

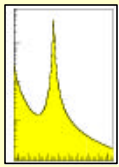
A Fresh Look at Gauge Coupling Unification

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Motivation I

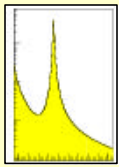
- By 1991 the *weak coupling was measured with much higher precision than the strong one* at LEP
- In a famous paper U. Amaldi et al. (PL B260 (1991) 447) showed that in contrast to the SM the MSSM leads to a single unification scale

$$M_{\text{SUSY}} = 10^{3.0 \pm 1.0} \text{ GeV}$$

$$M_{\text{GUT}} = 10^{16.0 \pm 0.3} \text{ GeV}$$

$$1/\alpha_{\text{GUT}} = 25.7 \pm 1.7$$

- At the time the relative error in $\alpha(M_Z)$, $\sin^2\theta_{\text{MS}}$ and $\alpha_s(M_Z)$ was 0.24 %, 0.77 % and 4.6 %



Motivation II

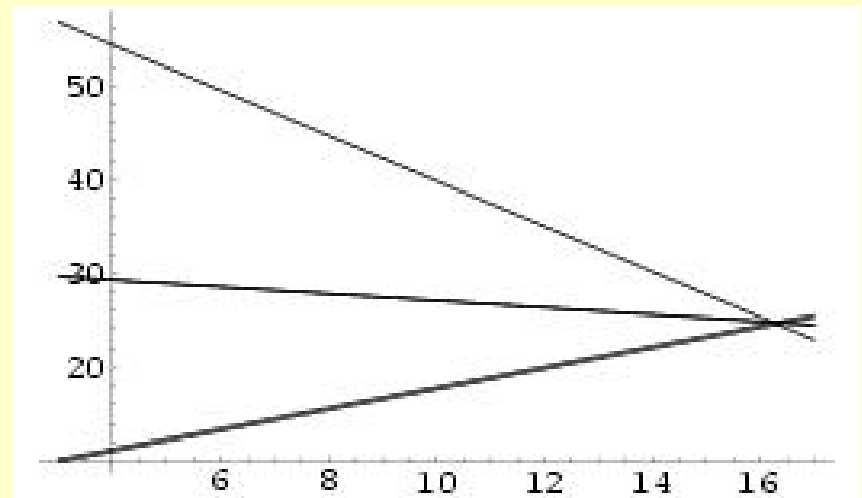
- From RPP 2004 the relative error in $\alpha(M_Z)$, $\sin^2\theta_{\text{MS}}$ and $\alpha_s(M_Z)$ is 0.014%, 0.065% and 1.7%, so we have improved by **more** than an order of magnitude **except for the strong coupling (less than 3 times)**
- So it's time for new analyses (one presented here)

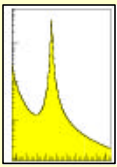
Recent analyses:

W. de Boer, C.Sander,
hep-ph/0307049

N.Arkani-Hamed, S.Dimopoulos,
hep-th/0405159

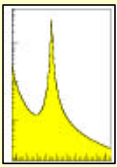
Example of a fit with $M_{\text{SUSY}} = M_{\text{TOP}}$





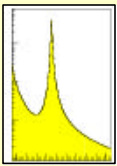
Side Remark

- It is amazing that we are trying to extrapolate from 10^2 to 10^{16} GeV
- From experiments we now that even **interpolation** or modest extrapolations can be non-trivial
- We measure the “offsets” around the Z peak and rely on theory to give us the “slopes” of the running couplings – **without errors** – up to the GUT scale
- This may be an illusion e.g. extra dimensions could modify the running already at TeV scales, so the “unification” point could be imaginary



Experimental Inputs

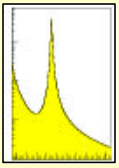
- From Review of Particle Properties 2004
- $1/\alpha(M_Z) = 127.918 \pm 0.018$
- $\sin^2\theta_{MS} = 0.23120 \pm 0.00015$
- $\alpha_s(M_Z)$ is a different story (larger statistical errors and theory uncertainties); will use both the high and low values
 - $\alpha_s(M_Z) = 0.1224 \pm 0.0038$ from $\Gamma_h / \Gamma_\mu @ Z$
 - $\alpha_s(M_Z) = 0.1213 \pm 0.0018$ Global Electroweak Fit
 - $\alpha_s(M_Z) = 0.1187 \pm 0.0020$ RPP QCD section
 - $\alpha_s(M_Z) = 0.1183 \pm 0.0027$ S.Bethke (LEP/SLC, DIS ...)



