Quantum Corrections to Newton's Law

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Outline:

- **Introduction**
- **Review of Feynman's Formulation of Einstein's Theory**
- **Resummed Quantum Gravity**
- **Newton's Law**
- **Conclusions**

Papers by B.F.L. Ward, S. Jadach *et al.*: hep-ph/0204102;MPL**A17** (2002) 2371, CPC**102** (1997) 229, CPC**124** (2000) 233, CPC**130** (2000) 260, CPC**140** (2001) 432, CPC**140** (2001) 475, and references therein.

Motivation

- NEWTON'S LAW: MOST BASIC ONE IN PHYSICS – TAUGHT TO ALL BEGINNING STUDENTS

- ALBERT EINSTEIN: SPECIAL CASE OF THE SOLUTIONS OF THE CLASSICAL FIELD EQUATIONS OF THE GENERAL THEORY OF RELATIVITY

  $g_{00} = 1 + 2\varphi \Rightarrow \nabla^2\varphi = 4\pi G_N \rho$

  from

  $R^{\alpha\gamma} - \frac{1}{2} g^{\alpha\gamma} R = -8\pi G_N T^{\alpha\gamma}$, etc.,

- HEISENBERG & SCHROEDINGER, FOLLOWING BOHR: QUANTUM MECHANICS

  $\Leftrightarrow$ EVEN WITH TREMENDOUS PROGRESS: QUANTUM FIELD THEORY, SUPERSTRINGS, ETC.,

  NO SATISFACTORY TREATMENT OF THE QUANTUM MECHANICS OF NEWTON'S LAW IS KNOWN TO BE PHENOMENOLOGICALLY CORRECT

TODAY'S TALK

- WE TRY A NEW APPROACH, BUILDING ON PREVIOUS WORK:

  R.P. FEYNMAN: Acta Pys. Pol. **24** (1963) 697; *FEYNMAN LECTURES ON GRAVITATION*, eds. F.B. Moringo and W.G. Wagner, (Caltech, Pasadena, 1971).

- BASIC IDEA: QUANTUM GRAVITY IS A POINT PARTICLE QUANTUM FIELD THEORY AND ITS APPARENT BAD UV BEHAVIOR IS DUE TO OUR NAIVETE –
  NOTHING FUNDAMENTAL PREVENTS
  THE UNION OF BOHR AND EINSTEIN.

- WEINBERG, IN *GENERAL RELATIVITY*, eds. S.W. Hawking and W. Israel,(Cambridge Univ. Press, Cambridge, 1979) p.790 FOUR APPROACHES TO UV BEHAVIOR OF QUANTUM GRAVITY (QG)

  - Extended Theories Of Gravitation: Supersymmetric Theories - Superstrings, Loop Quantum Gravity
  - Resummation$\Leftarrow$ TODAY'S TALK – NEW VERSION
  - Composite Gravitons
  - Asymptotic Safety: Fixed Point Theory (See Lautscher & Reuter, hep-th/0205062)

Review of Feynman's Formulation of Einstein's Theory

For the known world, we have the generally covariant Lagrangian

$$\mathcal{L}(x) = -\frac{1}{2\kappa^2}\sqrt{-g}R + \sqrt{-g}L^{\mathcal{G}}_{SM}(x) \tag{1}$$

- R is the curvature scalar,
- $-g = -det g_{\mu\nu}$
- $\kappa = \sqrt{8\pi G_N} \equiv \sqrt{8\pi/M^2_{Pl}}$, where G_N is Newton's constant,
- SM Lagrangian density $= L^{\mathcal{G}}_{SM}(x)$

One gets $L^{\mathcal{G}}_{SM}(x)$ from the usual SM Lagrangian density as follows:

- Note that $\partial_\mu \phi(x)$ is already generally covariant for any scalar field ϕ.
- Note that the only derivatives of the vector fields in the SM Lagrangian density occur in their curls, $\partial_\mu A^J_\nu(x) - \partial_\nu A^J_\mu(x)$, which are also already generally covariant.
- Thus, we only need to give a rule for making the fermionic terms in usual SM

Lagrangian density generally covariant. $\Rightarrow$

We introduce a differentiable structure with $\{\xi^a(x)\}$ as locally inertial coordinates and an attendant vierbein field $e_\mu^a \equiv \partial\xi^a/\partial x^\mu$ with indices that carry the vector representation for the flat locally inertial space, a, and for the manifold of space-time, μ, with the identification of the space-time base manifold metric as $g_{\mu\nu} = e_\mu^a e_{a\nu}$ where the flat locally inertial space indices are to be raised and lowered with Minkowski's metric η_{ab} as usual.

