A FIELD THEORETICAL MODEL IN NONCOMMUTATIVE MINKOWSKI SUPERSPACE

Vahagn Nazaryan and Carl E. Carlson

Nuclear and Particle Theory Group,
Department of Physics
College of William and Mary

Meeting of the Division of Particles and Fields

August 30 2004

Outline

- ★ **An Introduction**
- ★ **The Noncommutative SUSY Algebra**
- ★ **The Star Product**
- ★ **The Wess-Zumino Lagrangian**
  - ▶ **Chiral and Antichiral Superfields**
  - ▶ **Non-associativity and Weyl ordering**
  - ▶ **The Lagrangian**

I: Non(anti)commutativity from String Theory

★ **Consider standard Nambu-Goto string action with extra term $B^{\mu\nu}$, which is constant.**

$$\mathbf{S} = \frac{\mathbf{1}}{\mathbf{2}\pi\alpha'}\int \mathbf{d}\sigma\,\mathbf{d}\tau\left(\mathbf{g^{ab}}\eta_{\mu\nu}\partial_{\mathbf{a}}\mathbf{x}^{\mu}\partial_{\mathbf{b}}\mathbf{x}^{\nu} + \epsilon^{\mathbf{ab}}\mathbf{B}_{\mu\nu}\partial_{\mathbf{a}}\mathbf{x}^{\mu}\partial_{\mathbf{b}}\mathbf{x}^{\nu}\right)$$

► **quantize string in the presence of a background field** $B^{\mu\nu}$ $\Rrightarrow$

$$[\hat{x}^{\mu}, \hat{x}^{\nu}] = 2\pi\alpha'\left((1 - B^2)^{-1}B\right)^{\mu\nu} \equiv i\Theta^{\mu\nu}\,.$$

I: Non(anti)commutativity from Superstring Theory

Ooguri, Vafa :hep-th/0302109, Seiberg :hep-th/0305248...

★ **Consider superstrings in a self-dual graviphoton background $F^{\alpha\beta}$, keep $F^{\dot{\alpha}\dot{\beta}} = 0$** $\Rightarrow$

work in flat Euclidean superspace $\Rightarrow$

I: Non(anti)commutativity from Superstring Theory

Ooguri, Vafa :hep-th/0302109, Seiberg :hep-th/0305248

★ **Consider superstrings in a self-dual graviphoton background $F^{\alpha\beta}$, keep $F^{\dot\alpha\dot\beta} = 0 \Rrightarrow$**

work in flat Euclidean superspace $\Rrightarrow$

$$\{\hat\theta^\alpha, \hat\theta^\beta\} = (\alpha')^2 F^{\alpha\beta} \equiv C^{\alpha\beta} \,,$$

$$\{\hat{\bar\theta}^{\dot\alpha}, \hat{\bar\theta}^{\dot\beta}\} = 0 \,,$$

$$[\hat y^\mu, \hat\theta^\alpha] = [\hat y^\mu, \hat{\bar\theta}^{\dot\alpha}] = [\hat y^\mu, \hat y^\nu] = 0$$

$$[\hat x^\mu, \hat\theta^\alpha] = iC^{\alpha\beta}\sigma^\mu_{\beta\dot\alpha}\bar\theta^{\dot\alpha} \,, \quad [\hat x^\mu, \hat x^\nu] = \bar\theta\bar\theta C^{\mu\nu} \,.$$

I: Non(anti)commutativity from Superstring Theory

★ Lorentz invariant Wass-Zumino lagrangian, but nonhermitian

★ deformation of supersymmetry algebra is chirally asymmetric (Berenstein, Rey: hep-th/0308049)

⇛ operators induced by deformation are nonhermitian,

⇛ assign non-canonical scaling dimensions for the Grassman coordinates, and superfields,

⇛ proof for renormalizability of 4-dimensional QFT's with $\mathcal{N} = 1/2$ supersymmetry.

