

HEAVY QUARKONIUM  
UPSILON PRODUCTION IN HADRON COLLIDER

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1. Motivation — **Upsilon new data in Tevatron**
2. Factorization in Upsilon Production
3. Resummation in small  $p_T$
4. Color evaporation model (CEM)
5. Upsilon results in CEM.

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## Motivation — Upsilon new data in Tevatron

Perturbation Theory of QCD are very successful in high  $p_T$   
New Data from Tevatron test QCD to a new level of accuracy.

$p_T \sim$  small,

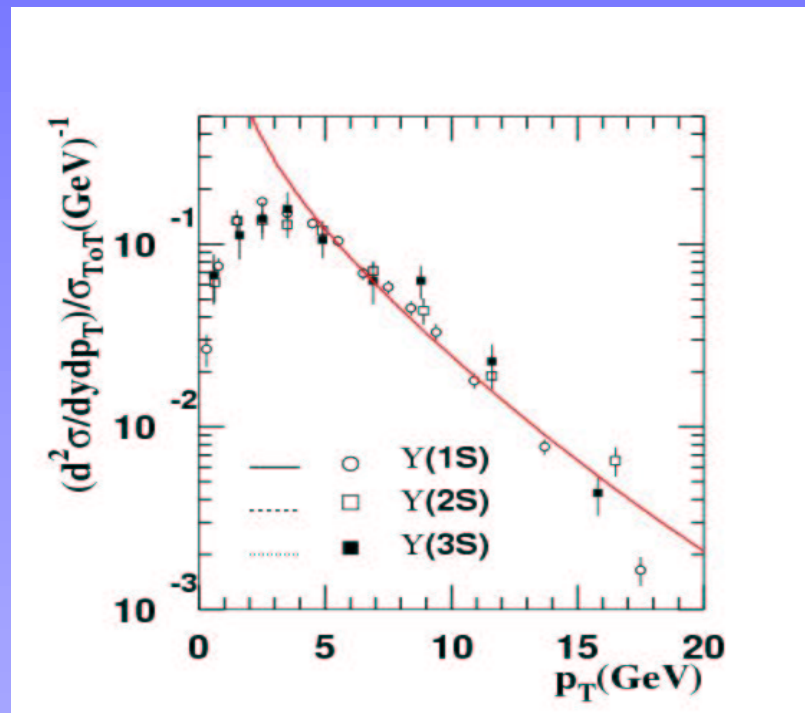
Soft gluon radiation

Perturbative value

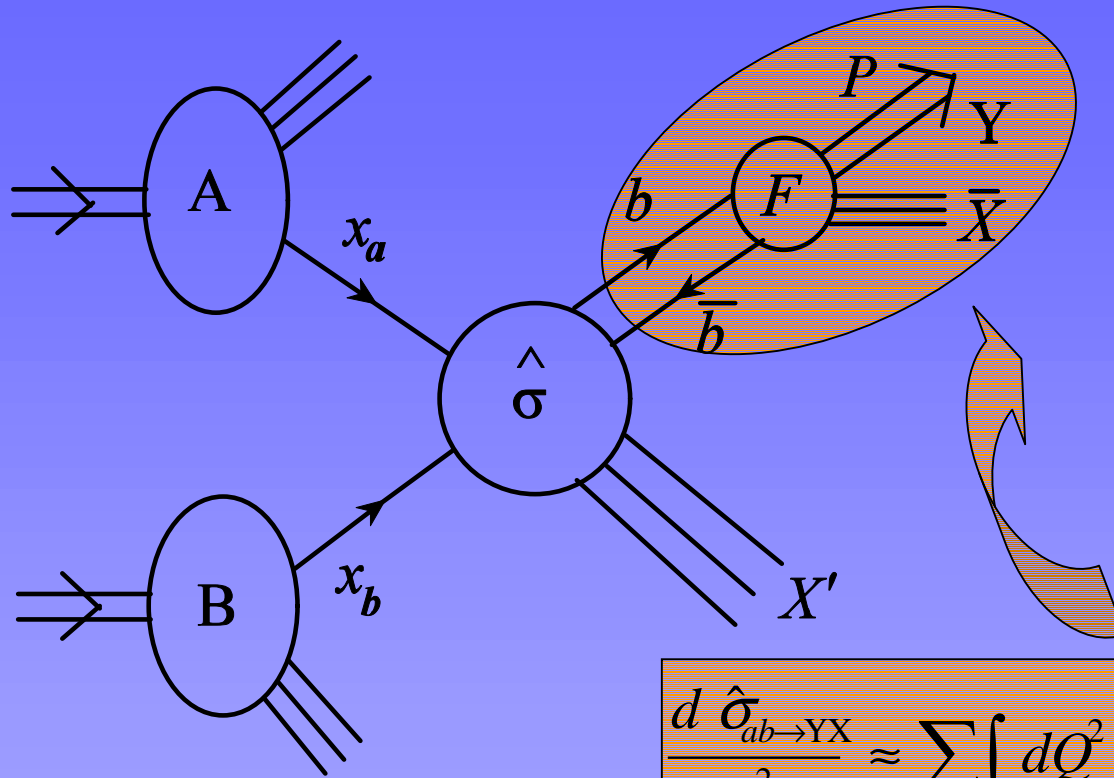
— infinite

Need to be resummed

to all orders in  $\alpha_s$



# Factorization in Upsilon production — two steps



$$\frac{d \hat{\sigma}_{ab \rightarrow YX}}{dp_T^2 dy} \approx \sum_{[b\bar{b}]} \int dQ^2 \left[ \frac{d \hat{\sigma}_{ab \rightarrow [b\bar{b}](Q)X}}{dQ^2 dp_T^2 dy} \right] F_{[b\bar{b}] \rightarrow YX}(Q^2)$$