Associating the usual Dirac gamma matrices $\{\gamma_a\}$ with the flat locally inertial space at x, we define base manifold Dirac gamma matrices by $\Gamma_\mu(x) = e_\mu^a(x)\gamma_a$.

Then the spin connection, $\omega_{\mu b}^a = -\frac{1}{2}e^{a\nu}\left(\partial_\mu e_\nu^b - \partial_\nu e_\mu^b\right) + \frac{1}{2}e^{b\nu}\left(\partial_\mu e_\nu^a - \partial_\nu e_\mu^b\right) + \frac{1}{2}e^{a\rho}e^{b\sigma}\left(\partial_\rho e_{c\sigma} - \partial_\sigma e_{c\rho}\right)e_\mu^c$ when there is no torsion, allows us to identify the generally covariant Dirac operator for the SM fields by the substitution $i\not\partial \rightarrow i\Gamma(x)^\mu\left(\partial_\mu + \frac{1}{2}\omega_{\mu b}{}^a\Sigma^b{}_a\right)$, where we have $\Sigma^b{}_a = \frac{1}{4}\left[\gamma^b, \gamma_a\right]$ everywhere in the SM Lagrangian density. This will generate $L^{\mathcal{G}}_{SM}(x)$ from the usual SM Lagrangian density $L_{SM}(x)$ as it is given in the papers of Hollik, Bardin, Passarino, etc., for example.

SM $\Leftrightarrow$ UV Renormalizable.

Is QG also calculable?

To study this question, we follow Feynman and treat spin as an inessential complication, as the question of whether a point particle with mass coupled to QG is a calculable theory should not depend too severely on whether or not it is spinning. We can come back to a spin-dependent analysis elsewhere.

We replace $L^{\mathcal{G}}_{SM}(x)$ in (1) with the simplest case for our question, that of a free scalar field, a free physical Higgs field, $\varphi(x)$, with a rest mass believed to be less than 400 GeV and known to be greater than 114.4 GeV with a 95% CL. We are then led to consider the representative model $\{$**R.P. Feynman, Acta Phys. Pol. 24 (1963) 697;** *Feynman Lectures on Gravitation*, **eds. F.B. Moringo and W.G. Wagner, (Caltech, Pasadena, 1971).** $\}$

$$
\begin{aligned}
\mathcal{L}(x) &= -\frac{1}{2\kappa^2} R\sqrt{-g} + \frac{1}{2}\left(g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - m_o^2\varphi^2\right)\sqrt{-g} \\
&= \quad \frac{1}{2}\left\{h^{\mu\nu,\lambda}\bar{h}_{\mu\nu,\lambda} - 2\eta^{\mu\mu'}\eta^{\lambda\lambda'}\bar{h}_{\mu_\lambda,\lambda'}\eta^{\sigma\sigma'}\bar{h}_{\mu'\sigma,\sigma'}\right\} \\
&\quad + \frac{1}{2}\left\{\varphi_{,\mu}\varphi^{,\mu} - m_o^2\varphi^2\right\} - \kappa h^{\mu\nu}\left[\overline{\varphi_{,\mu}\varphi_{,\nu}} + \frac{1}{2}m_o^2\varphi^2\eta_{\mu\nu}\right] \\
&\quad - \kappa^2\left[\frac{1}{2}h_{\lambda\rho}\bar{h}^{\rho\lambda}\left(\varphi_{,\mu}\varphi^{,\mu} - m_o^2\varphi^2\right) - 2\eta_{\rho\rho'}h^{\mu\rho}\bar{h}^{\rho'\nu}\varphi_{,\mu}\varphi_{,\nu}\right] + \cdots
\end{aligned}
\tag{2}
$$