II: The Noncommutative SUSY Algebra

★ **Consider 4D Minkowski space,** $\bar{\theta}^{\dot{\alpha}} \equiv (\theta^{\alpha})^{\dagger}$**,**

$\hat{y}^{\mu} \equiv \hat{x}^{\mu} + i\hat{\theta}\sigma^{\mu}\hat{\bar{\theta}}$**,** $\quad \hat{\bar{y}}^{\mu} \equiv \hat{x}^{\mu} - i\hat{\theta}\sigma^{\mu}\hat{\bar{\theta}}$**,** $\;\bar{C}^{\dot{\alpha}\dot{\beta}} = (C^{\beta\alpha})^{\dagger}$**,**

II: The Noncommutative SUSY Algebra

★ **Consider 4D Minkowski space,** $\bar{\theta}^{\dot{\alpha}} \equiv (\theta^{\alpha})^{\dagger}$**,**

$\hat{y}^{\mu} \equiv \hat{x}^{\mu} + i\hat{\theta}\sigma^{\mu}\hat{\bar{\theta}}$**,** $\quad \hat{\bar{y}}^{\mu} \equiv \hat{x}^{\mu} - i\hat{\theta}\sigma^{\mu}\hat{\bar{\theta}}$**,** $\quad \bar{C}^{\dot{\alpha}\dot{\beta}} = (C^{\beta\alpha})^{\dagger}$**,**

Require $\qquad \{\hat{\theta}^{\alpha}, \hat{\theta}^{\beta}\} = C^{\alpha\beta} \quad \Rightarrow \quad \{\hat{\bar{\theta}}^{\dot{\alpha}}, \hat{\bar{\theta}}^{\dot{\beta}}\} = \bar{C}^{\dot{\alpha}\dot{\beta}}$

II: The Noncommutative SUSY Algebra

★ **Consider 4D Minkowski space,** $\bar{\theta}^{\dot{\alpha}} \equiv (\theta^{\alpha})^{\dagger}$**,**

$\hat{y}^{\mu} \equiv \hat{x}^{\mu} + i\hat{\theta}\sigma^{\mu}\hat{\bar{\theta}}$**,** $\quad \hat{\bar{y}}^{\mu} \equiv \hat{x}^{\mu} - i\hat{\theta}\sigma^{\mu}\hat{\bar{\theta}}$**,** $\quad \bar{C}^{\dot{\alpha}\dot{\beta}} = (C^{\beta\alpha})^{\dagger}$**,**

Require $\quad \{\hat{\theta}^{\alpha}, \hat{\theta}^{\beta}\} = C^{\alpha\beta} \quad \Rightarrow \quad \{\hat{\bar{\theta}}^{\dot{\alpha}}, \hat{\bar{\theta}}^{\dot{\beta}}\} = \bar{C}^{\dot{\alpha}\dot{\beta}}$

Require $\quad [\hat{y}^{\mu}, \hat{\theta}^{\alpha}] = 0 \quad \Rightarrow \quad [\hat{\bar{y}}^{\mu}, \hat{\bar{\theta}}^{\dot{\alpha}}] = 0$

II: The Noncommutative SUSY Algebra

★ **Consider 4D Minkowski space,** $\bar{\theta}^{\dot{\alpha}} \equiv (\theta^{\alpha})^{\dagger}$**,**

$\hat{y}^{\mu} \equiv \hat{x}^{\mu} + i\hat{\theta}\sigma^{\mu}\hat{\bar{\theta}}$**,** $\quad \hat{\bar{y}}^{\mu} \equiv \hat{x}^{\mu} - i\hat{\theta}\sigma^{\mu}\hat{\bar{\theta}}$**,** $\bar{C}^{\dot{\alpha}\dot{\beta}} = (C^{\beta\alpha})^{\dagger}$**,**

Require $\quad \{\hat{\theta}^{\alpha}, \hat{\theta}^{\beta}\} = C^{\alpha\beta} \quad \Rightarrow \quad \{\hat{\bar{\theta}}^{\dot{\alpha}}, \hat{\bar{\theta}}^{\dot{\beta}}\} = \bar{C}^{\dot{\alpha}\dot{\beta}}$

Require $\quad [\hat{y}^{\mu}, \hat{\theta}^{\alpha}] = 0 \quad \Rightarrow \quad [\hat{\bar{y}}^{\mu}, \hat{\bar{\theta}}^{\dot{\alpha}}] = 0$

Require $\quad \{\hat{\bar{\theta}}^{\dot{\alpha}}, \hat{\theta}^{\alpha}\} = 0 \quad \Rightarrow$

II: The Noncommutative SUSY Algebra

★ **Consider 4D Minkowski space,** $\bar{\theta}^{\dot{\alpha}} \equiv (\theta^{\alpha})^{\dagger}$**,**

$\hat{y}^{\mu} \equiv \hat{x}^{\mu} + i\hat{\theta}\sigma^{\mu}\hat{\bar{\theta}}$**,** $\quad \hat{\bar{y}}^{\mu} \equiv \hat{x}^{\mu} - i\hat{\theta}\sigma^{\mu}\hat{\bar{\theta}}$**,** $\quad \bar{C}^{\dot{\alpha}\dot{\beta}} = (C^{\beta\alpha})^{\dagger}$**,**