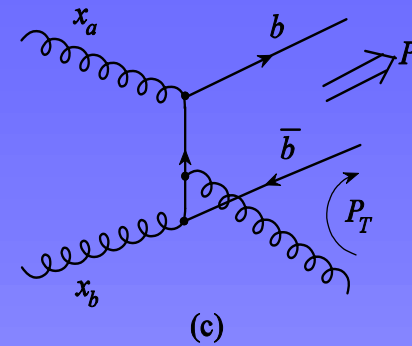
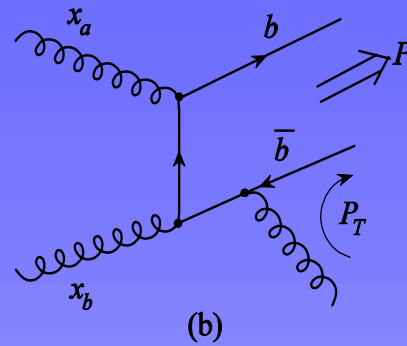
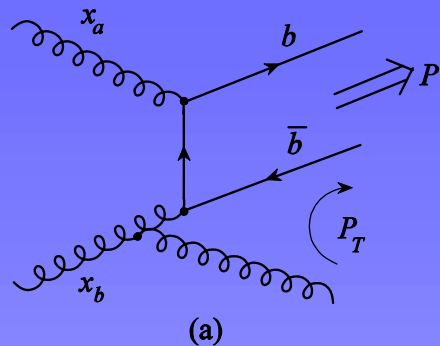
$$\frac{d\sigma_{AB \rightarrow YX}}{dp_T^2 dy} = \sum_{a,b} \int dx_a \phi_{a/A}(x_a) dx_b \phi_{b/B}(x_b) \frac{d\hat{\sigma}_{ab \rightarrow YX}}{dp_T^2 dy}$$

$$\frac{d\sigma_{AB \rightarrow YX}}{dQ^2 dp_T^2 dy} = \sum_{a,b} \int dx_a \phi_{a/A}(x_a) dx_b \phi_{b/B}(x_b)$$

$$\sum_{[bb]} \left[ \frac{d \hat{\sigma}_{ab \rightarrow [bb]}(Q, X')}{dQ^2 dp_T^2 dy} \right] F_{[bb] \rightarrow YX}(Q^2)$$

$$\frac{d \sigma_{AB \rightarrow YX}}{dQ^2 dp_T^2 dy} \approx \begin{cases} \frac{d\sigma_{AB \rightarrow YX}^{resum}}{dQ^2 dp_T^2 dy} & p_T < p_{T_M} \\ \frac{d\sigma_{AB \rightarrow YX}^{pert}}{dQ^2 dp_T^2 dy} & p_T \geq p_{T_M} \end{cases}$$

