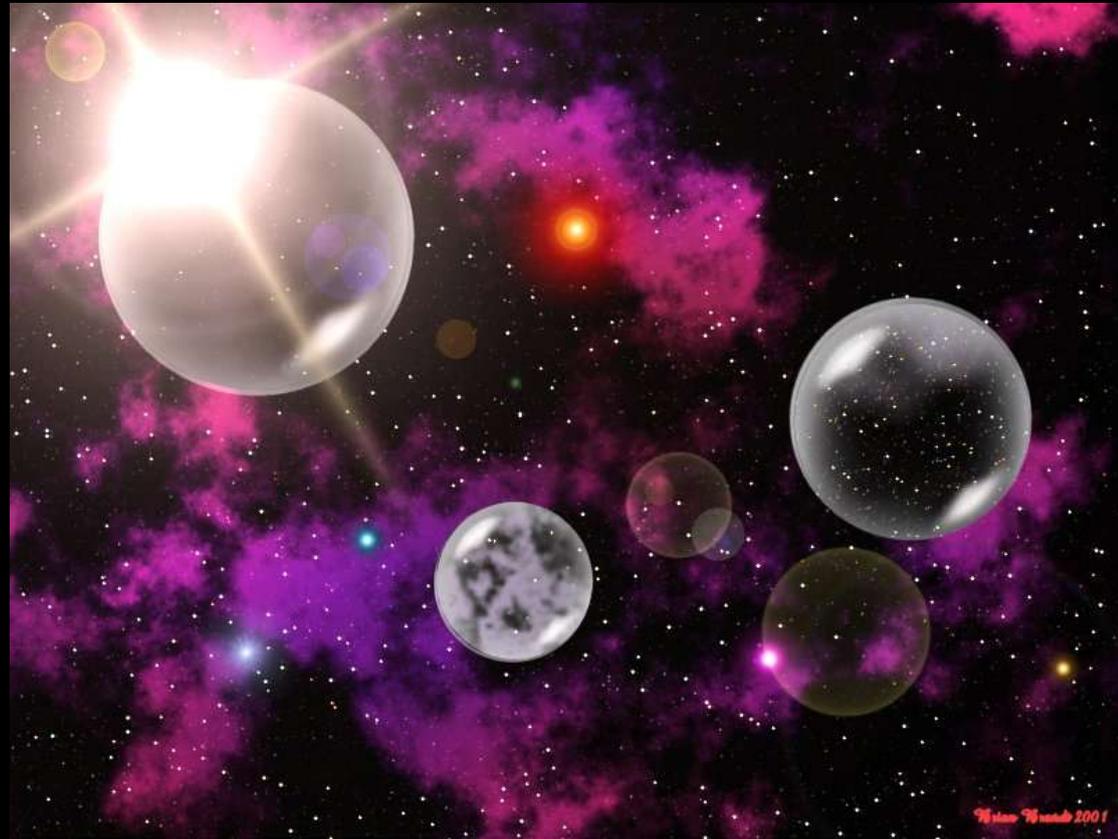


The multiverse and environmental selection (the "anthropic principle")

Imagine that:

- There are many disconnected "universes."
- They each have a different vacuum energy.

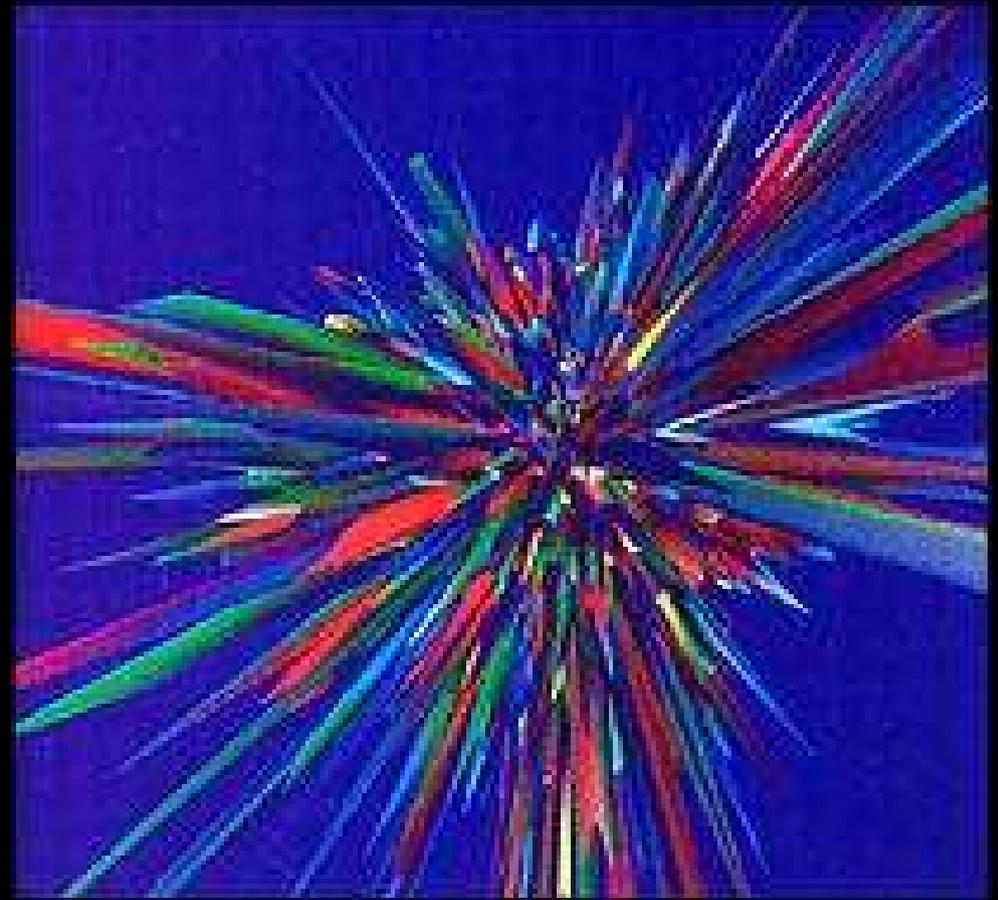
Then we could never observe regions where the vacuum energy is large enough to rip us to shreds - the ultimate selection effect.