Require $\quad \{\hat{\theta}^{\alpha}, \hat{\theta}^{\beta}\} = C^{\alpha\beta} \quad \Rightarrow \quad \{\hat{\bar{\theta}}^{\dot{\alpha}}, \hat{\bar{\theta}}^{\dot{\beta}}\} = \bar{C}^{\dot{\alpha}\dot{\beta}}$

Require $\quad [\hat{y}^{\mu}, \hat{\theta}^{\alpha}] = 0 \quad \Rightarrow \quad [\hat{\bar{y}}^{\mu}, \hat{\bar{\theta}}^{\dot{\alpha}}] = 0$

Require $\quad \{\hat{\bar{\theta}}^{\dot{\alpha}}, \hat{\theta}^{\alpha}\} = 0 \quad \Rightarrow$

$$[\hat{\bar{y}}^{\mu}, \hat{\theta}^{\alpha}] = 2iC^{\alpha\beta}\sigma^{\mu}_{\beta\dot{\beta}}\hat{\bar{\theta}}^{\dot{\beta}},$$

$$[\hat{y}^{\mu}, \hat{\bar{\theta}}^{\dot{\alpha}}] = 2i\bar{C}^{\dot{\alpha}\dot{\beta}}\theta^{\beta}\sigma^{\mu}_{\beta\dot{\beta}}.$$

II: The Noncommutative SUSY Algebra

★ $$[\hat{y}^{\mu}, \hat{y}^{\nu}] - [\hat{\bar{y}}^{\mu}, \hat{\bar{y}}^{\nu}] = 4(\bar{C}^{\dot{\alpha}\dot{\beta}}\hat{\theta}^{\alpha}\hat{\theta}^{\beta} - C^{\alpha\beta}\hat{\bar{\theta}}^{\dot{\alpha}}\hat{\bar{\theta}}^{\dot{\beta}})\sigma^{\mu}_{\alpha\dot{\alpha}}\sigma^{\nu}_{\beta\dot{\beta}}\,.$$

II: The Noncommutative SUSY Algebra

★ $$[\hat{y}^\mu, \hat{y}^\nu] - [\hat{\bar{y}}^\mu, \hat{\bar{y}}^\nu] = 4(\bar{C}^{\dot{\alpha}\dot{\beta}}\hat{\theta}^\alpha\hat{\theta}^\beta - C^{\alpha\beta}\hat{\bar{\theta}}^{\dot{\alpha}}\hat{\bar{\theta}}^{\dot{\beta}})\sigma^\mu_{\alpha\dot{\alpha}}\sigma^\nu_{\beta\dot{\beta}} \,.$$

If $[\hat{y}^\mu, \hat{y}^\nu] = [\hat{\bar{y}}^\mu, \hat{\bar{y}}^\nu] = 0$ **,**

II: The Noncommutative SUSY Algebra

★ $$[\hat{y}^\mu, \hat{y}^\nu] - [\hat{\bar{y}}^\mu, \hat{\bar{y}}^\nu] = 4(\bar{C}^{\dot{\alpha}\dot{\beta}}\hat{\theta}^\alpha\hat{\theta}^\beta - C^{\alpha\beta}\hat{\bar{\theta}}^{\dot{\alpha}}\hat{\bar{\theta}}^{\dot{\beta}})\sigma^\mu_{\alpha\dot{\alpha}}\sigma^\nu_{\beta\dot{\beta}}\,.$$

If $[\hat{y}^\mu, \hat{y}^\nu] = [\hat{\bar{y}}^\mu, \hat{\bar{y}}^\nu] = 0$ **,**

$$\Rightarrow \quad \bar{C}^{\dot{\alpha}\dot{\beta}}\hat{\theta}^\alpha\sigma^\mu_{\alpha\dot{\alpha}}\hat{\theta}^\beta\sigma^\nu_{\beta\dot{\beta}} = C^{\alpha\beta}\sigma^\mu_{\alpha\dot{\alpha}}\hat{\bar{\theta}}^{\dot{\alpha}}\sigma^\nu_{\beta\dot{\beta}}\hat{\bar{\theta}}^{\dot{\beta}}\,.$$

II: The Noncommutative SUSY Algebra

★ $$[\hat{y}^\mu, \hat{y}^\nu] - [\hat{\bar{y}}^\mu, \hat{\bar{y}}^\nu] = 4(\bar{C}^{\dot{\alpha}\dot{\beta}}\hat{\theta}^\alpha\hat{\theta}^\beta - C^{\alpha\beta}\hat{\bar{\theta}}^{\dot{\alpha}}\hat{\bar{\theta}}^{\dot{\beta}})\sigma^\mu_{\alpha\dot{\alpha}}\sigma^\nu_{\beta\dot{\beta}}\,.$$

