B Meson Wave Function in k_T Factorization

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Contents

Introduction and Motivation -1

Available theories for exclusive B decays:

- PQCD – Keum, Li, Sanda;
- QCDF – Beneke, Buchalla, Neubert, Sachrajda;
- SCEF – Bauer, Fleming, Pirjol, Stewart;
- LCSR – Ball, Khodjamirian, Melic, Ruckl.

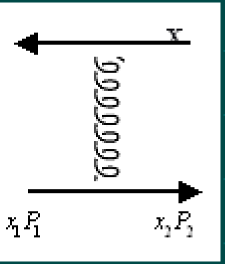
LCDA and WF in common / in differnce:

- governing the nonperturbative dynamics of exclusive decays
- universal and process-independent
- originated from different factorization theorems:
  LCDA -- collinear factorization theorem
  WF-- k_T factorization theorem
- giving different predictions

Introduction and Motivation -2

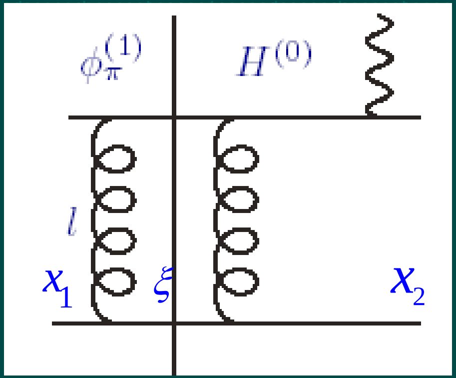
$\pi(P_1) + \gamma^*(q) \to \pi(P_2)$

In coll. Factorization



$$P_1 = (P_1^+, 0, 0_T), \quad P_2 = (0, P_2^-, 0_T), \quad Q^2 = 2P_1^+ P_2^-$$

At $O(\alpha_s)$, $H^{(0)}(\xi, x_2) \propto \dfrac{1}{(\xi P_1 - x_2 P_2)^2} = \dfrac{-1}{(\xi x_2 Q^2)}$, $\phi^{(0)}(x_1, \xi) = \delta(x_1 - \xi)$



At $O(\alpha_s^2)$,

$$l \,//\, P_1 \Rightarrow l^+ \sim P_1^+ \gg l_T \sim \Lambda \gg l^- \sim \frac{\Lambda^2}{P_1^+}, \quad P_1^2 \sim l^2 \sim O(\Lambda^2)$$

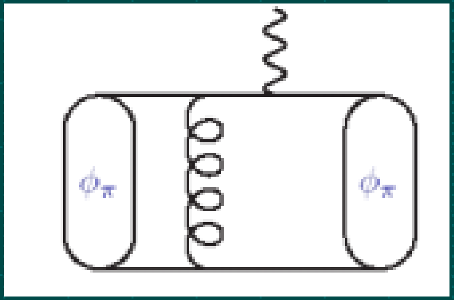
$$H^{(0)}(\xi_1, x_2) \propto \frac{1}{(x_1 P_1 - x_2 P_2 + l)^2}$$

$$\sim \frac{-1}{2\,(x_1 + l^+/P_1^+)\,x_2 P_1^+ P_2^-} \equiv \frac{-1}{\xi_1 x_2 Q^2}$$

$\phi_\pi^{(1)}(\xi_1)$ - contains the integration over l^- and l_T

- describes the probability of a parton carrying ξ

$$F_\pi \approx \int d\xi_1 d\xi_2 \;\phi_\pi(\xi_1) H(\xi_1, \xi_2) \phi_\pi(\xi_2)$$



Introduction and Motivation -3

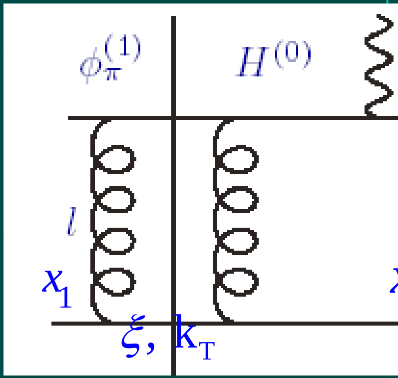
$\pi(P_1)+\gamma^*(q)\rightarrow\pi(P_2)$

In kT Factorization

$$H^{(0)}(\xi_1,x_2,l_T)\propto\frac{-1}{2(x_1+l^+/P_1^+)x_2P_1^+P_2^-+l_T^{\,2}}\equiv\frac{-1}{\xi_1x_2Q^2+l_T^{\,2}}$$

$\phi_\pi^{(1)}(\xi_1,k_{1T})$ - contains the integration over l^-

- describes the probability of a parton carrying ξ and k_T



$$F_\pi\approx\int d\xi_1d\xi_2\,d^2k_{1T}d^2k_{2T}\phi(\xi_1,k_{1T})H(\xi_1,\xi_2,k_{1T},k_{2T})\phi(\xi_2,k_{2T})$$

Introduction and Motivation -4

Motivation:

B. O. Lange and M.Neubert,

``Renormalization-Group Evolution of the B-Meson Light-Cone Distribution Amplitude''

Phys.\ Rev.\ Lett {\bf 91}, 102001 (2003) [hep-ph/0303082].