String theory might plausibly predict that there can be regions of space with utterly different physical properties. Perhaps 10^{500} different vacuum states.

The idea of **eternal inflation** says that inflation always continues in some parts of the universe. Our observable universe is just an infinitesimal patch in the big picture; elsewhere, the universe can be in other vacuum states.

Here is the hope:
In this large ensemble of “universes,” all parameters of physics and cosmology take on different values. But the likelihood that intelligent life arises is strongly concentrated in regions with certain calculable characteristics, e.g. low vacuum energy.



[Linde; Garriga & Vilenkin;
Martel, Shapiro & Weinberg]

What you should think about the anthropic principle

Statement you certainly agree with:

Intelligent life only arises under conditions that allow for the existence of intelligent life.

Statement you really should agree with:

If there are different conditions in various parts of the universe, we will only ever observe those consistent with the existence of intelligent life.

Statements that may someday be true, but certainly not yet:

Current theories predict the existence and distribution of countless regions outside our observable universe, each with very different conditions.

We understand what “intelligent life” is, and when it can exist.

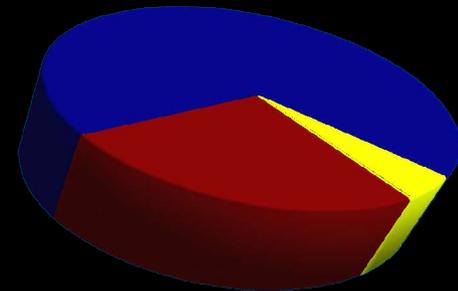
We can use the above information to predict likely values of observed quantities such as the cosmological constant.

“It makes me sad to think that the vacuum energy might be a random variable whose observed value is determined by a selection effect. Isn't that a good argument against this idea?”

No.

Conclusions

- The last hundred years have given us a remarkable picture of the universe; the last ten years have brought it into sharp focus.
- We are blessed with puzzles about the evolution, composition, and origin of the universe.
- Theoretical work is being driven by data, but also reaches out to string theory and quantum gravity. New experiments promise to keep us on our toes.
- We know much, we understand very little. What more could you ask for?



Complementarity: try to detect ambient dark matter

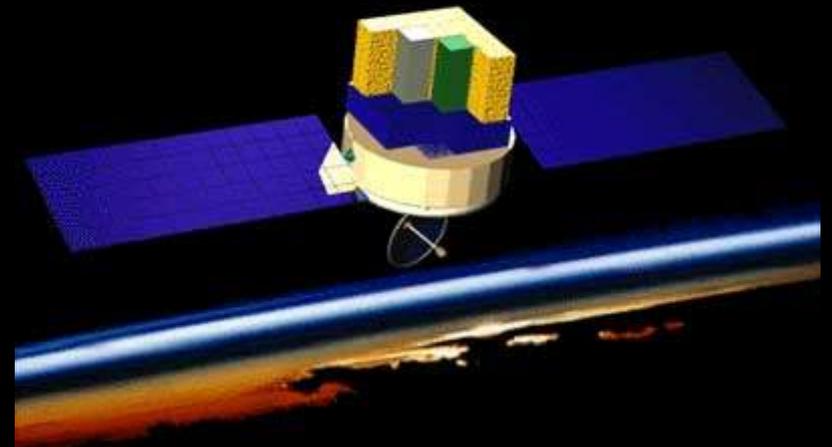
- **Directly:**
look for signs of WIMP scattering off of a cryogenic detector