where $\varphi_{,\mu} \equiv \partial_\mu \varphi$ and we have

- $g_{\mu\nu}(x) = \eta_{\mu\nu} + 2\kappa h_{\mu\nu}(x)$,
  $\eta_{\mu\nu} = diag\{1, -1, -1, -1\}$
- $\bar{y}_{\mu\nu} \equiv \frac{1}{2}\left(y_{\mu\nu} + y_{\nu\mu} - \eta_{\mu\nu} y_\rho{}^\rho\right)$ for any tensor $y_{\mu\nu}$
- Feynman rules already worked-out by Feynman (*op. cit.*), where we use his gauge, $\partial^\mu \bar{h}_{\nu\mu} = 0$

$\Leftrightarrow$ Quantum Gravity is just another quantum field theory where the metric now has quantum fluctuations as well.

For example, the one-loop corrections to the graviton propagator due to matter loops is just given by the diagrams in Fig. 1.

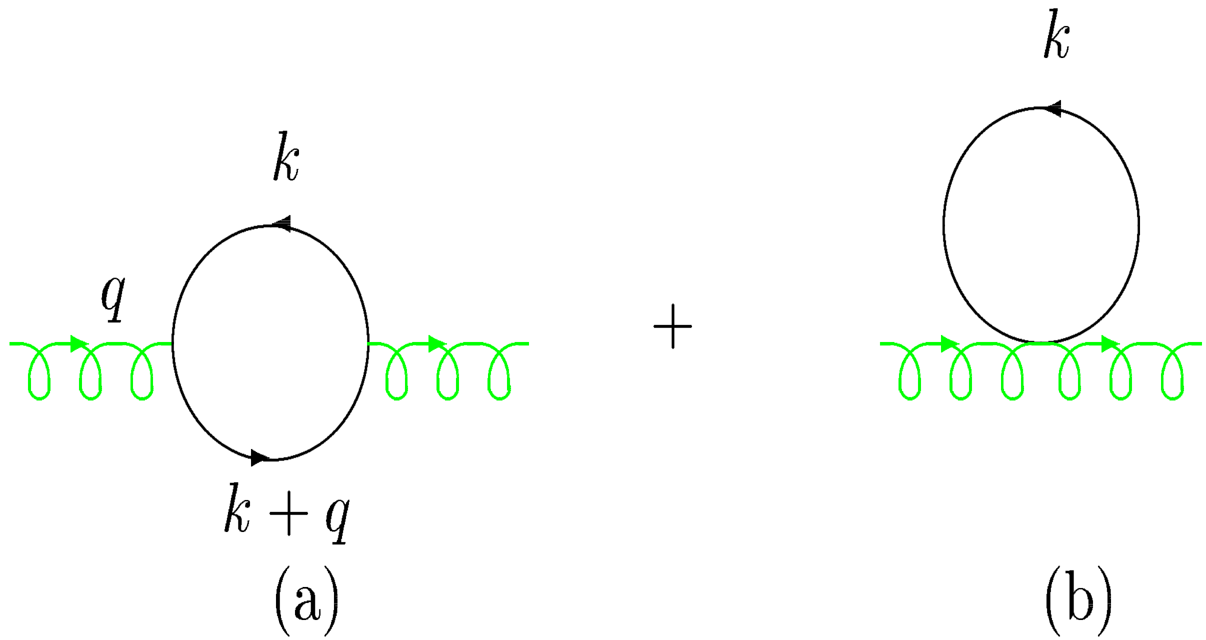

Figure 1: The scalar one-loop contribution to the graviton propagator. q is the 4-momentum of the graviton.

We return to these graphs shortly.