# Resummation in small $p_T$



Absorbed into  
 $F_{[b\bar{b}] \rightarrow YX}(Q^2)$

No included for  
 Tevatron data

Resummed an infinite series of soft gluon emissions

$$\sigma \approx \sigma^{(0)} e^{-S \left( \ln \frac{M_Y^2}{p_T^2} \right)}$$

## Collins, Soper and Sterman (CSS) b-space approach

$$\frac{d\sigma_{AB \rightarrow [b\bar{b}]X}^{(resum)}}{dQ^2 dp_T^2 dy} = \frac{1}{(2\pi)^2} \int d^2b e^{i \vec{p}_T \cdot \vec{b}} W_{AB \rightarrow [b\bar{b}]X}(b, Q, x_A, x_B) \\ \int \frac{db}{2\pi} J_0(p_T b) b W_{AB \rightarrow [b\bar{b}]X}(b, Q, x_A, x_B)$$

Two subprocesses:  $gg \rightarrow b\bar{b}$  and  $q\bar{q} \rightarrow b\bar{b}$

$$W_{AB \rightarrow [b\bar{b}]X}(b, Q, x_A, x_B) \equiv \sum_q w_{q\bar{q}}(b, Q, x_A, x_B) \frac{d\hat{\sigma}_{q\bar{q} \rightarrow [b\bar{b}]X}^{(LO)}}{dQ^2} \\ + w_{gg}(b, Q, x_A, x_B) \frac{d\hat{\sigma}_{gg \rightarrow [b\bar{b}]X}^{(LO)}}{dQ^2}$$

$b < 1 \text{ GeV}^{-1}$

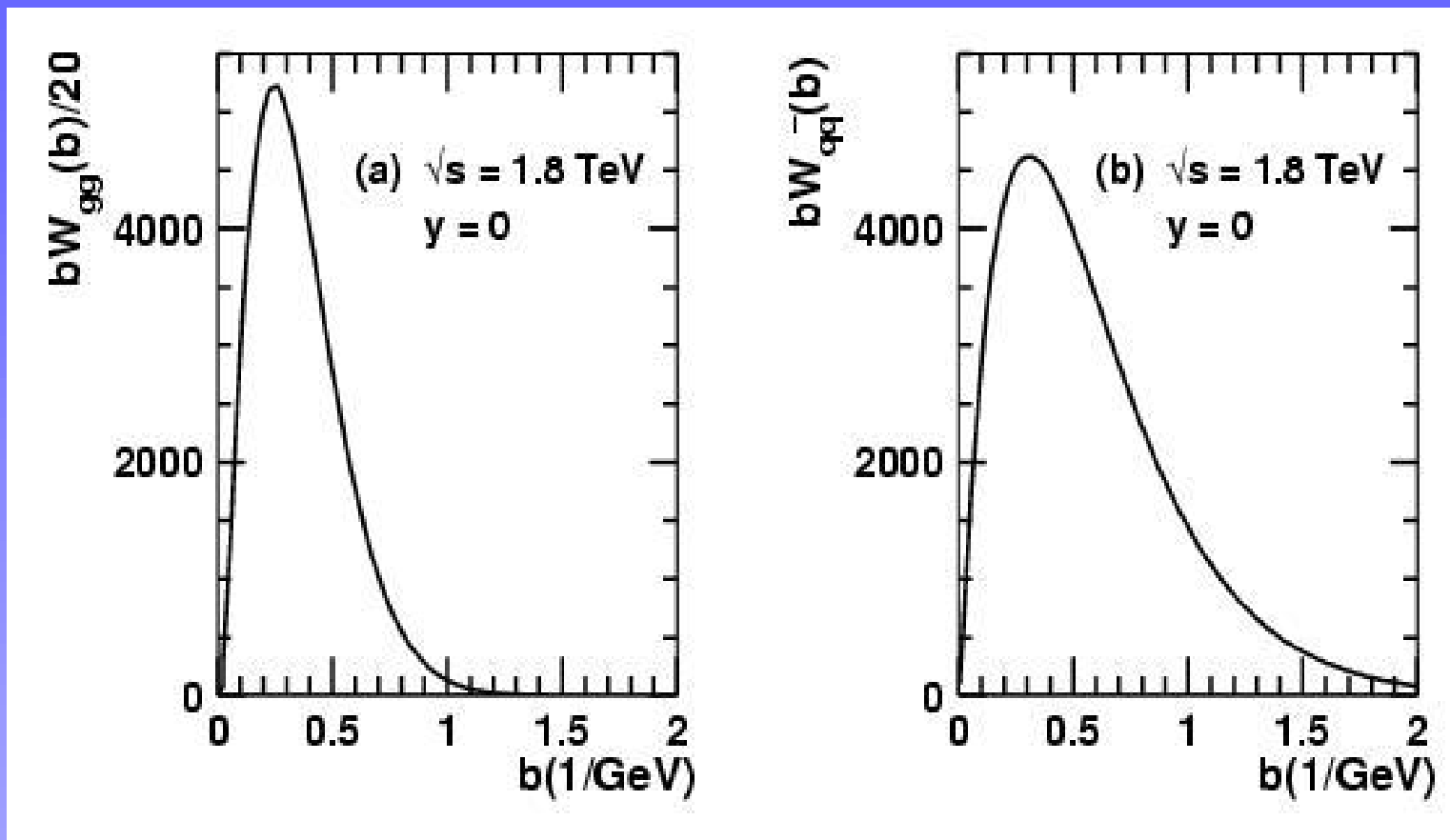
$$w_{q\bar{q}}(b, Q, x_A, x_B) \equiv e^{-S_q(b, Q)} w_{q\bar{q}}\left(b, \frac{c}{b}, x_A, x_B\right)$$