If $[\hat{y}^\mu, \hat{y}^\nu] = [\hat{\bar{y}}^\mu, \hat{\bar{y}}^\nu] = 0$ **,**

$$\Rightarrow \quad \bar{C}^{\dot{\alpha}\dot{\beta}}\hat{\theta}^\alpha\sigma^\mu_{\alpha\dot{\alpha}}\hat{\theta}^\beta\sigma^\nu_{\beta\dot{\beta}} = C^{\alpha\beta}\sigma^\mu_{\alpha\dot{\alpha}}\hat{\bar{\theta}}^{\dot{\alpha}}\sigma^\nu_{\beta\dot{\beta}}\hat{\bar{\theta}}^{\dot{\beta}}\,.$$

Thus, for consistency we choose,

$$[\hat{y}^\mu, \hat{y}^\nu] = (4\bar{C}^{\dot{\alpha}\dot{\beta}}\hat{\theta}^\alpha\hat{\theta}^\beta - 2C^{\alpha\beta}\bar{C}^{\dot{\alpha}\dot{\beta}})\sigma^\mu_{\alpha\dot{\alpha}}\sigma^\nu_{\beta\dot{\beta}}\,,$$

$$[\hat{\bar{y}}^\mu, \hat{\bar{y}}^\nu] = (4C^{\alpha\beta}\hat{\bar{\theta}}^{\dot{\alpha}}\hat{\bar{\theta}}^{\dot{\beta}} - 2C^{\alpha\beta}\bar{C}^{\dot{\alpha}\dot{\beta}})\sigma^\mu_{\alpha\dot{\alpha}}\sigma^\nu_{\beta\dot{\beta}}$$

$$\Rightarrow \quad [\hat{y}^\mu, \hat{\bar{y}}^\nu] = 2C^{\alpha\beta}\bar{C}^{\dot{\alpha}\dot{\beta}}\sigma^\mu_{\alpha\dot{\alpha}}\sigma^\nu_{\beta\dot{\beta}}\,.$$

II: The Noncommutative SUSY Algebra

$$\{\hat{\theta}^\alpha, \hat{\theta}^\beta\} = C^{\alpha\beta}\,,$$

$$\{\hat{\bar{\theta}}^{\dot{\alpha}}, \hat{\bar{\theta}}^{\dot{\beta}}\} = \bar{C}^{\dot{\alpha}\dot{\beta}}\,,$$

$$\{\hat{\bar{\theta}}^{\dot{\alpha}}, \hat{\theta}^\alpha\} = 0\,,$$

$$[\hat{x}^\mu, \hat{\bar{\theta}}^{\dot{\alpha}}] = i\bar{C}^{\dot{\alpha}\dot{\beta}}\hat{\theta}^\beta \sigma^\mu_{\beta\dot{\beta}}\,,$$

$$[\hat{x}^\mu, \hat{\theta}^\alpha] = iC^{\alpha\beta}\sigma^\mu_{\beta\dot{\beta}}\hat{\bar{\theta}}^{\dot{\beta}}\,,$$

$$[\hat{x}^\mu, \hat{x}^\nu] = (C^{\alpha\beta}\hat{\bar{\theta}}^{\dot{\alpha}}\hat{\bar{\theta}}^{\dot{\beta}} - \bar{C}^{\dot{\alpha}\dot{\beta}}\hat{\theta}^\beta\hat{\theta}^\alpha)\sigma^\mu_{\alpha\dot{\alpha}}\sigma^\nu_{\beta\dot{\beta}}\,.$$

II: The Star Product

★ **If** $\{\hat{\theta}^{\alpha}, \hat{\theta}^{\beta}\} = C^{\alpha\beta}$**, and** $\{\theta^{\alpha}, \theta^{\beta}\} = 0$ **, and choose Weyl ordering in noncommutative space, then**

