

# Massive Elementary Particles and Black Holes

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## Outline:

- Introduction
- Review of Feynman's Formulation of Einstein's Theory
- Resummed Quantum Gravity
- Newton's Law
- Massive Elementary Particles and Black Holes
- Conclusions

Papers by **B.F.L. Ward**, **S. Jadach et al.**: **JCAP0402(2004)011**; **MPLA19(2004)143**; **A17(2002) 2371**, **CPC102 (1997) 229**, **CPC124 (2000) 233**, **CPC130 (2000) 260**, **CPC140 (2001) 432**, **CPC140 (2001) 475**, and references therein.

## Motivation

- NEWTON'S LAW: MOST BASIC ONE IN PHYSICS – TAUGHT TO ALL BEGINNING STUDENTS
- ALBERT EINSTEIN: SPECIAL CASE OF THE SOLUTIONS OF THE CLASSICAL FIELD EQUATIONS OF THE GENERAL THEORY OF RELATIVITY

$$g_{00} = 1 + 2\varphi \Rightarrow \nabla^2 \varphi = 4\pi G_N \rho$$

from

$$R^{\alpha\gamma} - \frac{1}{2}g^{\alpha\gamma}R = -8\pi G_N T^{\alpha\gamma}, \text{ etc.},$$

- HEISENBERG & SCHROEDINGER, FOLLOWING BOHR: QUANTUM MECHANICS  
 $\Leftrightarrow$  EVEN WITH TREMENDOUS PROGRESS: QUANTUM FIELD THEORY, SUPERSTRINGS, LOOP QUANTUM GRAVITY, ETC.,  
NO SATISFACTORY TREATMENT OF THE QUANTUM MECHANICS OF NEWTON'S LAW IS KNOWN TO BE PHENOMENOLOGICALLY CORRECT

## TODAY'S TALK

- WE APPLY A NEW APPROACH(MPLA17(2002)2371;A19(2004)143;JCAP0402(2004)011), BUILDING ON PREVIOUS WORK:  
R.P. FEYNMAN: *Acta Pys. Pol.* **24** (1963) 697; *FEYNMAN LECTURES ON GRAVITATION*, eds. F.B. Moringo and W.G. Wagner, (Caltech, Pasadena, 1971).
- BASIC IDEA: QUANTUM GRAVITY IS A POINT PARTICLE QUANTUM FIELD THEORY AND ITS APPARENT BAD UV BEHAVIOR IS DUE TO OUR NAIVETE – NOTHING FUNDAMENTAL PREVENTS THE UNION OF BOHR AND EINSTEIN.
- WEINBERG, IN *GENERAL RELATIVITY*, eds. S.W. Hawking and W. Israel,( Cambridge Univ. Press, Cambridge, 1979) p.790  
FOUR APPROACHES TO UV BEHAVIOR OF QUANTUM GRAVITY (QG)
  - Extended Theories Of Gravitation: Supersymmetric Theories
    - Superstrings; Loop Quantum Gravity
  - Resummation  $\Leftarrow$  TODAY'S TALK – NEW VERSION
  - Composite Gravitons
  - Asymptotic Safety: Fixed Point Theory (See Lautscher & Reuter, hep-th/0205062;Bonnanno & Reuter, PRD62(2000)043008)

## NEW APPROACH: RESUMMED QUANTUM GRAVITY

IN *Mod. Phys. Lett.* **A17**(2002)2371, WE SHOWED THAT  
RESUMMATION CURES THE BAD UV BEHAVIOR OF  
EINSTEIN'S THEORY



MANY CONSEQUENCES AND TESTS

- A SM MASSIVE POINT PARTICLE OF MASS  $m$  HAS A NONZERO SCHWARZSCHILD RADIUS  $r_S = 2(m/M_{Pl}^2)$  AND BY EINSTEIN'S THEORY IS CLASSICALLY A BLACK HOLE— WE CAN ONLY SEE ITS HAWKING RADIATION.
- TODAY, WE ADDRESS THE FATE OF SUCH SM PARTICLES IN OUR NEW APPROACH TO QG.

## Review of Feynman's Formulation of Einstein's Theory

For the known world, we have the generally covariant Lagrangian

$$\mathcal{L}(x) = -\frac{1}{2\kappa^2} \sqrt{-g} R + \sqrt{-g} L_{SM}^{\mathcal{G}}(x) \quad (1)$$

- $R$  is the curvature scalar,
- $-g = -\det g_{\mu\nu}$
- $\kappa = \sqrt{8\pi G_N} \equiv \sqrt{8\pi / M_{Pl}^2}$ , where  $G_N$  is Newton's constant,
- SM Lagrangian density =  $L_{SM}^{\mathcal{G}}(x)$

One gets  $L_{SM}^{\mathcal{G}}(x)$  from the usual SM Lagrangian density as follows:

- Note that  $\partial_\mu \phi(x)$  is already generally covariant for any scalar field  $\phi$ .
- Note that the only derivatives of the vector fields in the SM Lagrangian density occur in their curls,  $\partial_\mu A_\nu^J(x) - \partial_\nu A_\mu^J(x)$ , which are also already generally covariant.
- Thus, we only need to give a rule for making the fermionic terms in usual SM

Lagrangian density generally covariant.  $\Rightarrow$

We introduce a differentiable structure with  $\{\xi^a(x)\}$  as locally inertial coordinates and an attendant vierbein field  $e_\mu^a \equiv \partial\xi^a/\partial x^\mu$  with indices that carry the vector representation for the flat locally inertial space,  $a$ , and for the manifold of space-time,  $\mu$ , with the identification of the space-time base manifold metric as  $g_{\mu\nu} = e_\mu^a e_{a\nu}$  where the flat locally inertial space indices are to be raised and lowered with Minkowski's metric  $\eta_{ab}$  as usual.