RESUMMED QUANTUM GRAVITY

WE WILL YFS RESUM THE PROPAGATORS IN THE THEORY:

$\Rightarrow$ **FROM THE EW RESUMMED FORMULA OF YFS**

$$\Sigma_F(p) = e^{\alpha B''_\gamma}\left[\Sigma'_F(p) - S_F^{-1}(p)\right] + S_F^{-1}(p), \tag{3}$$

WHICH $\Rightarrow$

$$iS'_F(p) = \frac{ie^{-\alpha B''_\gamma}}{S_F^{-1}(p) - \Sigma'_F(p)}, \tag{4}$$

FOR

$$\Sigma'_F(p) = \sum_{n=1}^{\infty} \Sigma'_{Fn}, \tag{5}$$

WE NEED TO FIND FOR QUANTUM GRAVITY THE ANALOGUE OF

$$\alpha B''_\gamma = \int d^4\ell \frac{S''(k,k,\ell)}{\ell^2 - \lambda^2 + i\epsilon} \tag{6}$$

WHERE $\lambda \equiv$ IR CUT-OFF AND

$$S''(k,k,\ell) = \frac{-i8\alpha}{(2\pi)^3}\frac{kk'}{(\ell^2-2\ell k+\Delta+i\epsilon)(\ell^2-2\ell k'+\Delta'+i\epsilon)}\Big|_{k=k'}, \tag{7}$$

$\Delta = k^2 - m^2$, $\Delta' = {k'}^2 - m^2$.

TO THIS END, NOTE ALSO

$$\alpha B''_\gamma = \int \frac{d^4\ell}{(2\pi)^4}\frac{-i\eta^{\mu\nu}}{(\ell^2-\lambda^2+i\epsilon)}\frac{-ie(2ik_\mu)}{(\ell^2-2\ell k+\Delta+i\epsilon)}\frac{-ie(2ik'_\nu)}{(\ell^2-2\ell k'+\Delta'+i\epsilon)}\Big|_{k=k'}. \tag{8}$$

$\Rightarrow$ WE FOLLOW WEINBERG/THE FEYNMAN RULES

AND IDENTIFY THE CONSERVED GRAVITON CHARGES AS

$e \longrightarrow \kappa k_\rho$ FOR SOFT EMISSION FROM k

$\Rightarrow$ WE GET THE ANALOGUE ,$-B''_g(k)$, OF $\alpha B''_\gamma$ BY

- REPLACING THE γ PROPAGATOR IN (8) BY THE GRAVITON

PROPAGATOR,

$$\frac{i\frac{1}{2}(\eta^{\mu\nu}\eta^{\bar\mu\bar\nu}+\eta^{\mu\bar\nu}\eta^{\bar\mu\nu}-\eta^{\mu\bar\mu}\eta^{\nu\bar\nu})}{\ell^2-\lambda^2+i\epsilon}$$

,

- **BY REPLACING THE QED CHARGES BY THE CORRESPONDING GRAVITY CHARGES** $\kappa k_{\bar\mu},\ \kappa k'_{\bar\nu}$

$\Rightarrow$

$$B''_g(k)=-2i\kappa^2k^4\frac{\int d^4\ell}{16\pi^4}\frac{1}{\ell^2-\lambda^2+i\epsilon}\frac{1}{(\ell^2+2\ell k+\Delta+i\epsilon)^2} \tag{9}$$

AND

$$i\Delta'_F(k)|_{Resummed}=\frac{ie^{B''_g(k)}}{(k^2-m^2-\Sigma'_s+i\epsilon)} \tag{10}$$

THIS IS THE BASIC RESULT.

NOTE THE FOLLOWING:

- Σ'_s **STARTS IN** $\mathcal{O}(\kappa^2)$**, SO WE MAY DROP IT IN CALCULATING ONE-LOOP EFFECTS.**

- EXPLICIT EVALUATION GIVES, FOR THE DEEP UV REGIME,

$$B_g''(k) = \frac{\kappa^2|k^2|}{8\pi^2}\ln\left(\frac{m^2}{m^2+|k^2|}\right), \tag{11}$$

$\Rightarrow$ THE RESUMMED PROPAGATOR FALLS FASTER THAN ANY POWER OF $|k^2|$!