Analysis technique



- χ^2 minimization with 3 parameters: fit with MINUIT
 $M_{\text{SUSY}}, M_{\text{GUT}}, 1/\alpha_{\text{GUT}}$
- Strong correlation (> 0.999) between M_{GUT} and $1/\alpha_{\text{GUT}}$: problem can be re-factored with 2 parameters, taking $1/\alpha_{\text{GUT}}$ as the weighted average of the 3 couplings at any given scale (results are numerically the same except the error on the GUT coupling)
- Even so the correlation $M_{\text{SUSY}} - M_{\text{GUT}}$ is > 0.96
- Threshold corrections around the GUT scale (-4%)



Running Couplings

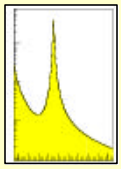


1-loop Renormalization Group (RG) running – can solve analytically

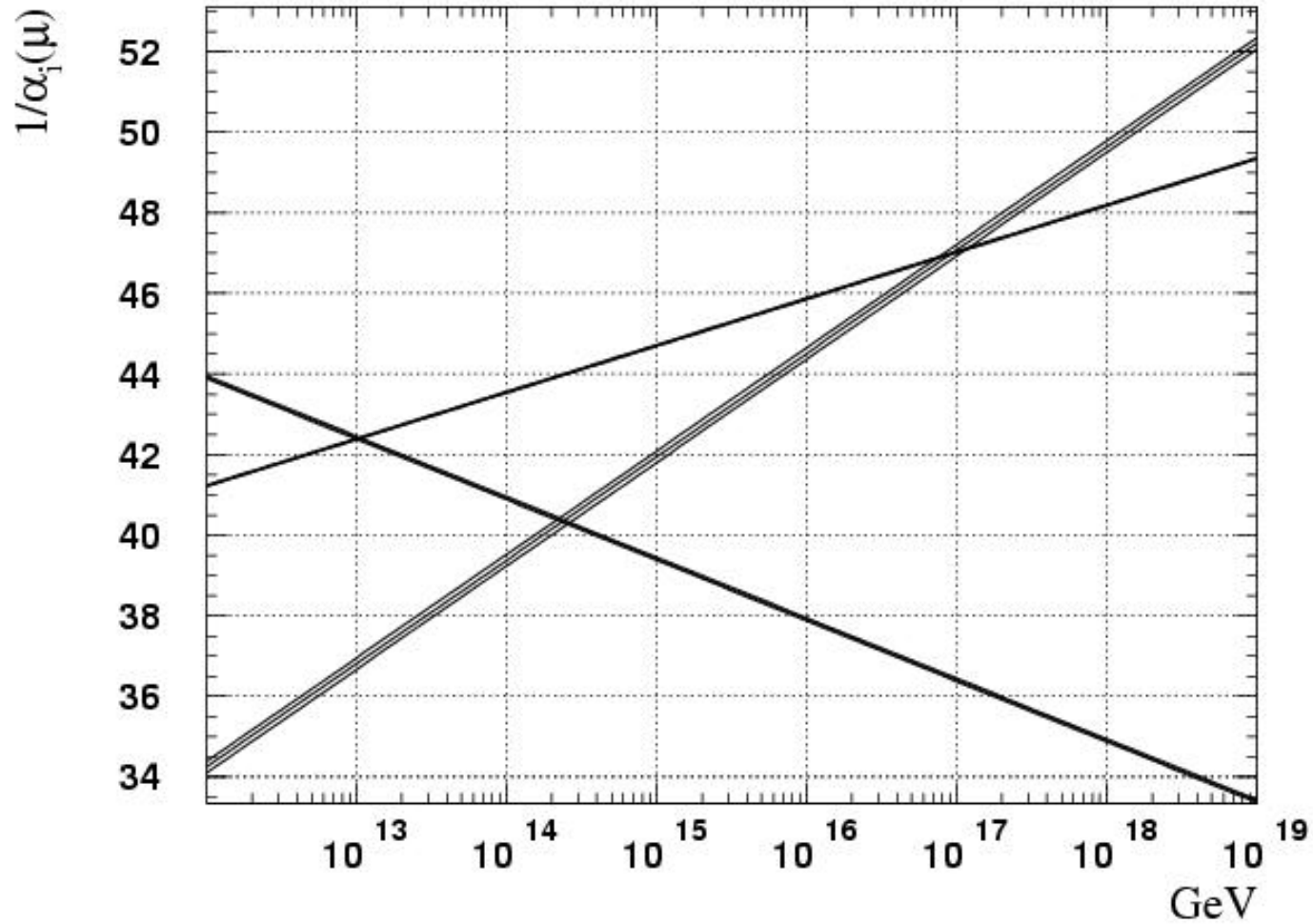
$$1/\alpha(v) = 1/\alpha(\mu) - (b_i/2\pi) \cdot \ln(v/\mu)$$