Associating the usual Dirac gamma matrices  $\{\gamma_a\}$  with the flat locally inertial space at  $x$ , we define base manifold Dirac gamma matrices by

$$\Gamma_\mu(x) = e_\mu^a(x)\gamma_a.$$

Then the spin connection,  $\omega_{\mu b}^a = -\frac{1}{2}e^{a\nu}(\partial_\mu e_\nu^b - \partial_\nu e_\mu^b) + \frac{1}{2}e^{b\nu}(\partial_\mu e_\nu^a - \partial_\nu e_\mu^a) + \frac{1}{2}e^{a\rho}e^{b\sigma}(\partial_\rho e_{c\sigma} - \partial_\sigma e_{c\rho})e_\mu^c$  when there is no torsion, allows us to identify the generally covariant Dirac operator for the SM fields by the substitution  $i \not{\partial} \rightarrow i\Gamma(x)^\mu (\partial_\mu + \frac{1}{2}\omega_{\mu b}^a \Sigma^b_a)$ , where we have  $\Sigma^b_a = \frac{1}{4}[\gamma^b, \gamma_a]$  everywhere in the SM Lagrangian density. This will generate  $L_{SM}^{\mathcal{G}}(x)$  from the usual SM Lagrangian density  $L_{SM}(x)$  as it is given in the papers of Hollik, Bardin, Passarino, etc., for example.

SM  $\Leftrightarrow$  Many Massive Point Particles.

Are they black holes in our new approach to quantum gravity?

To study this question, we follow Feynman and treat spin as **an inessential complication**, as the question of whether a point particle with mass is or is not a black hole should not depend too severely on whether or not it is spinning. We can come back to a spin-dependent analysis elsewhere.

We replace  $L_{SM}^{\mathcal{G}}(x)$  in (1) with the simplest case for our question, that of a free scalar field, a free physical Higgs field,  $\varphi(x)$ , with a rest mass believed to be less than 400 GeV and known to be greater than 114.4 GeV with a 95% CL. We are then led to consider the representative model {R.P. Feynman, *Acta Phys. Pol.* 24 (1963) 697; *Feynman Lectures on Gravitation*, eds. F.B. Moringo and W.G. Wagner, (Caltech, Pasadena, 1971). }

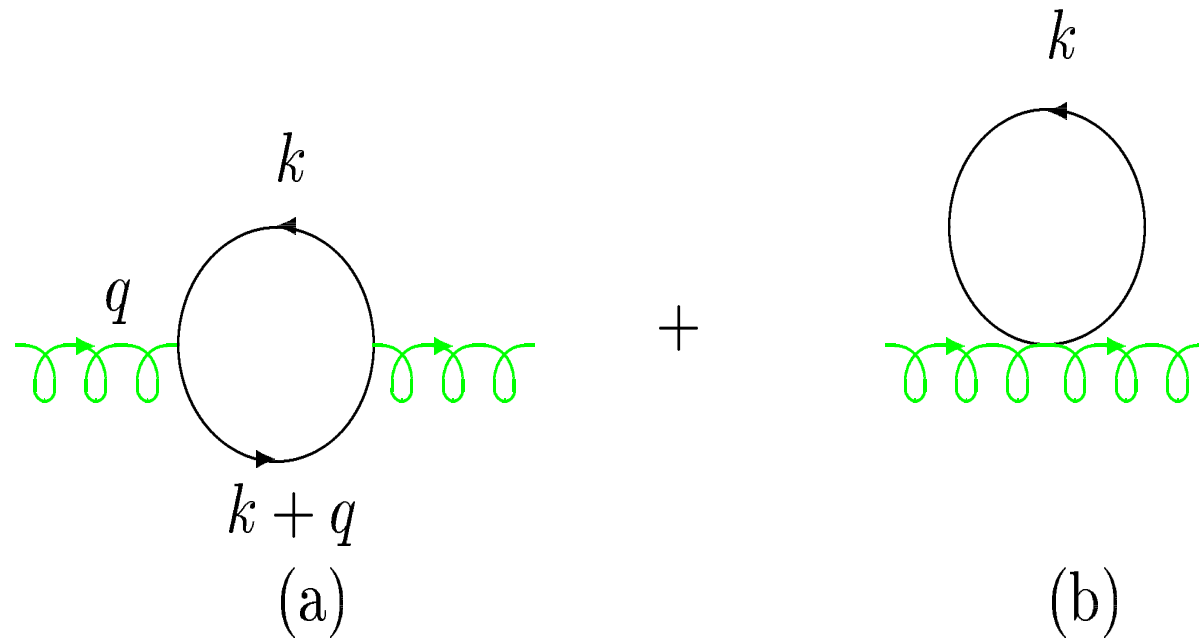
$$\begin{aligned}
 \mathcal{L}(x) &= -\frac{1}{2\kappa^2} R\sqrt{-g} + \frac{1}{2} (g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - m_o^2 \varphi^2) \sqrt{-g} \\
 &= \frac{1}{2} \left\{ h^{\mu\nu,\lambda} \bar{h}_{\mu\nu,\lambda} - 2\eta^{\mu\mu'} \eta^{\lambda\lambda'} \bar{h}_{\mu\lambda,\lambda'} \eta^{\sigma\sigma'} \bar{h}_{\mu'\sigma,\sigma'} \right\} \\
 &\quad + \frac{1}{2} \left\{ \varphi_{,\mu} \varphi^{,\mu} - m_o^2 \varphi^2 \right\} - \kappa h^{\mu\nu} \left[ \overline{\varphi_{,\mu} \varphi_{,\nu}} + \frac{1}{2} m_o^2 \varphi^2 \eta_{\mu\nu} \right] \\
 &\quad - \kappa^2 \left[ \frac{1}{2} h_{\lambda\rho} \bar{h}^{\rho\lambda} (\varphi_{,\mu} \varphi^{,\mu} - m_o^2 \varphi^2) - 2\eta_{\rho\rho'} h^{\mu\rho} \bar{h}^{\rho'\nu} \varphi_{,\mu} \varphi_{,\nu} \right] + \dots
 \end{aligned} \tag{2}$$