$$w_{gg}(b, Q, x_A, x_B) \equiv e^{-S_g(b, Q)} w_{gg}\left(b, \frac{c}{b}, x_A, x_B\right)$$

$$S_{q, g}(b, Q) = \int_{c^2/b^2}^{Q^2} \frac{d\mu^2}{\mu^2} \left[ \ln\left(\frac{Q^2}{\mu^2}\right) A_{q, g}(\alpha_S(\mu)) + B_{q, g}(\alpha_S(\mu)) \right]$$

$$w_{ij}\left(b, \frac{c}{b}, x_A, x_B\right) = f_{i/A}\left(x_A, \mu, \frac{c}{b}\right) f_{j/B}\left(x_B, \mu, \frac{c}{b}\right)$$

$$f_{i/A}\left(x_A, \mu, \frac{c}{b}\right) = \sum_a \int_{x_A}^1 \frac{d\xi}{\xi} \phi_{a/A}(\xi, \mu) C_{a \rightarrow i}\left(\frac{x_A}{\xi}, \mu, \frac{c}{b}\right)$$



$$w_{ij}(b, Q, x_A, x_B) = \begin{cases} w_{ij}(b, Q, x_A, x_B) & b \leq b_{\max} \\ w_{ij}(b, Q, x_A, x_B) F_{ij}^{NP}(b, Q, b_{\max}) & b > b_{\max} \end{cases}$$

$$F_{ij}^{NP} = \exp \left\{ -\ln \left( \frac{Q^2 b_{\max}^2}{c^2} \right) \left[ g_1 \left( (b^2)^\alpha - (b_{\max}^2)^\alpha \right) - g_2 (b^2 - b_{\max}^2) - \bar{g}_2 (b^2 - b_{\max}^2) \right] \right\}$$

$g_1$  and  $\alpha$  are from  $x_A$  and  $x_B$  dependences

**fixed by continuous**

$g_2$  and  $\bar{g}_2$  are from power corrections

**fixed by fitting data**

## Color evaporation model (CEM) hep-ph/0404158

Ignore detail of the formation of Upsilon from Quarkonium

$$F_{[b\bar{b}] \rightarrow Y}(Q^2) = \begin{cases} C_Y & M_Y^2 \leq Q^2 \leq M_B^2 \\ 0 & \text{otherwise} \end{cases}$$

$$\frac{d \hat{\sigma}_{ab \rightarrow YX}^{CEM}}{dp_T^2 dy} \approx C_Y \int_{M_Y^2}^{M_B^2} dQ^2 \left[ \frac{d \hat{\sigma}_{ab \rightarrow [b\bar{b}]}(Q)}{dQ^2 dp_T^2 dy} \right]$$