A non-normalizable light-cone distribution amplitude is concluded.

How is the normalizability of the corresponding nonperturbative object in kT factorization?

Non-normalizable LCDA -1

- B-meson light-cone distribution amplitudes (LCDAs)
  - first introduced in a study of the asymptotic behavior of heavy-meson form factors at large momentum transfer

- The factorization formula:

$$\int_0^\infty \frac{d\omega}{\omega} T(\omega,\mu)\,\phi_+^B(\omega,\mu)$$

The definition of the LCDA:

$$\langle 0\,|\,\bar{q}_s(z)\,S_n(z,0)\,\not{n}\,\Gamma\,h(0)\,|\bar{B}(v)\rangle = -\frac{iF(\mu)}{2}\,\tilde{\phi}_+^B(\tau,\mu)\,\mathrm{tr}\left(\not{n}\,\Gamma\,\frac{1+\not{v}}{2}\,\gamma_5\right)$$

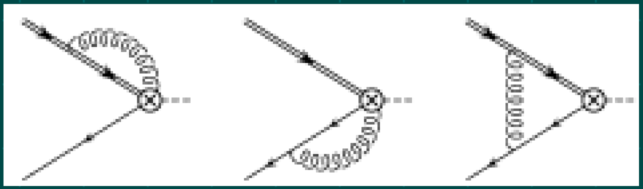
nd

$$\phi_+^B(\omega,\mu) = \frac{1}{2\pi}\int d\tau\, e^{i\omega\tau}\,\tilde{\phi}_+^B(\tau,\mu)$$

$$O_+^{\mathrm{ren}}(\omega,\mu) = \int d\omega'\, Z_+(\omega,\omega',\mu)\, O_+^{\mathrm{bare}}(\omega')$$

Non-normalizable LCDA -2

Considering the one-loop diagrams,



The counterterm corresponding to the non-normalizable LCDA is

$$-\frac{\alpha_s C_F}{2\pi}\frac{1}{\epsilon}\left[\frac{k^+}{k'^+}\frac{\theta(k'^+-k^+)}{(k'^+-k^+)_+}+\frac{\theta(k^+-k'^+)}{(k^+-k'^+)_+}\right]$$

After solving the evolution equation, the asymptotic behavior of B-LCDA was obtained:

$$\phi_+(k^+,\mu)\sim 1/k^+,\ \text{for}\ k^+\to\infty$$

$$\int_0^\infty \frac{d\omega}{\omega}T(\omega,\mu)\,\phi_+^B(\omega,\mu)$$

- Not an obstacle in practice!!

$$\langle 0\,|\,\bar{q}_s(z)\,S_n(z,0)\,\not{n}\,\Gamma\,h(0)\,|\bar{B}(v)\rangle$$

$$= -\frac{iF(\mu)}{2}\,\tilde{\phi}_+^B(\tau,\mu)\,\mathrm{tr}\left(\not{n}\,\Gamma\,\frac{1+\not{v}}{2}\,\gamma\right.$$

- **not theoretically desirable!!!**

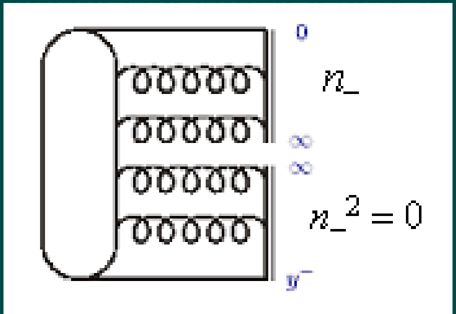
Normalizable WF –1

Definition of the Wave Function:

$$\Phi_B(x,b) = i\int \frac{dy^-}{2\pi} e^{-ixP_1^+ y^-} \langle 0 | \bar{q}(y) W_y(n_-)^+ I_{n_-;y,0} W_0(n_-) \not{n}_- \Gamma h(0) | \bar{B}(v) \rangle$$

where $W_y(n_-) = P\exp\{-ig\int_0^\infty dz n_- \bullet A(y+zn_-)\}$

and $I_{n_-;y,0} = P\exp\{-ig\int_\infty^{\infty+y} dz n_- \bullet A(zn_-)\}, y = (0, y^-, b)$



The additional collinear divergence $l // n_-$ cannot be cancelled owing to

$$\int dl^+ \{\Phi_B^{(1)}(l^+, l_T) H^{(0)}(x, k_T) - \Phi_B^{(1)}(l^+, l_T) H^{(0)}(x + l^+/P_1^+, k_T + l_T)\}$$