II: The Star Product

★ **If** $\{\hat{\theta}^\alpha, \hat{\theta}^\beta\} = C^{\alpha\beta}$**, and** $\{\theta^\alpha, \theta^\beta\} = 0$ **, and choose Weyl ordering in noncommutative space, then**

$$\begin{aligned}
\theta^\alpha\theta^\beta &\Rrightarrow \mathbf{W}(\hat{\theta}^\alpha\hat{\theta}^\beta)\,, & \hat{\theta}^\alpha\hat{\theta}^\beta &\Rrightarrow \theta^\alpha * \theta^\beta\,,\\
&= -\hat{\theta}^\beta\hat{\theta}^\alpha + \frac{1}{2}C^{\alpha\beta}\,, & &= \theta^\beta\theta^\alpha + \frac{1}{2}C^{\alpha\beta}\,,\\
&\Rrightarrow -\theta^\beta * \theta^\alpha + \frac{1}{2}C^{\alpha\beta}\,, & &\Rrightarrow -\mathbf{W}(\hat{\theta}^\beta\hat{\theta}^\alpha) + \frac{1}{2}C^{\alpha\beta}\,,\\
&= \theta^\alpha\theta^\beta\,, & &= \hat{\theta}^\alpha\hat{\theta}^\beta\,.
\end{aligned}$$

II: The Star Product

★ **If** $\{\hat{\theta}^\alpha, \hat{\theta}^\beta\} = C^{\alpha\beta}$**, and** $\{\theta^\alpha, \theta^\beta\} = 0$ **, and choose Weyl ordering in noncommutative space, then**

$$\begin{aligned} \theta^\alpha\theta^\beta &\Rrightarrow \mathbf{W}(\hat{\theta}^\alpha\hat{\theta}^\beta)\,, \\ &= -\hat{\theta}^\beta\hat{\theta}^\alpha + \frac{1}{2}C^{\alpha\beta}\,, \\ &\Rrightarrow -\theta^\beta * \theta^\alpha + \frac{1}{2}C^{\alpha\beta}\,, \\ &= \theta^\alpha\theta^\beta\,, \end{aligned} \qquad \begin{aligned} \hat{\theta}^\alpha\hat{\theta}^\beta &\Rrightarrow \theta^\alpha * \theta^\beta\,, \\ &= \theta^\beta\theta^\alpha + \frac{1}{2}C^{\alpha\beta}\,, \\ &\Rrightarrow -\mathbf{W}(\hat{\theta}^\beta\hat{\theta}^\alpha) + \frac{1}{2}C^{\alpha\beta}\,, \\ &= \hat{\theta}^\alpha\hat{\theta}^\beta\,. \end{aligned}$$

★ **An important requirement: reality condition**

$$(f_1 * f_2)^\dagger = f_2^\dagger * f_1^\dagger$$

II: The Star Product

$$\hat{f}\hat{g} \Rrightarrow f(x,\theta,\bar{\theta}) * g(x,\theta,\bar{\theta}) = fg +$$

$$f\Big[-\frac{C^{\alpha\beta}}{2}\overleftarrow{Q}_\alpha\overrightarrow{Q}_\beta - \frac{\bar{C}^{\dot{\alpha}\dot{\beta}}}{2}\overleftarrow{\bar{Q}}_{\dot{\alpha}}\overrightarrow{\bar{Q}}_{\dot{\beta}}$$

$$+\frac{C^{\alpha\beta}C^{\gamma\delta}}{8}\overleftarrow{Q}_\alpha\overleftarrow{Q}_\gamma\overrightarrow{Q}_\delta\overrightarrow{Q}_\beta + \frac{\bar{C}^{\dot{\alpha}\dot{\beta}}\bar{C}^{\dot{\gamma}\dot{\delta}}}{8}\overleftarrow{\bar{Q}}_{\dot{\alpha}}\overleftarrow{\bar{Q}}_{\dot{\gamma}}\overrightarrow{\bar{Q}}_{\dot{\delta}}\overrightarrow{\bar{Q}}_{\dot{\beta}}$$

$$+\frac{C^{\alpha\beta}\bar{C}^{\dot{\alpha}\dot{\beta}}}{4}\left(\overleftarrow{\bar{Q}}_{\dot{\alpha}}\overleftarrow{Q}_\alpha\overrightarrow{\bar{Q}}_{\dot{\beta}}\overrightarrow{Q}_\beta - \overleftarrow{Q}_\alpha\overleftarrow{\bar{Q}}_{\dot{\alpha}}\overrightarrow{Q}_\beta\overrightarrow{\bar{Q}}_{\dot{\beta}}\right)\Big]g$$