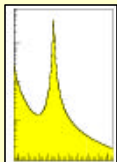
The coefficients for the 3 couplings are **independent** and given by the SM or MSSM

2-loop-RG running: additional terms so the 3 couplings **depend on each other** – solved numerically; the errors depend on the **scale** (for typical GUT scales they grow by 4, 12 and 6 % for the 3 couplings)

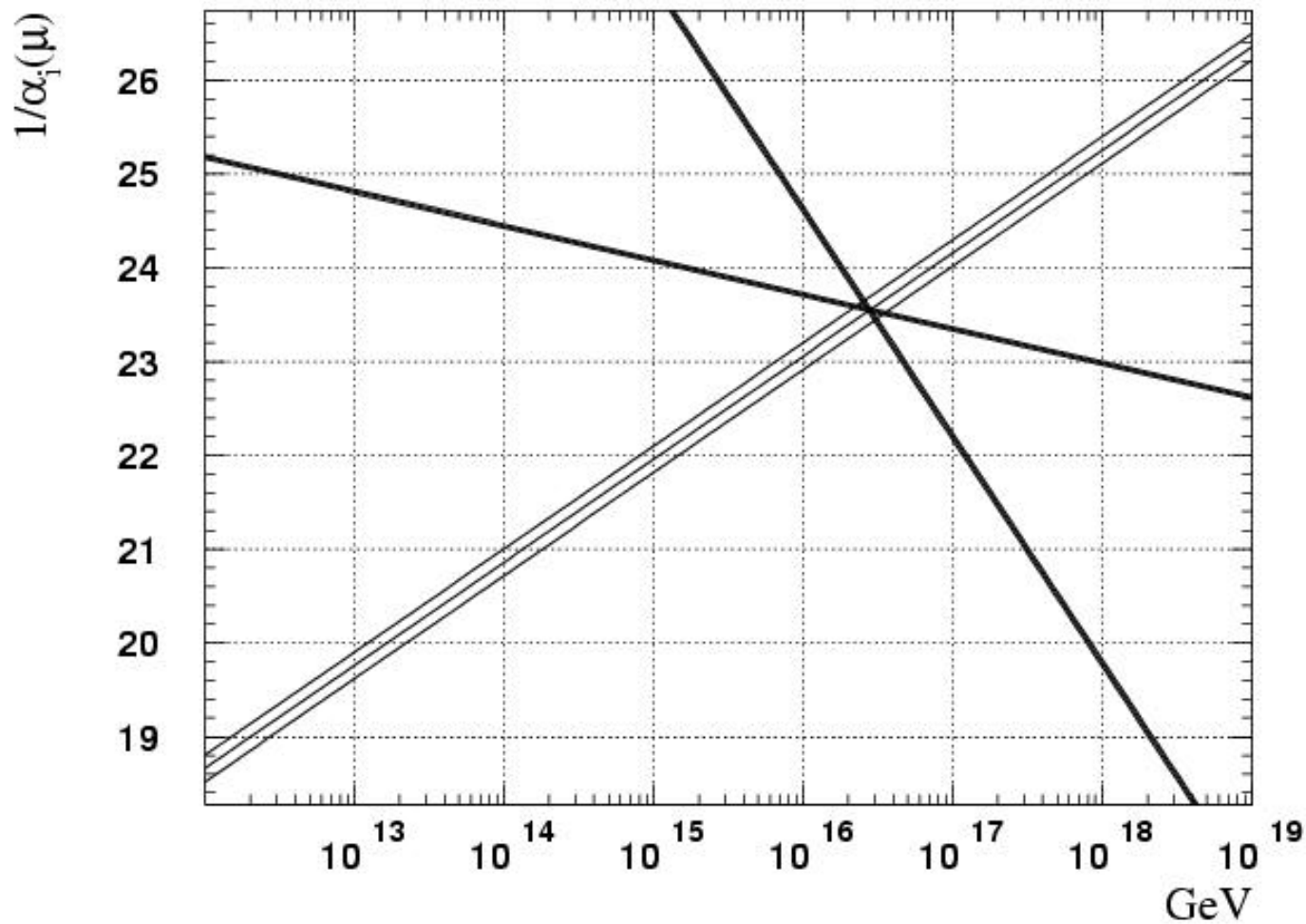


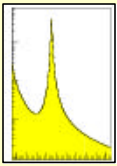
Results - SM





Example of SUSY Fit





Results



Low $\alpha_s(M_Z) \cong 0.118-0.119$

- RPP 2004 QCD

$$M_{\text{SUSY}} = 10^{1.5 \pm 0.4} \text{ GeV}$$

$$M_{\text{GUT}} = 10^{16.7 \pm 0.1} \text{ GeV}$$

$$1/\alpha_{\text{GUT}} = 22.3 \pm 0.7$$

And threshold correction (-3 %)

$$M_{\text{SUSY}} = 10^{2.05 \pm 0.4} \text{ GeV}$$

$$M_{\text{GUT}} = 10^{16.5 \pm 0.1} \text{ GeV}$$

$$1/\alpha_{\text{GUT}} = 23.3 \pm 0.7$$

- S. Bethke 2002-2004

$$M_{\text{SUSY}} = 10^{1.55 \pm 0.5} \text{ GeV}$$

$$M_{\text{GUT}} = 10^{16.65 \pm 0.15} \text{ GeV}$$

$$1/\alpha_{\text{GUT}} = 22.4 \pm 0.9$$

High $\alpha_s(M_Z) \cong 0.121 - 0.122$

- RPP 2004 EW Global Fit