[CDMS]



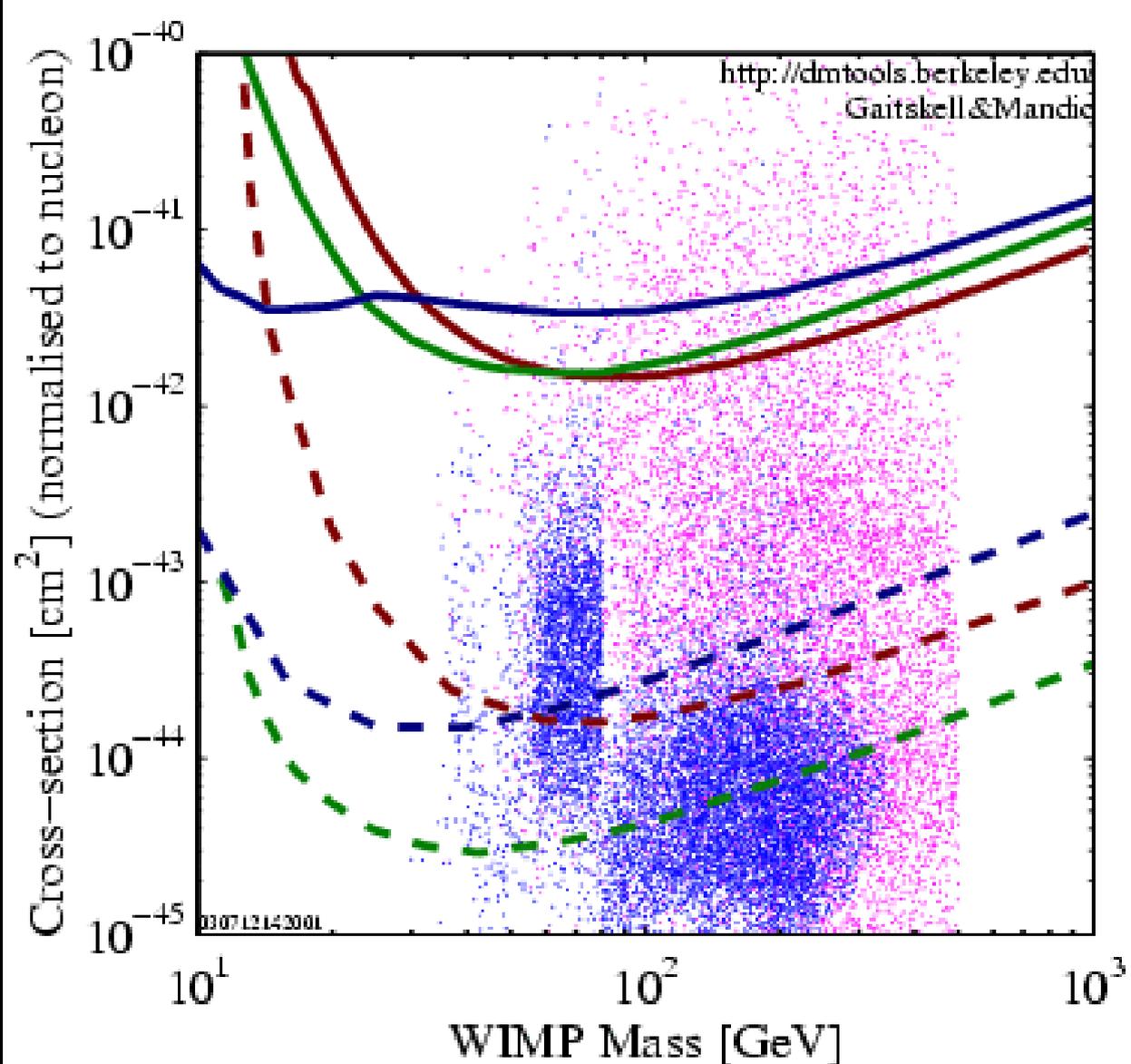
- **Indirectly:**
look for annihilation products (e.g. γ -rays) of DM in galaxy

[GLAST]



State of the art:

- beginning to cut into interesting parameter space
- will do much better
- won't ever cover all of interesting parameter space



- DATA listed top to bottom on plot
 - CDMS June 2003, bkgd subtracted
 - ZEPLIN I Preliminary 2002 result
 - Edelweiss, 32 kg-days Ge 2000+2002+2003 limit
 - - Edelweiss 2 projection
 - - CDMS, projected at Soudan mine
 - - ZEPLIN + projection
 - V. Bednyakov et al., Z.Phys.A 357 (1997) 339 SUSY MSSM
 - Bottino et al., hep-ph/0001309 SUSY
- 03071214.2001