## Non-relativistic QCD Model (NRQCD)

$$\frac{d \hat{\sigma}_{ab \rightarrow YX}}{dp_T^2 dy} \approx \sum_{[b\bar{b}]} \left[ \frac{d \hat{\sigma}_{ab \rightarrow [b\bar{b}]}(Q)}{dQ^2 dp_T^2 dy} \left( Q^2 = M_Y^2 \right) \right] \int dQ^2 F_{[b\bar{b}] \rightarrow YX}(Q^2)$$

$$[b\bar{b}] \Rightarrow {}^1S_0^{[1,8]}, {}^3S_1^{[1,8]}, {}^1P_1^{[1,8]} \dots \quad \langle \hat{O}_{[b\bar{b}]} \rangle$$

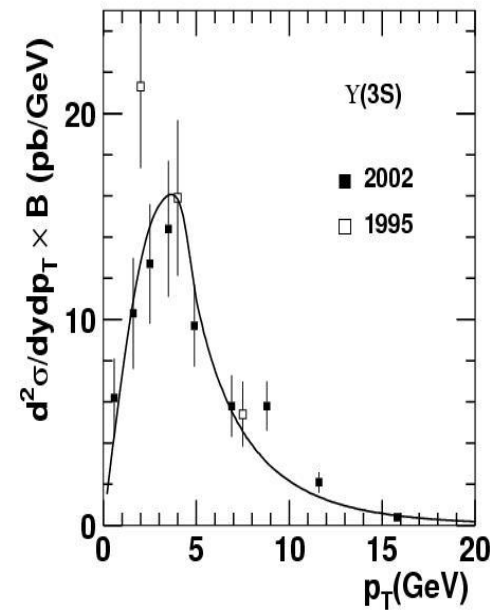
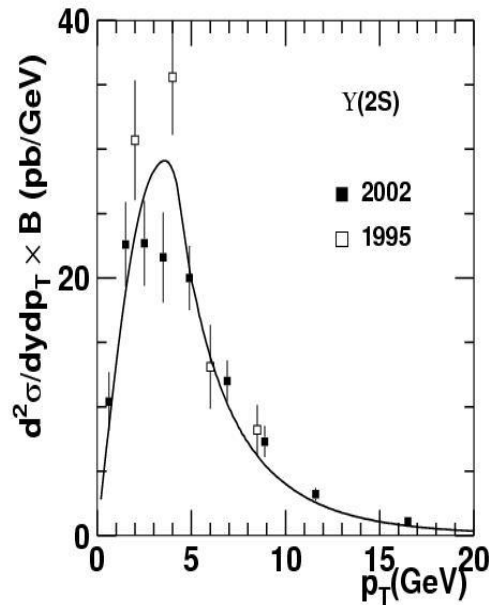
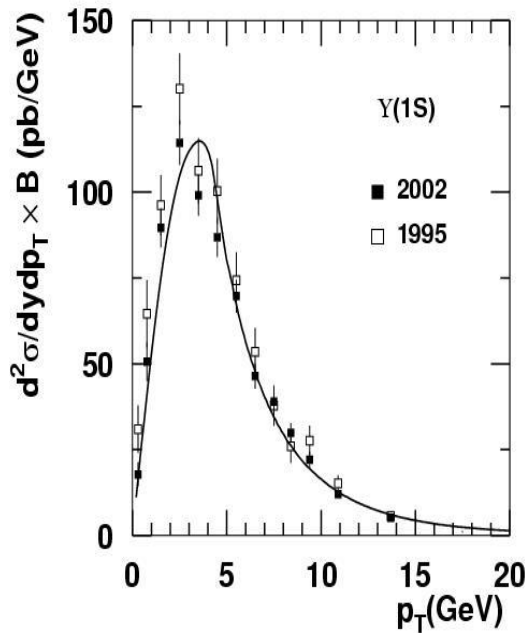
## Upsilon results in color evaporation model (CEM)

$$\frac{d\sigma_{AB \rightarrow Y(nS)X}}{dp_T^2 dy} = C_Y \int_{M_Y^2}^{4M_B^2} dQ^2 \sum_{a,b} \int dx_a \phi_{a/A}(x_a) dx_b \phi_{b/B}(x_b) \frac{d\hat{\sigma}_{ab \rightarrow [b\bar{b}](Q)X}}{dQ^2 dp_T^2 dy}$$