$$M_{\text{SUSY}} = 10^{1.1 \pm 0.3} \text{ GeV}$$

$$M_{\text{GUT}} = 10^{16.8 \pm 0.1} \text{ GeV}$$

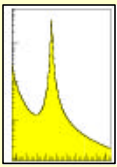
$$1/\alpha_{\text{GUT}} = 21.6 \pm 0.55$$

- $\Gamma_h / \Gamma_\mu @ Z$

$$M_{\text{SUSY}} = 10^{0.95 \pm 0.6} \text{ GeV}$$

$$M_{\text{GUT}} = 10^{16.85 \pm 0.2} \text{ GeV}$$

$$1/\alpha_{\text{GUT}} = 21.3 \pm 1.1$$



Conclusions

- The MSSM still provides coupling unification at a GUT scale well below the Planck scale for the full set of precise 2004 measurements
- The strong coupling is a key for interpreting the results: the “high” values require an uncomfortably low SUSY scale ~ 10 GeV – excluded by experiment
- The “low” $\alpha_s(M_Z)$ values favor a higher SUSY scale:
 $M_{\text{SUSY}} < 185(660)$ GeV at 95 % CL for threshold corrections of $-4(-3)\%$ \Rightarrow well in the LHC (TEVATRON?) direct discovery range
- The GUT scale is higher: $M_{\text{GUT}} \approx 10^{16.7 \pm 0.1}$ GeV & pushing the proton lifetime ($\sim 10^{36}$ years) well beyond the experimental limits ($\sim 5 \times 10^{33}$ years)