II: The Star Product

$$f(\theta) \, * \, g(\theta) = f(\theta)\mathbf{e}xp\left(-\frac{C^{\alpha\beta}}{2}\frac{\overleftarrow{\partial}}{\partial\theta^\alpha}\frac{\overrightarrow{\partial}}{\partial\theta^\beta}\right) g(\theta)$$
$$= f(\theta)\left(1 - \frac{C^{\alpha\beta}}{2}\frac{\overleftarrow{\partial}}{\partial\theta^\alpha}\frac{\overrightarrow{\partial}}{\partial\theta^\beta} - \mathbf{d}etC\frac{\overleftarrow{\partial}}{\partial\theta\theta}\frac{\overrightarrow{\partial}}{\partial\theta\theta}\right) g(\theta)\,,$$

II: The Star Product

$$f(\theta) * g(\theta) = f(\theta)\mathbf{e}xp\left(-\frac{C^{\alpha\beta}}{2}\frac{\overleftarrow{\partial}}{\partial\theta^\alpha}\frac{\overrightarrow{\partial}}{\partial\theta^\beta}\right)g(\theta)$$

$$= f(\theta)\left(1 - \frac{C^{\alpha\beta}}{2}\frac{\overleftarrow{\partial}}{\partial\theta^\alpha}\frac{\overrightarrow{\partial}}{\partial\theta^\beta} - \mathbf{d}etC\frac{\overleftarrow{\partial}}{\partial\theta\theta}\frac{\overrightarrow{\partial}}{\partial\theta\theta}\right)g(\theta)\,,$$

and

$$f(\bar{\theta}) * g(\bar{\theta}) = f(\bar{\theta})\mathbf{e}xp\left(-\frac{\bar{C}^{\dot{\alpha}\dot{\beta}}}{2}\frac{\overleftarrow{\partial}}{\partial\bar{\theta}^{\dot{\alpha}}}\frac{\overrightarrow{\partial}}{\partial\bar{\theta}^{\dot{\beta}}}\right)g(\bar{\theta})$$

$$= f(\bar{\theta})\left(1 - \frac{\bar{C}^{\dot{\alpha}\dot{\beta}}}{2}\frac{\overleftarrow{\partial}}{\partial\bar{\theta}^{\dot{\alpha}}}\frac{\overrightarrow{\partial}}{\partial\bar{\theta}^{\dot{\beta}}} - \mathbf{d}et\bar{C}\frac{\overleftarrow{\partial}}{\partial\bar{\theta}\bar{\theta}}\frac{\overrightarrow{\partial}}{\partial\bar{\theta}\bar{\theta}}\right)g(\bar{\theta})\,,$$

II: The Star Product

$$\{\theta^\alpha, \theta^\beta\}_* = C^{\alpha\beta}\,, \qquad [x^\mu, \theta^\alpha]_* = iC^{\alpha\beta}\sigma^\mu_{\beta\dot\beta}\bar\theta^{\dot\beta}\,,$$

$$\{\bar\theta^{\dot\alpha}, \bar\theta^{\dot\beta}\}_* = \bar C^{\dot\alpha\dot\beta}\,, \qquad [x^\mu, \bar\theta^{\dot\alpha}]_* = i\bar C^{\dot\alpha\dot\beta}\theta^\beta\sigma^\mu_{\beta\dot\beta}\,,$$

$$\{\bar\theta^{\dot\alpha}, \theta^\alpha\}_* = 0\,, \qquad [x^\mu, x^\nu]_* = \bar\theta\bar\theta C^{\mu\nu} + \theta\theta\bar C^{\mu\nu}\,.$$

where,

$$C^{\mu\nu} \equiv C^{\alpha\beta}\epsilon_{\beta\gamma}(\sigma^{\mu\nu})_\alpha{}^\gamma\,,$$

$$\bar C^{\mu\nu} \equiv \bar C^{\dot\alpha\dot\beta}\epsilon_{\dot\beta\dot\gamma}(\bar\sigma^{\mu\nu})^{\dot\gamma}{}_{\dot\alpha}\,.$$

III: Supercharges and Covariant Derivatives

★ Q's and D's have their canonical forms,

$$\{Q_\alpha, Q_\beta\} = -4\bar{C}^{\dot{\alpha}\dot{\beta}} \sigma^\mu_{\alpha\dot{\alpha}} \sigma^\nu_{\beta\dot{\beta}} \frac{\partial^2}{\partial\bar{y}^\mu \partial\bar{y}^\nu},$$

$$\{\bar{Q}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\} = -4C^{\alpha\beta} \sigma^\mu_{\alpha\dot{\alpha}} \sigma^\nu_{\beta\dot{\beta}} \frac{\partial^2}{\partial y^\mu \partial y^\nu},$$

$$\{\overrightarrow{Q}_\alpha, \overrightarrow{\bar{Q}}_{\dot{\alpha}}\} = 2i\sigma^\mu_{\alpha\dot{\alpha}} \frac{\partial}{\partial y^\mu}.$$