order of  $\alpha$  corrections

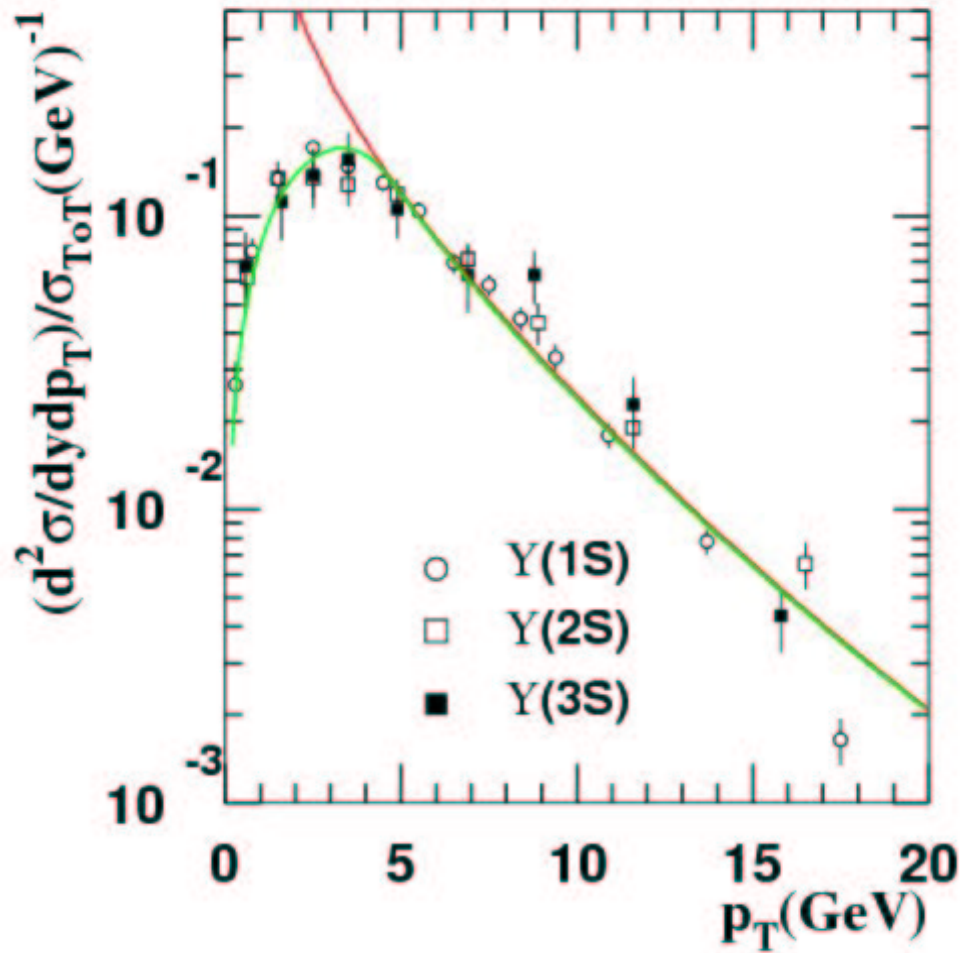
$$\frac{d\sigma_{AB \rightarrow Y(nS)X}}{dp_T^2 dy} \approx \begin{cases} \kappa_Y \frac{d\sigma_{AB \rightarrow Y(nS)X}^{resum}}{dp_T^2 dy} & p_T < p_{T_M} \\ \frac{d\sigma_{AB \rightarrow Y(nS)X}^{pert}}{dp_T^2 dy} & p_T \geq p_{T_M} \end{cases}$$

resummed and perturbative parts match at  $p_{T_M} \leq M_Y$



$p_{T_M} \sim 4.27 \text{ GeV},$
$\kappa_Y = 1.22 \pm 0.02$

	$C_Y$
Y(1S)	0.044
Y(2S)	0.040
Y(3S)	0.041



## Conclusion

- Upsilon is derived from a pair of bottom quarks.
- resummed large logarithmic contributions from initial-state gluon showers at small  $p_T$ .
- Results are in good agreement with data from Tevatron.