QED $\otimes$ QCD Threshold Corrections at the LHC

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Outline:

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- **QED $\otimes$ QCD Threshold Corrections at the LHC**
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Papers by S. Jadach and B.F.L. Ward, S. Jadach, *et al.*, M. Phys. Lett. A **14** (1999) 491, hep-ph/0205062; *ibid.* **12** (1997) 2425;hep-ph/0404087

Motivation

- FNAL/RHIC $t\bar{t}$ PRODUCTION; POLARIZED pp PROCESSES; $b\bar{b}$ PRODUCTION; J/Ψ PRODUCTION: SOFT n(G) EFFECTS ALREADY NEEDED

  $\Delta m_t = 5.1$ GeV with SOFT n(G) UNCERTAINTY $\sim$ 2-3 GeV, ..., ETC.

- FOR THE LHC/TESLA/LC, THE REQUIREMENTS WILL BE EVEN MORE DEMANDING AND OUR QCD SOFT n(G) MC EXPONENTIATION RESULTS WILL BE AN IMPORTANT PART OF THE NECESSARY THEORY – YFS EXPONENTIATED $\mathcal{O}(\alpha_s^2)L$, ON AN EVENT-BY-EVENT BASIS

- HOW RELEVANT ARE QED HIGHER ORDER CORRECTIONS WHEN QCD IS CONTROLLED AT $\sim 1\%$ PRECISION?

- CROSS CHECK OF QCD LITERATURE:
  1. PHASE SPACE – CATANI, CATANI-SEYMOUR, ALL INITIAL PARTONS MASSLESS
  2. RESUMMATION – CATANI ET AL., BERGER ET AL., ...
  3. NO-GO THEOREMS

- CROSS CHECK OF QED LITERATURE:
  1. ESTIMATES BY SPIESBERGER, STIRLING, ROTH and WEINZIERL – FEW PER MILLE EFFECTS FROM QED CORRECTIONS TO STR. FN. EVOLUTION.

2. WELL-KNOWN POSSIBLE ENHANCEMENT OF QED CORRECTIONS AT THRESHOLD, ESPECIALLY IN RESONANCE PRODUCTION

$\Rightarrow$ HOW BIG ARE THESE EFFECTS AT THE LHC?

- TREAT QED AND QCD SIMULTANEOUSLY IN THE YFS EXPONENTIATION TO ESTIMATE THE ROLE OF THE QED.

PRELIMINARIES

- WE USE THE GPS CONVENTIONS OF JWW FOR SPINORS.
- PHOTON-GLUON POLARIZATION VECTORS FOLLOW THEREFROM:

$$(\epsilon^{\mu}_{\sigma}(\beta))^{*} = \frac{\bar{u}_{\sigma}(k)\gamma^{\mu}u_{\sigma}(\beta)}{\sqrt{2}\;\bar{u}_{-\sigma}(k)u_{\sigma}(\beta)}, \quad (\epsilon^{\mu}_{\sigma}(\zeta))^{*} = \frac{\bar{u}_{\sigma}(k)\gamma^{\mu}\mathfrak{u}_{\sigma}(\zeta)}{\sqrt{2}\;\bar{u}_{-\sigma}(k)\mathfrak{u}_{\sigma}(\zeta)}, \quad (1)$$

- REPRESENTATIVE PROCESSES

$pp \to V + n(\gamma) + m(g) + X \to \bar{\ell}\ell' + n'(\gamma) + m(g) + X$,

where $V = W^{\pm}, Z$,and $\ell = e, \mu,\ \ell' = \nu_e, \nu_\mu(e,\mu)$ respectively for $V = W^{+}(Z)$, and $\ell = \nu_e, \nu_\mu,\ \ell' = e, \mu$ respectively for $V = W^{-}$.

Review of YFS Theory and Its Extension to QCD

QED CASE – S. Jadach et al., YFS2, YFS3, BHLUMI, BHWIDE, KORALZ, KKMC, YFSWW3, YFSZZ, KoralW

For $e^+(p_1)e^-(q_1) \to \bar{f}(p_2)f(q_2) + n(\gamma)(k_1, \cdot, k_n)$**, renormalization group improved YFS theory (PRD36(1987)939) gives**

$$d\sigma_{exp} = e^{2\alpha\, Re\, B + 2\alpha\, \tilde{B}} \sum_{n=0}^{\infty} \frac{1}{n!} \int \prod_{j=1}^{n} \frac{d^3 k_j}{k_j^0} \int \frac{d^4 y}{(2\pi)^4} e^{iy(p_1+q_1-p_2-q_2-\sum_j k_j)+D}$$

$$\bar{\beta}_n(k_1, \ldots, k_n) \frac{d^3 p_2 d^3 q_2}{p_2^0 q_2^0}$$

where the YFS real infrared function $\tilde{B}$ and the virtual infrared function B are known and where we note the usual connections