where  $\varphi_{,\mu} \equiv \partial_\mu \varphi$  and we have

- $g_{\mu\nu}(x) = \eta_{\mu\nu} + 2\kappa h_{\mu\nu}(x)$ ,  
 $\eta_{\mu\nu} = \text{diag}\{1, -1, -1, -1\}$
- $\bar{y}_{\mu\nu} \equiv \frac{1}{2} (y_{\mu\nu} + y_{\nu\mu} - \eta_{\mu\nu} y_\rho{}^\rho)$  for any tensor  $y_{\mu\nu}$
- Feynman rules already worked-out by Feynman (*op. cit.*), where we use his gauge,  $\partial^\mu \bar{h}_{\nu\mu} = 0$

⇔ Quantum Gravity is just another quantum field theory where the metric now has quantum fluctuations as well.

For example, the one-loop corrections to the graviton propagator due to matter loops is just given by the diagrams in Fig. 1.



**Figure 1: The scalar one-loop contribution to the graviton propagator.  $q$  is the 4-momentum of the graviton.**

**We return to these graphs shortly.**

## RESUMMED QUANTUM GRAVITY

WE WILL YFS RESUM THE PROPAGATORS IN THE THEORY:

⇒ FROM THE EW RESUMMED FORMULA OF YFS

$$\Sigma_F(p) = e^{\alpha B''_\gamma} [\Sigma'_F(p) - S_F^{-1}(p)] + S_F^{-1}(p), \quad (3)$$

WHICH ⇒

$$iS'_F(p) = \frac{ie^{-\alpha B''_\gamma}}{S_F^{-1}(p) - \Sigma'_F(p)}, \quad (4)$$

FOR

$$\Sigma'_F(p) = \sum_{n=1}^{\infty} \Sigma'_{Fn}, \quad (5)$$

WE NEED TO FIND FOR QUANTUM GRAVITY THE ANALOGUE OF

$$\alpha B''_\gamma = \int d^4\ell \frac{S''(k, k, \ell)}{\ell^2 - \lambda^2 + i\epsilon} \quad (6)$$

WHERE  $\lambda \equiv$  IR CUT-OFF AND

$$S''(k, k, \ell) = \frac{-i8\alpha}{(2\pi)^3} \frac{kk'}{(\ell^2 - 2\ell k + \Delta + i\epsilon)(\ell^2 - 2\ell k' + \Delta' + i\epsilon)} \Big|_{k=k'}, \quad (7)$$

$$\Delta = k^2 - m^2, \Delta' = k'^2 - m^2.$$

TO THIS END, NOTE ALSO

$$\alpha B''_\gamma = \int \frac{d^4\ell}{(2\pi)^4} \frac{-i\eta^{\mu\nu}}{(\ell^2 - \lambda^2 + i\epsilon)} \frac{-ie(2ik_\mu)}{(\ell^2 - 2\ell k + \Delta + i\epsilon)} \frac{-ie(2ik'_\nu)}{(\ell^2 - 2\ell k' + \Delta' + i\epsilon)} \Big|_{k=k'}. \quad (8)$$