III: Supercharges and Covariant Derivatives

★ Q's and D's have their canonical forms,

$$\{Q_\alpha, Q_\beta\} = -4\bar{C}^{\dot{\alpha}\dot{\beta}}\sigma^\mu_{\alpha\dot{\alpha}}\sigma^\nu_{\beta\dot{\beta}}\frac{\partial^2}{\partial\bar{y}^\mu\partial\bar{y}^\nu},$$
$$\{\bar{Q}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\} = -4C^{\alpha\beta}\sigma^\mu_{\alpha\dot{\alpha}}\sigma^\nu_{\beta\dot{\beta}}\frac{\partial^2}{\partial y^\mu\partial y^\nu},$$
$$\{\overrightarrow{Q}_\alpha, \overrightarrow{\bar{Q}}_{\dot{\alpha}}\} = 2i\sigma^\mu_{\alpha\dot{\alpha}}\frac{\partial}{\partial y^\mu}.$$

★ Supersymmetry is broken by the star product,

$$\overrightarrow{Q}(\Psi_1 * \Psi_2) \neq \overrightarrow{Q}(\Psi_1) * \Psi_2 + (-1)^{p(\Psi_1)}\Psi_1 * \overrightarrow{Q}(\Psi_2),$$
$$\overrightarrow{\bar{Q}}(\Psi_1 * \Psi_2) \neq \overrightarrow{\bar{Q}}(\Psi_1) * \Psi_2 + (-1)^{p(\Psi_1)}\Psi_1 * \overrightarrow{\bar{Q}}(\Psi_2).$$

III: Supercharges and Covariant Derivatives

$$\{D_\alpha, D_\beta\} = 0\,,$$

$$\{\bar{D}_{\dot\alpha}, \bar{D}_{\dot\beta}\} = 0\,,$$

$$\{\overrightarrow{D}_\alpha, \overrightarrow{\bar{D}}_{\dot\alpha}\} = -2i\sigma^\mu_{\alpha\dot\alpha}\frac{\partial}{\partial y^\mu}\,.$$

III: Supercharges and Covariant Derivatives

$$\{D_\alpha, D_\beta\} = 0\,,$$

$$\{\bar{D}_{\dot\alpha}, \bar{D}_{\dot\beta}\} = 0\,,$$

$$\{\overrightarrow{D}_\alpha, \overrightarrow{\bar{D}}_{\dot\alpha}\} = -2i\sigma^\mu_{\alpha\dot\alpha}\frac{\partial}{\partial y^\mu}\,.$$

★ $\{D_\alpha, Q_\beta\} = \{\bar{D}_{\dot\alpha}, Q_\beta\} = \{D_\alpha, \bar{Q}_{\dot\beta}\} = \{\bar{D}_{\dot\alpha}, \bar{Q}_{\dot\beta}\} = 0\,.$

III: Supercharges and Covariant Derivatives

$$\{D_\alpha, D_\beta\} = 0\,,$$

$$\{\bar{D}_{\dot\alpha}, \bar{D}_{\dot\beta}\} = 0\,,$$

$$\{\overrightarrow{D}_\alpha, \overrightarrow{\bar{D}}_{\dot\alpha}\} = -2i\sigma^\mu_{\alpha\dot\alpha}\frac{\partial}{\partial y^\mu}\,.$$

★ $\{D_\alpha, Q_\beta\} = \{\bar{D}_{\dot\alpha}, Q_\beta\} = \{D_\alpha, \bar{Q}_{\dot\beta}\} = \{\bar{D}_{\dot\alpha}, \bar{Q}_{\dot\beta}\} = 0\,.$

Can keep the same definitions for $\Phi(y,\theta)$, and $\bar\Phi(\bar y,\bar\theta)$,

$$\bar{D}_{\dot\alpha}\Phi(y,\theta) = 0\,,$$

$$D_\alpha\bar\Phi(\bar y,\bar\theta) = 0\,.$$

IV: Chiral and Antichiral Superfields

$$\Phi(\hat{y},\hat{\theta}) = A(\hat{y}) + \sqrt{2}\hat{\theta}\psi(\hat{\theta}) + \hat{\theta}\hat{\theta}F(\hat{y})\,,$$

$$\bar{\Phi}(\hat{\bar{y}},\hat{\bar{\theta}}) = A(\hat{\bar{y}}) + \sqrt{2}\hat{\bar{\theta}}\bar{\psi}(\hat{\bar{y}}) + \hat{\bar{\theta}}\hat{\bar{\theta}}\bar{F}(\hat{\bar{y}})\,.$$