$$2\alpha\, \tilde{B} = \int^{k \le K_{max}} \frac{d^3 k}{k_0} \tilde{S}(k)$$

$$D = \int d^3 k \frac{\tilde{S}(k)}{k^0} \left(e^{-iy\cdot k} - \theta(K_{max} - k)\right) \quad (2)$$

for the standard YFS infrared emission factor

$$\tilde{S}(k) = \frac{\alpha}{4\pi^2}\left[Q_f Q_{(\bar{f})'}\left(\frac{p_1}{p_1\cdot k} - \frac{q_1}{q_1\cdot k}\right)^2 + (\ldots)\right] \tag{3}$$

if Q_f is the electric charge of f in units of the positron charge. For example, the YFS hard photon residuals $\bar{\beta}_i$ in (1), $i = 0, 1, 2$, are given in S. Jadach *et al.*,CPC102(1997)229 for BHLUMI 4.04 $\Rightarrow$ YFS exponentiated exact $\mathcal{O}(\alpha)$ and LL $\mathcal{O}(\alpha^2)$ cross section for Bhabha scattering via a corresponding Monte Carlo realization of (1).

In hep-ph/0210357(ICHEP02), Acta Phys.Polon.B33,1543-1558,2002, we have extended the YFS theory to QCD:

$$d\hat{\sigma}_{\rm exp} = \sum_n d\hat{\sigma}^n$$
$$= e^{\rm SUM_{IR}(QCD)} \sum_{n=0}^{\infty} \int \prod_{j=1}^{n} \frac{d^3 k_j}{k_j} \int \frac{d^4 y}{(2\pi)^4} e^{iy\cdot(P_1+P_2-Q_1-Q_2-\sum k_j)+D_{\rm QCD}}$$
$$* \tilde{\bar{\beta}}_n(k_1,\ldots,k_n)\frac{d^3P_2}{P_2^{\,0}}\frac{d^3Q_2}{Q_2^{\,0}} \quad (4)$$

where now the hard gluon residuals $\tilde{\bar{\beta}}_n(k_1,\ldots,k_n)$ defined by

$$\tilde{\bar{\beta}}_n(k_1,\ldots,k_n) = \sum_{\ell=0}^{\infty} \tilde{\bar{\beta}}_n^{(\ell)}(k_1,\ldots,k_n)$$

are free of all infrared divergences to all orders in $\alpha_s(Q)$.

- **We stress that the arguments in the earlier papers (DeLaney *et al.* PRD52(1995)108, PLB342(1995)239) are not really sufficient to derive the respective analog of eq.(4); for, they did not really expose the compensation between the left over genuine non-Abelian IR virtual and real singularities between $\int dPh\bar{\beta}_n$ and $\int dPh\bar{\beta}_{n+1}$ respectively that really allows us to isolate $\tilde{\bar{\beta}}_j$ and distinguishes QCD from QED, where no such compensation occurs.**

- **Note the following: In**

$$d\sigma_{exp}(pp \to V + X \to \bar{\ell}\ell' + X') = \sum_{i,j} \int dx_i dx_j F_i(x_i) F_j(x_j) d\hat{\sigma}_{exp}(x_i x_j s), \tag{5}$$

WE DO NOT ATTEMPT TO REPLACE HERWIG and/or PYTHIA – WE INTEND TO COMBINE OUR EXACT YFS CALCULUS WITH HERWIG and/or PYTHIA BY USING THE LATTER IN LIEU OF THE $\{F_i\}$.

- THIS COMBINATION OF THEORETICAL CONSTRUCTS CAN BE SYSTEMATICALLY IMPROVED WITH EXACT RESULTS ORDER-BY-ORDER IN α_s.
- THE RECENT ALTERNATIVE PARTON SHOWER ALGORITHM BY JADACH and SKRZYPEK,hep-ph/0312355, Acta. Phys. Pol.B35, 745 (2004), CAN ALSO BE USED.
- DUE TO ITS LACK OF THE APPROPRIATE COLOR COHERENCE, WE DO NOT CONSIDER ISAJET HERE.

Extension to QED⊗QCD and QCED

Simultaneous exponentiation of QED and QCD higher order effects,

hep-ph/0404087,

gives

$$\begin{aligned} B^{nls}_{QCD} &\rightarrow B^{nls}_{QCD} + B^{nls}_{QED} \equiv B^{nls}_{QCED}, \\ \tilde{B}^{nls}_{QCD} &\rightarrow \tilde{B}^{nls}_{QCD} + \tilde{B}^{nls}_{QED} \equiv \tilde{B}^{nls}_{QCED}, \\ \tilde{S}^{nls}_{QCD} &\rightarrow \tilde{S}^{nls}_{QCD} + \tilde{S}^{nls}_{QED} \equiv \tilde{S}^{nls}_{QCED} \end{aligned} \tag{6}$$