$\Rightarrow$  WE FOLLOW WEINBERG/THE FEYNMAN RULES  
AND IDENTIFY THE CONSERVED GRAVITON CHARGES AS

$e \rightarrow \kappa k_\rho$  FOR SOFT EMISSION FROM  $k$

$\Rightarrow$  WE GET THE ANALOGUE,  $-B''_g(k)$ , OF  $\alpha B''_\gamma$  BY

- REPLACING THE  $\gamma$  PROPAGATOR IN (8) BY THE GRAVITON

PROPAGATOR,

$$\frac{i\frac{1}{2}(\eta^{\mu\nu}\eta^{\bar{\mu}\bar{\nu}} + \eta^{\mu\bar{\nu}}\eta^{\bar{\mu}\nu} - \eta^{\mu\bar{\mu}}\eta^{\nu\bar{\nu}})}{\ell^2 - \lambda^2 + i\epsilon}$$

,

- BY REPLACING THE QED CHARGES BY THE CORRESPONDING GRAVITY CHARGES  $\kappa k_{\bar{\mu}}, \kappa k'_{\bar{\nu}}$

⇒

$$B_g''(k) = -2i\kappa^2 k^4 \frac{\int d^4\ell}{16\pi^4} \frac{1}{\ell^2 - \lambda^2 + i\epsilon} \frac{1}{(\ell^2 + 2\ell k + \Delta + i\epsilon)^2} \quad (9)$$

AND

$$i\Delta'_F(k)|_{Resummed} = \frac{ie^{B_g''(k)}}{(k^2 - m^2 - \Sigma'_s + i\epsilon)} \quad (10)$$

THIS IS THE BASIC RESULT.

NOTE THE FOLLOWING:

- $\Sigma'_s$  STARTS IN  $\mathcal{O}(\kappa^2)$ , SO WE MAY DROP IT IN CALCULATING ONE-LOOP EFFECTS.

- EXPLICIT EVALUATION GIVES, FOR THE DEEP UV REGIME,

$$B_g''(k) = \frac{\kappa^2 |k^2|}{8\pi^2} \ln \left( \frac{m^2}{m^2 + |k^2|} \right), \quad (11)$$

⇒ THE RESUMMED PROPAGATOR FALLS FASTER THAN ANY POWER OF  $|k^2|$ !

- IF  $m$  VANISHES, USING THE USUAL  $-\mu^2$  NORMALIZATION POINT WE GET  $B_g''(k) = \frac{\kappa^2 |k^2|}{8\pi^2} \ln \left( \frac{\mu^2}{|k^2|} \right)$  WHICH AGAIN VANISHES FASTER THAN ANY POWER OF  $|k^2|$ !

THIS MEANS THAT ONE-LOOP CORRECTIONS ARE FINITE!

INDEED, ALL QUANTUM GRAVITY LOOPS ARE UV FINITE!

**ALL ORDERS PROOF**

Consider the entire theory from (2) to all orders in  $\kappa$ :

$$\mathcal{L}(x) = \mathcal{L}_0(x) + \sum_{n=1}^{\infty} \kappa^n \mathcal{L}_I^{(n)}(x) \quad (12)$$

– the interactions, including the ghost interactions, are the terms of  $\mathcal{O}(\kappa^n)$ ,  $n \geq 1$ .

$$\mathcal{L}_I^{(n)}(x) = \sum_{\ell=1}^{m_n} \mathcal{L}_{I,\ell}^{(n)}(x). \quad (13)$$

$\mathcal{L}_{I,\ell}^{(n)}$  has dimension  $d_{n,\ell}$ .

Let  $d_n^M = \max_{\ell} \{d_{n,\ell}\}$ .  $\Rightarrow$  The maximum power of momentum at any vertex in  $\mathcal{L}_I^{(n)}$  is  $\bar{d}_n^M = \min\{d_n^M - 3, 2\}$  and is finite.

Note, in any gauge,

$$i\mathcal{D}_{F\alpha_1\dots;\alpha'_1\dots}^{(0)}(k)|_{YFS-resummed} = \frac{iP_{\alpha_1\dots;\alpha'_1\dots}e^{B_g''(k)}}{(k^2 - m^2 + i\epsilon)}, \quad (14)$$

so that it is also exponentially damped at high energy in the deep Euclidean regime (DER).

Now consider any 1PI vertex  $\Gamma_N$  with  $[N] \equiv n_1 + n_2$  amputated external legs, where  $N = (n_1, n_2)$ , when  $n_1(n_2)$  is the respective number of graviton(scalar) external lines.

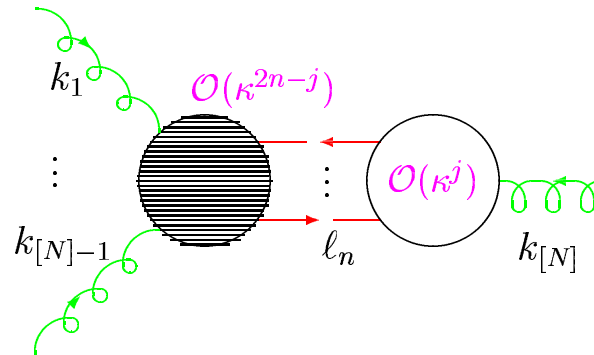
At its zero-loop order, there are only tree contributions which are manifestly UV finite.