For the star product of two chiral fields we find,

$$\begin{aligned}\Phi_1(y,\theta) * \Phi_2(y,\theta) = \Phi_1(y,\theta)\Phi_2(y,\theta) - C^{\alpha\beta}\psi_{1\alpha}\psi_{2\beta} - \underline{\mathbf{det}CF_1F_2} \\ + \sqrt{2}\theta^\gamma C^{\alpha\beta}\big[\epsilon_{\beta\gamma}(\psi_{1\alpha}F_2 - \psi_{2\alpha}F_1) \\ + \bar{C}^{\dot{\alpha}\dot{\beta}}\sigma^\mu_{\alpha\dot{\alpha}}\sigma^\nu_{\gamma\dot{\beta}}(\partial_\mu A_1\partial_\nu\psi_{2\beta} - \partial_\mu A_2\partial_\nu\psi_{1\beta})\big] \\ + \theta\theta\big[2\bar{C}^{\mu\nu}\partial_\mu A_1\partial_\nu A_2 \\ + C^{\alpha\beta}\bar{C}^{\dot{\alpha}\dot{\beta}}\sigma^\mu_{\alpha\dot{\alpha}}\sigma^\nu_{\beta\dot{\beta}}(\partial_\mu A_1\partial_\nu F_2 - \partial_\mu A_2\partial_\nu F_1)\big]\,,\end{aligned}$$

V: Non-associativity and Weyl ordering

$$\Phi_1 * \Phi_2 \neq \Phi_2 * \Phi_1$$

$$\bar{\Phi}_1 * \bar{\Phi}_2 \neq \bar{\Phi}_2 * \bar{\Phi}_1$$

V: Non-associativity and Weyl ordering

$$\Phi_1 * \Phi_2 \neq \Phi_2 * \Phi_1$$

$$\bar{\Phi}_1 * \bar{\Phi}_2 \neq \bar{\Phi}_2 * \bar{\Phi}_1$$

★ **limit discussion to symmetrized via Weyl ordering products of superfields.** (**Seiberg: hep-th/0305248**)

V: Non-associativity and Weyl ordering

$$\Phi_1 * \Phi_2 \neq \Phi_2 * \Phi_1$$

$$\bar{\Phi}_1 * \bar{\Phi}_2 \neq \bar{\Phi}_2 * \bar{\Phi}_1$$

★ **limit discussion to symmetrized via Weyl ordering products of superfields.** **(Seiberg: hep-th/0305248)**

$$\Phi_1 * (\Phi_2 * \Phi_3) \neq (\Phi_1 * \Phi_2) * \Phi_3$$

V: Non-associativity and Weyl ordering

$$\Phi_1 * \Phi_2 \neq \Phi_2 * \Phi_1$$

$$\bar{\Phi}_1 * \bar{\Phi}_2 \neq \bar{\Phi}_2 * \bar{\Phi}_1$$

★ **limit discussion to symmetrized via Weyl ordering products of superfields.** (**Seiberg: hep-th/0305248**)

$$\Phi_1 * (\Phi_2 * \Phi_3) \neq (\Phi_1 * \Phi_2) * \Phi_3$$

★ **Double Weyl ordering**

VI: The Wess-Zumino Lagrangian

★ **We find the following result for the Wess-Zumino lagrangian**

$$\mathcal{L} = \mathbf{w}\Big[\int d^2\theta\theta d^2\bar\theta\bar\theta\,\bar\Phi * \Phi + \int d^2\theta\,\left(\frac{1}{2}m\Phi * \Phi + \frac{1}{3}g\Phi * \Phi * \Phi\right) + \int d^2\bar\theta\,\left(\frac{1}{2}\bar m\bar\Phi * \bar\Phi + \frac{1}{3}\bar g\bar\Phi * \bar\Phi * \bar\Phi\right)\Big]$$

$$= \mathcal{L}(C=0) - \underline{\frac{1}{3}\mathbf{det}CF^3 - \frac{1}{3}\mathbf{det}\bar C\bar F^3} + \textbf{total derivatives}\,.$$

VII: Summery

★ **We worked out a consistent algebra in Minkowski space,**

★ **Presented a star product that reproduces the algebra of deformed parameter space in its *entirety*, satisfies the reality condition, and preserves the chirality of a product of superfields,**

★ **Obtained Wess-Zumino lagrangian that is both Lorentz invariant, and Hermitian.**