- IF m VANISHES, USING THE USUAL $-\mu^2$ NORMALIZATION POINT WE GET $B_g''(k) = \frac{\kappa^2|k^2|}{8\pi^2}\ln\left(\frac{\mu^2}{|k^2|}\right)$ WHICH AGAIN VANISHES FASTER THAN ANY POWER OF $|k^2|$!

THIS MEANS THAT ONE-LOOP CORRECTIONS ARE FINITE!

INDEED, ALL QUANTUM GRAVITY LOOPS ARE UV FINITE!

ALL ORDERS PROOF

Consider the entire theory from (2) to all orders in κ:

$$\mathcal{L}(x) = \mathcal{L}_0(x) + \sum_{n=1}^{\infty} \kappa^n \mathcal{L}_I^{(n)}(x) \tag{12}$$

– the interactions, including the ghost interactions, are the terms of $\mathcal{O}(\kappa^n), n \geq 1$.

$$\mathcal{L}_I^{(n)}(x) = \sum_{\ell=1}^{m_n} \mathcal{L}_{I,\ell}^{(n)}(x). \tag{13}$$

$\mathcal{L}_{I,\ell}^{(n)}$ **has dimension $d_{n,\ell}$.**

Let $d_n^M = \max_\ell\{d_{n,\ell}\}$. $\Rightarrow$ The maximum power of momentum at any vertex in $\mathcal{L}_I^{(n)}$ is $\bar{d}_n^M = \min\{d_n^M - 3, 2\}$ and is finite.

Note, in any gauge,

$$iD^{(0)}_{F\alpha_1\cdots;\alpha_1'\cdots}(k)|_{YFS-resummed} = \frac{iP_{\alpha_1\cdots;\alpha_1'\cdots}e^{B_g''(k)}}{(k^2-m^2+i\epsilon)}, \quad (14)$$

so that it is also exponentially damped at high energy in the deep Euclidean regime (DER).

Now consider any 1PI vertex Γ_N with $[N] \equiv n_1 + n_2$ amputated external legs, where $N = (n_1, n_2)$, when $n_1(n_2)$ is the respective number of graviton(scalar) external lines.

At its zero-loop order, there are only tree contributions which are manifestly UV finite.

Consider the first loop ($\mathcal{O}(\kappa^2)$) corrections to Γ_N. There must be at least one improved exponentially damped propagator in the respective loop contribution and at most two vertices so that the maximum power of momentum in the numerator of the loop due to the vertices is $\max\{2\bar{d}_1^M, \bar{d}_2^M\}$ and is finite.

The exponentially damped propagator $\Rightarrow$ the loop integrals finite $\Rightarrow$ the entire one-loop ($\mathcal{O}(\kappa^2)$) contribution is finite.

Corollary: If Γ_N vanishes in tree approximation, we can conclude that its first non-trivial contributions at one-loop are all finite, due to the exponentially damped propagator.

Induction Hypothesis:

Suppose all contributions to all $\{\Gamma_N\}$ for m-loop corrections ($\mathcal{O}(\kappa^{2m})), m < n,$ are finite.

At the n-loop ($\mathcal{O}(\kappa^{2n})$) level, when the exponentially damped improved Born propagators are taken into account, we argue that respective n-loop integrals are finite as follows.

By momentum conservation, if $\{\ell_1, \cdots, \ell_n\}$ are the respective Euclidean loop momenta, we may without loss of content assume that ℓ_n is precisely the momentum of one of the exponentially damped improved Born propagators.

The $n-1$ loop integrations over the remaining loop variables $\{\ell_1, \cdots, \ell_{n-1}\}$ for fixed ℓ_n then produces the contribution of a subgraph which if it is 1PI is a part of Γ_{N+2} and which if it is not 1PI is a product of the contributions to the respective $\{\Gamma_J\}$ and the respective improved YFS resummed Born propagator functions.

This is then finite by the induction hypothesis.

Standard arguments (see *Royden*) from Lebesgue integration theory (specifically, for any two measurable functions f, g, $f \leq g$ almost everywhere implies that $\int f \leq \int g$) in conjunction with Weinberg's theorem guarantees that this finite result behaves at most as a finite power of $|\ell_n|$ modulo Weinberg's logarithms for $|\ell_n| \to \infty$.