Consider the first loop ( $\mathcal{O}(\kappa^2)$ ) corrections to  $\Gamma_N$ . There must be at least **one improved exponentially damped propagator in the respective loop contribution** and at most **two vertices** so that the maximum power of momentum in the numerator of the loop due to the vertices is  $\max\{2\bar{d}_1^M, \bar{d}_2^M\}$  **and is finite.**

The exponentially damped propagator  $\Rightarrow$  the loop integrals finite  $\Rightarrow$  the entire **one-loop ( $\mathcal{O}(\kappa^2)$ ) contribution is finite.**

**Corollary:** If  $\Gamma_N$  vanishes in tree approximation, we can conclude that its first non-trivial contributions at one-loop are all finite, **due to the exponentially damped propagator.** Using induction on the number of loops, it is possible to **prove finiteness of all loop contributions**– See MPLA17(2002)2371.

Pictorially, we illustrate the type of situations we have in Fig. 2.



**Fig.2. The typical contribution we encounter in  $\Gamma_N$  at the  $n$ -loop level;  $l_n$  is the  $n$ -th loop momentum and is precisely the momentum of the indicated YFS-resummed improved Born propagator.**

## Explicit Finiteness of $\Sigma'^{(1)}$

$$\Sigma'_s{}^{(1)}(k) = \Sigma_s^{(1)}(k) - B_g''(k) \Delta_F^{-1}(k) \quad (15)$$

⇒

$$\begin{aligned} \Sigma'_s{}^{(1)}(k) = & -\kappa^2 \frac{\int d^4 \ell}{(2\pi)^4} \left\{ \left[ (2k^\mu k^\nu) \mathcal{P}_{\mu\nu; \mu' \nu'}(\ell) (2k^{\mu'} k^{\nu'}) \frac{\ell^2 + 2\ell k + 2(k^2 - m^2)}{\ell^2 + 2\ell k + k^2 - m^2 + i\epsilon} \right. \right. \\ & + \Delta V^{\mu\nu}(k, \ell) \mathcal{P}_{\mu\nu; \mu' \nu'}(\ell) (2k^{\mu'} k^{\nu'}) + (2k^\mu k^\nu) \mathcal{P}_{\mu\nu; \mu' \nu'}(\ell) \Delta V^{\mu' \nu'}(k, \ell) \\ & + \left. \Delta V^{\mu\nu}(k, \ell) \mathcal{P}_{\mu\nu; \mu' \nu'}(\ell) \Delta V^{\mu' \nu'}(k, \ell) \right] \frac{e^{\frac{\kappa^2 |(k+\ell)^2|}{8\pi^2} \ln\left(\frac{m^2}{m^2 + |(k+\ell)^2|}\right)}}{(k+\ell)^2 - m^2 + i\epsilon} \frac{e^{\frac{\kappa^2 |\ell^2|}{8\pi^2} \ln\left(\frac{\mu^2}{|\ell^2|}\right)}}{\ell^2 + i\epsilon} \\ & + \left[ \frac{1}{2}(k^2 - m^2) \left( \mathcal{P}_{\lambda\rho; \rho\lambda}(\ell) + \mathcal{P}_{\lambda\rho; \lambda\rho}(\ell) - \mathcal{P}_{\lambda\lambda; \lambda'\lambda'}(\ell) \right) \right. \\ & \left. - (2k^\mu k^\nu) \left( \mathcal{P}_{\mu\rho; \rho\nu}(\ell) + \mathcal{P}_{\mu\rho; \nu\rho}(\ell) - \mathcal{P}_{\mu\nu; \rho\rho}(\ell) \right) \right] \frac{e^{\frac{\kappa^2 |\ell^2|}{8\pi^2} \ln\left(\frac{\mu^2}{|\ell^2|}\right)}}{\ell^2 + i\epsilon} \right\}, \end{aligned} \quad (16)$$

where  $\Delta V^{\mu\nu}(k, \ell) = k^\mu \ell^\nu + k^\nu \ell^\mu - (k^2 - m^2 + k\ell)\eta^{\mu\nu}$ . ⇒ **UV FINITE!**

## NEWTON'S LAW

CONSIDER THE ONE-LOOP CORRECTIONS TO NEWTON'S LAW IMPLIED BY THE **DIAGRAMS IN FIG. 1**. THESE CORRECTIONS DIRECTLY IMPACT OUR BLACK HOLE ISSUE. INTRODUCING THE YFS RESUMMED PROPAGATORS INTO FIG. 1  $\Rightarrow$  ( HERE,  $k \rightarrow (ik^0, \vec{k})$  BY WICK ROTATION, AND WE WORK IN THE TRANSV.-TRACELESS SPACE )

$$i\Sigma(q)_{\bar{\mu}\bar{\nu};\mu\nu}^{1a} = i\kappa^2 \frac{\int d^4k}{2(2\pi)^4} \frac{(k'_{\bar{\mu}}k_{\bar{\nu}} + k'_{\bar{\nu}}k_{\bar{\mu}}) e^{\frac{\kappa^2|k'^2|}{8\pi^2} \ln\left(\frac{m^2}{m^2+|k'^2|}\right)}}{(k'^2 - m^2 + i\epsilon)} \tag{17}$$

$$\frac{(k'_{\mu}k_{\nu} + k'_{\nu}k_{\mu}) e^{\frac{\kappa^2|k^2|}{8\pi^2} \ln\left(\frac{m^2}{m^2+|k^2|}\right)}}{(k^2 - m^2 + i\epsilon)} .$$

$\Leftrightarrow$  CONVERGENT!! SO IS FIG 1b.