which leads to

$$\begin{aligned} d\hat{\sigma}_{\rm exp} = e^{\rm SUM_{IR}(QCED)} \sum_{n,m=0}^{\infty} \int \prod_{j_1=1}^{n} \frac{d^3 k_{j_1}}{k_{j_1}} \\ \prod_{j_2=1}^{m} \frac{d^3 {k'}_{j_2}}{{k'}_{j_2}} \int \frac{d^4 y}{(2\pi)^4} e^{iy\cdot(p_1+q_1-p_2-q_2-\sum k_{j_1}-\sum {k'}_{j_2})+D_{\rm QCED}} \\ \tilde{\bar{\beta}}_{n,m}(k_1,\ldots,k_n;k'_1,\ldots,k'_m)\frac{d^3p_2}{p_2^{\,0}}\frac{d^3q_2}{q_2^{\,0}}, \end{aligned} \tag{7}$$

where the new YFS residuals

$\tilde{\bar{\beta}}_{n,m}(k_1,\ldots,k_n;k'_1,\ldots,k'_m)$, **with n hard gluons and m hard photons,** represent the successive application of the YFS expansion first for QCD and subsequently for QED.

The infrared functions are now

$$\mathrm{SUM_{IR}(QCED)} = 2\alpha_s \Re B^{nls}_{QCED} + 2\alpha_s \tilde{B}^{nls}_{QCED}$$
$$D_{\mathrm{QCED}} = \int \frac{dk}{k^0}\left(e^{-iky} - \theta(K_{max} - k^0)\right)\tilde{S}^{nls}_{QCED} \tag{8}$$

where K_{max} is a dummy parameter – here the same for QCD and QED.

Infrared Algebra(QCED):

$x_{avg}(QED) \cong \gamma(QED)/(1+\gamma(QED))$

$x_{avg}(QCD) \cong \gamma(QCD)/(1+\gamma(QCD))$

$\gamma(A) = \frac{2\alpha_A \mathcal{C}_A}{\pi}(L_s - 1)$, $A = QED, QCD$

$\mathcal{C}_A = Q_f^2, C_F$, respectively, for $A = QED, QCD$

$\Rightarrow$ QCD dominant corrections happen an order of magnitude earlier than those for QED.

$\Rightarrow$ Leading $\tilde{\bar{\beta}}^{(0,0)}_{0,0}$ -level gives a good estimate of the size of the effects we study.

QED⊗QCD Threshold Corrections at the LHC

We shall apply the new simultaneous QED⊗QCD exponentiation calculus to the sinlge Z production with leptonic decay at the LHC (and at FNAL) to focus on the ISR alone, for definiteness. See also the work of Baur *et al.*, **Dittmaier and Kramer, Zykunov for exact $\mathcal{O}(\alpha)$ results and Hamberg** *et al.*, **van Neerven and Matsuura and Bern** *et al.* **for exact $\mathcal{O}(alpha_s^2)$ results.**

For the basic formula

$$d\sigma_{exp}(pp \to V + X \to \bar{\ell}\ell' + X') = \sum_{i,j} \int dx_i dx_j F_i(x_i) F_j(x_j) d\hat{\sigma}_{exp}(x_i x_j s) \tag{9}$$

we use the result in (7) here with semi-analytical methods and structure functions from Martin *et al.*.

We compute , with and without QED, the ratio

$r_{exp} = \sigma_{exp}/\sigma_{Born}$

to get the results (We stress that we *do not* use the narrow resonance approximation here.)

$$r_{exp} = \begin{cases} 1.1901 & , \textbf{QCED} \equiv \textbf{QCD+QED, LHC} \\ 1.1872 & , \textbf{QCD, LHC} \\ 1.1911 & , \textbf{QCED} \equiv \textbf{QCD+QED, Tevatron} \\ 1.1879 & , \textbf{QCD, Tevatron} \end{cases} \quad (10)$$

$\Rightarrow$ **QED IS AT .3% AT BOTH LHC and FNAL.**

THIS IS STABLE UNDER SCALE VARIATIONS.

WE AGREE WITH BAUR ET AL., HAMBERG ET AL., van NEERVEN and ZIJLSTRA.

QED EFFECT SIMILAR IN SIZE TO STR. FN. RESULTS

Conclusions

YFS THEORY (EEX AND CEEX) EXTENDS TO NON-ABELIAN GAUGE THEORY AND ALLOWS SIMULTANEOUS EXPN OF QED AND QCD.

FOR QED$\otimes$QCD

- FULL MC EVENT GENERATOR REALIZATION IS POSSIBLE.
- SEMI-ANALYTICAL RESULTS FOR QED (AND QCD) THRESHOLD EFFECTS AGREE WITH LITERATURE ON Z PRODUCTION
- AS QED IS AT THE .3% LEVEL, IT IS NEEDED FOR 1% LHC THEORY PREDICTIONS.
- A FIRM BASIS FOR THE COMPLETE $\mathcal{O}(\alpha_s^2, \alpha\alpha_s, \alpha^2)$ RESULTS NEEDED FOR THE FNAL/LHC/RHIC/TESLA/LC PHYSICS HAS BEEN DEMONSTRATED AND ALL THE LATTER IS IN PROGRESS.