It follows that the remaining integration over ℓ_n is damped into convergence by the already identified exponentially damped propagator with momentum ℓ_n. $\Rightarrow$ Each n-loop contribution to Γ_N is finite, from which it follows that Γ_N is finite at n-loop level.

Q.E.D.

Pictorially, we illustrate the type of situations we have in Fig. 2.

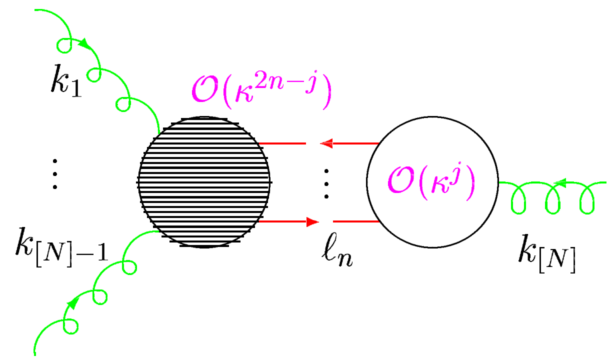

Fig.2. The typical contribution we encounter in Γ_N at the n-loop level; ℓ_n is the n-th loop momentum and is precisely the momentum of the indicated YFS-resummed improved Born propagator.

Explicit Finiteness of $\Sigma'^{(1)}$

$$\Sigma_s'^{(1)}(k) = \Sigma_s^{(1)}(k) - B_g''(k)\Delta_F^{-1}(k) \qquad (15)$$

$\Rightarrow$

$$\begin{aligned}\Sigma_s'^{(1)}(k) = -\kappa^2 \frac{\int d^4\ell}{(2\pi)^4}\Bigg\{ \Big[&(2k^\mu k^\nu)\mathcal{P}_{\mu\nu;\mu'\nu'}(\ell)(2k^{\mu'}k^{\nu'})\frac{\ell^2+2\ell k+2(k^2-m^2)}{\ell^2+2\ell k+k^2-m^2+i\epsilon} \\ &+ \Delta V^{\mu\nu}(k,\ell)\mathcal{P}_{\mu\nu;\mu'\nu'}(\ell)(2k^{\mu'}k^{\nu'}) + (2k^\mu k^\nu)\mathcal{P}_{\mu\nu;\mu'\nu'}(\ell)\Delta V^{\mu'\nu'}(k,\ell) \\ &+ \Delta V^{\mu\nu}(k,\ell)\mathcal{P}_{\mu\nu;\mu'\nu'}(\ell)\Delta V^{\mu'\nu'}(k,\ell)\Big] \frac{e^{\frac{\kappa^2|(k+\ell)^2|}{8\pi^2}\ln\left(\frac{m^2}{m^2+|(k+\ell)^2|}\right)}}{(k+\ell)^2-m^2+i\epsilon}\frac{e^{\frac{\kappa^2|\ell^2|}{8\pi^2}\ln\left(\frac{\mu^2}{|\ell^2|}\right)}}{\ell^2+i\epsilon} \\ &+ \Big[\frac{1}{2}(k^2-m^2)\left(\mathcal{P}_{\lambda\rho;}{}^{\rho\lambda}(\ell) + \mathcal{P}_{\lambda\rho;}{}^{\lambda\rho}(\ell) - \mathcal{P}_{\lambda}{}^{\lambda}{}_{;\lambda'}{}^{\lambda'}(\ell)\right) \\ &- (2k^\mu k^\nu)\left(\mathcal{P}_{\mu\rho;}{}^{\rho}{}_{\nu}(\ell) + \mathcal{P}_{\mu\rho;\nu}{}^{\rho}(\ell) - \mathcal{P}_{\mu\nu;\rho}{}^{\rho}(\ell)\right)\Big]\frac{e^{\frac{\kappa^2|\ell^2|}{8\pi^2}\ln\left(\frac{\mu^2}{|\ell^2|}\right)}}{\ell^2+i\epsilon}\Bigg\}, \end{aligned} \qquad (16)$$

where $\Delta V^{\mu\nu}(k,\ell) = k^\mu \ell^\nu + k^\nu \ell^\mu - (k^2-m^2+k\ell)\eta^{\mu\nu}$. $\Rightarrow$ **UV FINITE!**