CONTINUING TO WORK IN THE TRANSV.-TRACELESS SPACE, WE GET THE GRAVITON PROPAGATOR DENOMINATOR

$$q^2 + \frac{1}{2}q^4\Sigma^{T(2)} + i\epsilon \tag{18}$$

WHERE THE TRANSVERSE, TRACELESS SELF-ENERGY FUNCTION  $\Sigma^T(q^2)$  FOLLOWS FROM eq.(17) FIG. 1a AND ITS ANALOG FOR FIG. 1b BY THE STANDARD METHODS. FOR THE COEFFICIENT OF  $q^4$  IN  $\Sigma^T(q^2)$  FOR  $|q^2| \gg m^2$  WE GET

$$-\frac{1}{2}\Sigma^{T(2)} \simeq \frac{c_2}{360\pi M_{Pl}^2} \quad (19)$$

FOR

$$c_2 = \int_0^\infty dx x^3 (1+x)^{-4-\lambda_c x} \simeq 72.1 \quad (20)$$

WHERE  $\lambda_c = \frac{2m^2}{\pi M_{Pl}^2}$ . WHEN WE FOURIER TRANSFORM THE INVERSE OF (18) WE FIND THE POTENTIAL

$$\Phi_{Newton}(r) = -\frac{G_N M_1 M_2}{r} (1 - e^{-ar}) \quad (21)$$

WHERE  $a = 1/\sqrt{-\frac{1}{2}\Sigma^{T(2)}} \simeq 3.96 M_{Pl}$  WHERE FOR DEFINITENESS WE SET  $m \simeq 120 \text{GeV}$ .

WE NOTE FOR COMPLETENESS THAT

$$c_2 \cong \ln \frac{1}{\lambda_c} - \ln \ln \frac{1}{\lambda_c} - \frac{\ln \ln \frac{1}{\lambda_c}}{\ln \frac{1}{\lambda_c} - \ln \ln \frac{1}{\lambda_c}} - \frac{11}{6} \quad (22)$$

**AND WE USED THIS RESULT TO CHECK (20).** WITHOUT RESUMMATION,  $\lambda_c = 0$ , OUR RESULT IN (20) WOULD BE INFINITE **AND, AS THIS IS THE COEFFICIENT OF  $q^4$  IN THE INVERSE PROPAGATOR, NO RENORMALIZATION OF THE FIELD AND OF THE MASS COULD REMOVE SUCH AN INFINITY.** IN OUR NEW APPROACH, THIS INFINITY IS ABSENT.

OUR RESULT IN (19) IS GAUGE INVARIANT, AS OUR APPROACH INVOLVES THE EXACT RE-ARRANGEMENT OF THE FEYNMAN SERIES **(SEE MPLA17(2002)2371)** AND THE ORIGINAL SERIES IS GAUGE INVARIANT.

CROSS CHECK WITH 't HOOFT AND VELTMAN, **Ann. Inst. Henri Poincare XX, 69 (1974)**, WHERE THE COMPLETE RESULT OF THE ONE-LOOP DIVERGENCES OF OUR SCALAR FIELD COUPLED TO EINSTEIN'S GRAVITY HAVE BEEN COMPUTED.

**KEY OBSERVATION:** THE RESULT WHICH WE HAVE OBTAINED WOULD BE

UV DIVERGENT WITHOUT OUR RESUMMATION. THUS, THE DOMINANT TERMS WHICH WE ARE ISOLATING IN THIS TALK ARE PRECISELY THOSE THAT ARE GIVEN BY 't HOOFT AND VELTMAN, WHERE WE NEED TO MAKE THE CORRESPONDENCE BETWEEN THE POLES IN  $n$ , THE DIMENSION OF SPACE-TIME, AT  $n = 4$  CALCULATED BY THEM AND THE LEADING LOG  $\ln \frac{1}{\lambda_c}$ .  $\Rightarrow$  SET  $C_2$  EQUAL TO ITS VALUE WHEN  $\lambda_c = 0$  IN  $n$  DIMENSIONS AND ALLOW  $n \rightarrow 4$ .  $\Rightarrow$

$$\frac{1}{(2 - n/2)} \leftrightarrow c_2. \quad (23)$$

THIS MEANS THAT, IF WE LOOK AT THE LIMIT  $q^2 \rightarrow 0$ , WE GET THE RESULT THAT THE COEFFICIENT OF  $q^4$  IN (18) IS  $3/(2 - n/2)$  TIMES THE COEFFICIENT OF  $c_2$  ON THE RIGHT-HAND SIDE OF (19), AND THIS IS IN COMPLETE AGREEMENT WITH THE RESULT THAT IS IMPLIED BY EQ.(3.40) IN *Ann. Inst. Henri Poincare XX, 69 (1974)*, FOR EXAMPLE. OF COURSE, THE RESULTS IN OF 't HOOFT AND VELTMAN ARE ALSO GAUGE INVARIANT.