NEWTON'S LAW

CONSIDER THE ONE-LOOP CORRECTIONS TO NEWTON'S LAW IMPLIED BY THE DIAGRAMS IN FIG. 1. THESE CORRECTIONS DIRECTLY SHOW OUR CALULABILITY ISSUE. INTRODUCING THE YFS RESUMMED PROPAGATORS INTO FIG. 1⇒ (HERE, $k \to (ik^0, \vec{k})$ BY WICK ROTATION, AND WE WORK IN THE TRANSV.-TRACELESS SPACE)

$$i\Sigma(q)^{1a}_{\bar\mu\bar\nu;\mu\nu} = i\kappa^2 \frac{\int d^4k}{2(2\pi)^4} \frac{\left(k'_{\bar\mu}k_{\bar\nu} + k'_{\bar\nu}k_{\bar\mu}\right) e^{\frac{\kappa^2|k'^2|}{8\pi^2}\ln\left(\frac{m^2}{m^2+|k'^2|}\right)}}{\left(k'^2 - m^2 + i\epsilon\right)} \frac{\left(k'_{\mu}k_{\nu} + k'_{\nu}k_{\mu}\right) e^{\frac{\kappa^2|k^2|}{8\pi^2}\ln\left(\frac{m^2}{m^2+|k^2|}\right)}}{\left(k^2 - m^2 + i\epsilon\right)}. \tag{17}$$

⇔CONVERGENT!! SO IS FIG 1b.

CONTINUING TO WORK IN THE TRANSV.-TRACELESS SPACE, WE GET THE GRAVITON PROPAGATOR DENOMINATOR

$$q^2 + \frac{1}{2}q^4\Sigma^{T(2)} + i\epsilon \tag{18}$$

WHERE THE TRANSVERSE, TRACELESS SELF-ENERGY FUNCTION $\Sigma^T(q^2)$ FOLOWS FROM eq.(17) FIG. 1a AND ITS ANALOG FOR FIG. 1b BY THE STANDARD METHODS. FOR THE COEFFICIENT OF q^4 IN $\Sigma^T(q^2)$ FOR $|q^2| >> m^2$ WE GET

$$-\frac{1}{2}\Sigma^{T(2)} \cong \frac{c_2}{360\pi M_{Pl}^2} \tag{19}$$

FOR

$$c_2 = \int_0^\infty dx x^3 (1+x)^{-4-\lambda_c x} \cong 72.1 \tag{20}$$

WHERE $\lambda_c = \frac{2m^2}{\pi M_{Pl}^2}$. WHEN WE FOURIER TRANSFORM THE INVERSE OF (18) WE FIND THE POTENTIAL

$$\Phi_{Newton}(r) = -\frac{G_N M_1 M_2}{r}(1 - e^{-ar}) \tag{21}$$

WHERE $a = 1/\sqrt{-\frac{1}{2}\Sigma^{T(2)}} \simeq 3.96 M_{Pl}$ WHERE FOR DEFINITENESS WE SET $m \cong 120$GeV.

WE NOTE FOR COMPLETENESS THAT

$$c_2 \cong \ln\frac{1}{\lambda_c} - \ln\ln\frac{1}{\lambda_c} - \frac{\ln\ln\frac{1}{\lambda_c}}{\ln\frac{1}{\lambda_c} - \ln\ln\frac{1}{\lambda_c}} - \frac{11}{6} \quad (22)$$

AND WE USED THIS RESULT TO CHECK (20). WITHOUT RESUMMATION, $\lambda_c = 0$, OUR RESULT IN (20) WOULD BE INFINITE AND, AS THIS IS THE COEFFICIENT OF q^4 IN THE INVERSE PROPAGATOR, NO RENORMALIZATION OF THE FIELD AND OF THE MASS COULD REMOVE SUCH AN INFINITY. IN OUR NEW APPROACH, THIS INFINITY IS ABSENT.

OUR RESULT IN (19) IS GAUGE INVARIANT, AS OUR APPROACH INVOLVES THE EXACT RE-ARRANGEMENT OF THE FEYNMAN SERIES (SEE MPLA17(2002)2371) AND THE ORIGINAL SERIES IS GAUGE INVARIANT.

CROSS CHECK WITH ’t HOOFT AND VELTMAN, Ann. Inst. Henri Poincare XX, 69 (1974), WHERE THE COMPLETE RESULT OF THE ONE-LOOP DIVERGENCES OF OUR SCALAR FIELD COUPLED TO EINSTEIN’S GRAVITY HAVE BEEN COMPUTED.

KEY OBSERVATION: THE RESULT WHICH WE HAVE OBTAINED WOULD BE

UV DIVERGENT WITHOUT OUR RESUMMATION. THUS, THE DOMINANT TERMS WHICH WE ARE ISOLATING IN THIS TALK ARE PRECISELY THOSE THAT ARE GIVEN BY 't HOOFT AND VELTMAN, WHERE WE NEED TO MAKE THE CORRESPONDENCE BETWEEN THE POLES IN n, THE DIMENSION OF SPACE-TIME, AT $n = 4$ CALCULATED BY THEM AND THE LEADING LOG $\ln \frac{1}{\lambda_c}$. $\Rightarrow$ SET C_2 EQUAL TO ITS VALUE WHEN $\lambda_c = 0$ IN n DIMENSIONS AND ALLOW $n \to 4$. $\Rightarrow$

$$\frac{1}{(2 - n/2)} \leftrightarrow c_2. \tag{23}$$

THIS MEANS THAT, IF WE LOOK AT THE LIMIT $q^2 \to 0$, WE GET THE RESULT THAT THE COEFFICIENT OF q^4 IN (18) IS $3/(2 - n/2)$ TIMES THE COEFFICIENT OF c_2 ON THE RIGHT-HAND SIDE OF (19), AND THIS IS IN COMPLETE AGREEMENT WITH THE RESULT THAT IS IMPLIED BY EQ.(3.40) IN Ann. Inst. Henri Poincare XX, 69 (1974) , FOR EXAMPLE. OF COURSE, THE RESULTS IN OF 't HOOFT AND VELTMAN ARE ALSO GAUGE INVARIANT.

OBSERVATIONS

- **SUB ℓ_{Pl} ACCESSIBLE TO POINT PARTICLE FIELD THEORY $\Rightarrow$ CURRENT SUPERSTRING THEORIES MAY BE PHENOMENOLOGICAL MODELS FOR A MORE FUNDAMENTAL THEORY (TUT=The Ultimate Theory) JUST AS THE OLD STRING THEORY IS FOR QCD.**
- **OUR DEEP EUCLIDEAN STUDIES ARE COMPLEMENTARY TO THE LOW ENERGY STUDIES OF DONOGHUE(PRL 72 (1994) 2996; PRD 50 (1994) 3974).**
- **CUT-OFF AT M_{Pl} $\Rightarrow$ RENORM. QFT BELOW M_{Pl} UNAFFECTED**
- **NONRENORM. QFT'S GIVEN NEW LIFE HERE – THEY MAY HAVE OTHER PROBLEMS.**

Conclusions

YFS RESUMMATION RENDERS QUANTUM GRAVITY FINITE

- QUANTUM LOOP CORRECTIONS ARE NOW CUT OFF DYNAMICALLY.
- PHYSICS BELOW THE PLANCK SCALE ACCESSIBLE TO POINT PARTICLE QFT: (TUT)
- EARLY UNIVERSE STUDIES MAY BE ABLE TO TEST PREDICTIONS.
- RENORMALIZABLE QFT BELOW M_{Pl} UNAFFECTED
- NEW LIFE TO SOME NONRENORMALIZABLE QFT'S: THEY MAY HAVE OTHER PROBLEMS
- MINIMAL UNION OF BOHR AND EINSTEIN