## OBSERVATIONS

- SUB  $\ell_{Pl}$  ACCESSIBLE TO POINT PARTICLE FIELD THEORY  $\Rightarrow$  CURRENT SUPERSTRING THEORIES MAY BE PHENOMENOLOGICAL MODELS FOR A MORE FUNDAMENTAL THEORY (TUT=The Ultimate Theory) JUST AS THE OLD STRING THEORY IS FOR QCD.
- OUR DEEP EUCLIDEAN STUDIES ARE COMPLEMENTARY TO THE LOW ENERGY STUDIES OF DONOGHUE(PRL 72 (1994) 2996; PRD 50 (1994) 3974).
- CUT-OFF AT  $M_{Pl}$   $\Rightarrow$  RENORM. QFT BELOW  $M_{Pl}$  UNAFFECTED
- NONRENORM. QFT'S GIVEN NEW LIFE HERE – THEY MAY HAVE OTHER PROBLEMS.

**MASSIVE ELEMENTARY PARTICLES AND BLACK HOLES**

FOCUSING THE PREVIOUS RESULTS, NOTE THAT ,IN THE SM, THERE ARE NOW BELIEVED TO BE THREE MASSIVE NEUTRINOS, WITH MASSES THAT WE ESTIMATE AT  $\sim 3$  eV, AND THE REMAINING MEMBERS OF THE KNOWN THREE GENERATIONS OF DIRAC FERMIONS  $\{e, \mu, \tau, u, d, s, c, b, t\}$ , WITH MASSES GIVEN BY  $m_e \cong 0.51$  MeV,  $m_\mu \cong 0.106$  GeV,  $m_\tau \cong 1.78$  GeV,  $m_u \cong 5.1$  MeV,  $m_d \cong 8.9$  MeV,  $m_s \cong 0.17$  GeV,  $m_c \cong 1.3$  GeV,  $m_b \cong 4.5$  GeV and  $m_t \cong 174$  GeV, AS WELL AS THE MASSIVE VECTOR BOSONS  $W^\pm, Z$ , WITH MASSES  $M_W \cong 80.4$  GeV,  $M_Z \cong 91.19$  GeV.

USE THE GENERAL SPIN INDEPENDENCE OF THE GRAVITON COUPLING TO MATTER AT *RELATIVELY LOW MOMENTUM TRANSFERS*.  $\Rightarrow$  COUNT EACH DIRAC FERMION AS 4 DEGREES OF FREEDOM, EACH MASSIVE VECTOR BOSON AS 3 DEGREES OF FREEDOM AND REMEMBER THAT EACH QUARK HAS THREE COLORS.  $\Rightarrow$  THE RESULT (22) FOR EACH SM MASSIVE DEGREE OF FREEDOM YIELDS

$$c_{2,eff} \cong 9.26 \times 10^3 \tag{24}$$

SO THAT IN THE SM

$$a_{eff} \cong 0.349 M_{Pl}. \tag{25}$$

TO MAKE DIRECT CONTACT WITH BLACK HOLE PHYSICS, NOTE THAT, FOR  $r \rightarrow r_S$ ,  $a_{eff} r \ll 1$  SO THAT  $|2\Phi_{Newton}(r)|_{M_1=m/M_2}| \ll 1$ .

$\Rightarrow g_{00} \cong 1 + 2\Phi_{Newton}(r)|_{M_1=m/M_2}$  REMAINS POSITIVE AS WE PASS THROUGH THE SCHWARZSCHILD RADIUS.

IT CAN BE SHOWN THAT THIS POSITIVITY HOLDS TO  $r = 0$ . SIMILARLY,  $g_{rr}$  REMAINS NEGATIVE THROUGH  $r_S$  DOWN TO  $r = 0$ .

TO GET THESE RESULTS, NOTE THAT IN THE RELEVANT REGIME FOR R, THE SMALLNESS OF THE QUANTUM CORRECTED NEWTON POTENTIAL MEANS THAT WE CAN USE THE LINEARIZED EINSTEIN EQUATIONS FOR A SMALL SPHERICALLY SYMMETRIC STATIC SOURCE  $\rho(r)$  WHICH GENERATES  $\Phi_{Newton}(r)|_{M_1=m/M_2}$  VIA THE STANDARD POISSON'S EQUATION. THE USUAL RESULT (SEE MTW, ABS) FOR THE RESPECTIVE METRIC SOLUTION THEN GIVES

$g_{00} \cong 1 + 2\Phi_{Newton}(r)|_{M_1=m/M_2}$  AND  $g_{rr} \cong -1 + 2\Phi_{Newton}(r)|_{M_1=m/M_2}$  WHICH REMAIN RESPECTIVELY TIME-LIKE AND SPACE-LIKE TO  $r = 0$ .

QUANTUM CORRECTIONS HAVE OBIATED THE CLASSICAL CONCLUSION THAT A MASSIVE POINT PARTICLE IS A BLACK HOLE.

THE VALUE OF  $a_{eff}$  GIVEN HERE IS INCOMPLETE, AS THERE MAY BE AS YET UNKNOWN MASSIVE PARTICLES BEYOND THOSE ALREADY DISCOVERED.

INCLUDING MORE PARTICLES IN THE COMPUTATION OF  $a_{eff}$  WOULD MAKE IT SMALLER AND HENCE WOULD NOT CHANGE THE CONCLUSIONS OF OUR ANALYSIS. FOR EXAMPLE, IN THE MINIMAL SUPERSYMMETRIC STANDARD MODEL WE EXPECT APPROXIMATELY THAT  $a_{eff} \rightarrow \frac{1}{\sqrt{2}} a_{eff}$ .

IN ADDITION, WE POINT-OUT THAT, USING THE CORRESPONDENCE IN (23) ONE CAN ALSO USE THE RESULTS FOR THE COMPLETE ONE-LOOP UV DIVERGENT CORRECTIONS OF 'T HOOFT AND VELTMAN TO SEE THAT THE REMAINING INTERACTIONS AT ONE-LOOP ORDER NOT DISCUSSED HERE (VERTEX CORRECTIONS, PURE GRAVITY SELF-ENERGY CORRECTIONS, ETC. ) ALSO DO NOT INCREASE THE VALUE OF  $a_{eff}$ .  $\Rightarrow$

$a_{eff}$  IS A PARAMETER WHICH IS BOUNDED FROM ABOVE BY THE ESTIMATES WE GIVE ABOVE AND WHICH SHOULD BE DETERMINED FROM COSMOLOGICAL AND/OR OTHER CONSIDERATIONS. SUCH IMPLICATIONS WILL BE TAKEN UP ELSEWHERE.

## CONTACT WITH ASYMPTOTIC SAFETY APPROACH

- OUR RESULTS IMPLY

$$G_N(k) = G_N / \left(1 + \frac{k^2}{a_{eff}^2}\right)$$

⇒ **FIXED POINT BEHAVIOR FOR**

$$k^2 \rightarrow \infty,$$

**IN AGREEMENT WITH THE PHENOMENOLOGICAL ASYMPTOTIC SAFETY APPROACH OF BONNANNO & REUTER IN PRD62(2000) 043008.**

- OUR RESULT THAT AN ELEMENTARY PARTICLE HAS NO HORIZON ALSO AGREES WITH BONNANNO'S & REUTER'S RESULT THAT A BLACKHOLE WITH A MASS LESS THAN

$$M_{cr} \sim M_{Pl}$$

**HAS NO HORIZON. BASIC PHYSICS:  $G_N(k)$  VANISHES FOR  $k^2 \rightarrow \infty$ .**

- A FURTHER AGREEMENT: FINAL STATE OF HAWKING RADIATION OF AN ORIGINALLY VERY MASSIVE BLACKHOLE

BECAUSE OUR VALUE OF THE COEFFICIENT,

$$\frac{1}{a_{eff}^2},$$

OF  $k^2$  IN THE DENOMINATOR OF  $G_N(k)$

AGREES WITH THAT FOUND BY BONNANNO & REUTER,

IF WE USE THEIR PRESCRIPTION FOR THE

RELATIONSHIP BETWEEN  $k$  AND  $r$

IN THE REGIME WHERE THE LAPSE FUNCTION VANISHES,

WE GET THE SAME HAWKING RADIATION PHENOMENOLOGY AS THEY DO:

THE BLACK HOLE EVAPORATES UNTIL IT REACHES A MASS

$$M_{cr} \sim M_{Pl}$$

AT WHICH THE BEKENSTEIN-HAWKING TEMPERATURE VANISHES,

LEAVING A PLANCK SCALE REMNANT.

## Conclusions

### YFS RESUMMATION RENDERS QUANTUM GRAVITY FINITE

- QUANTUM LOOP CORRECTIONS ARE NOW CUT OFF DYNAMICALLY.
- PHYSICS BELOW THE PLANCK SCALE ACCESSIBLE TO POINT PARTICLE QFT: (TUT)
- EARLY UNIVERSE STUDIES MAY BE ABLE TO TEST PREDICTIONS.
- MINIMAL UNION OF BOHR AND EINSTEIN
- FIRST CHECKS:
  1. MASSIVE ELEMENTARY POINT PARTICLES ARE NOT BLACK HOLES.
  2. AGREEMENT WITH BONNANNO&REUTER ASYMPTOTIC SAFETY RESULTS
    - A. BLACK HOLES WITH  $M < M_{cr} \sim M_{Pl}$  HAVE NO HORIZON.
    - B. FINAL STATE OF HAWKING RAD. IS PLANCK SCALE REMNANT.