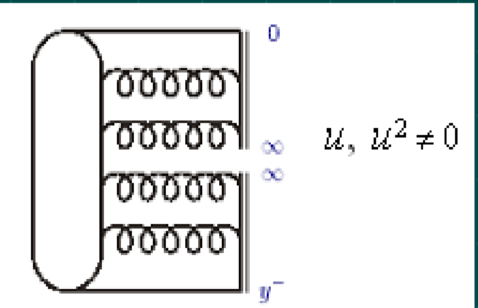
Normalizable WF –2

J. Collins ph/0304122

Two options to regularize the additional collinear divergence in a gauge invariant way:

1. Insert Wilson lines in non-light-like directions

$$\Phi_B(x,b) = i\int \frac{dy^-}{2\pi} e^{-ixP_1^+ y^-} \langle 0 | \bar{q}(y^-, b) W_y(u)^+ I_{u;y,0} W_0(u) \not{n}_- \Gamma h(0) | \bar{B}(v) \rangle$$



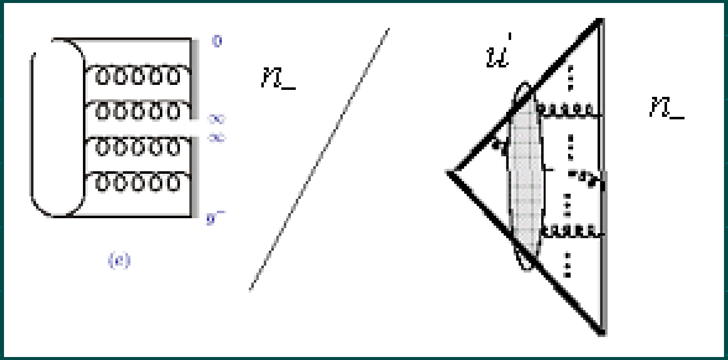
- The universality of the B WF is broken due to the appearanc of u in $\zeta = (k \bullet u)/\sqrt{u^2}$
- the UV structure of the quark-Wilson-line vertex correction changes:

$$N_c^{(1)} = \frac{\alpha_s C_F}{4\pi}\Gamma(\varepsilon)\left[2 - \left(\frac{4\zeta^2}{m_g^2}\right)^{-\varepsilon}\right]$$

Normalizable WF –3

2. Use the definition with light-like Wilson lines, but multiply it by a suitable gauge-invariant factor to cancel the additional divergence.

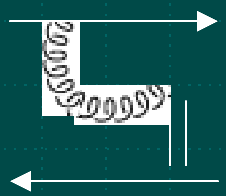
$$\Phi_B(x,b) = i\int \frac{dy^-}{2\pi} e^{-ixP_1^+y^-} \frac{\langle 0|\bar{q}(y^-,b)W_y(n_-)^+ I_{n_-;y,0} W_0(n_-)\, \not{n}_- \Gamma h(0)|B(P_1)\rangle}{\langle 0|W_y(n_-)^+ W_y(u')I_{n_-;y,0} I^+_{u';y,0} W_0(n_-)W_0(u')^+|0\rangle}$$



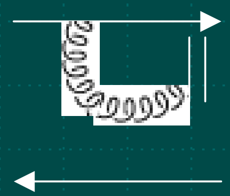
Normalizable WF –4

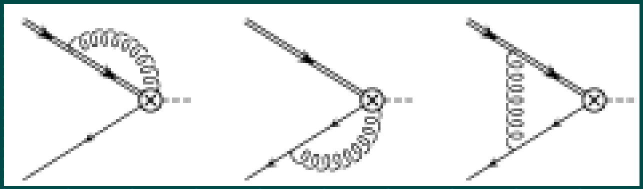
First of all, consider two of the relevant 1-loop diagrams as an illustration,

(a)



(b)





$$Z_{ab}^{(1)}(k^+, k'^+, b, \mu) = ig^2 C_F \mu^{2\epsilon} \int \frac{d^{4-2\epsilon} l}{(2\pi)^{4-2\epsilon}} \frac{n_- \cdot v}{v \cdot l l^2 n_- \cdot l} \times \left[\delta\left(k^+ - k'^+ + l^+\right) \exp(-i\mathbf{l}_T \cdot \mathbf{b}) - \delta(k^+ - k'^+)\right] .$$

The integral is UV finite:

$$-\frac{\alpha_s C_F}{\pi}\left[\frac{\theta(k'^+ - k^+)}{(k'^+ - k^+)_+} K_0\left((k'^+ - k^+)b\right) - \frac{\theta(k^+ - k'^+)}{(k^+ - k'^+)_+} K_0\left((k^+ - k'^+)b\right)\right]$$

➢At small b limit, K_0 remains finite!

Normalizable WF –5/

The evolution of B WF is given by

$$\Phi_+(k^+, b, \mu) = S(k^+, b, \nu)\phi_+(k^+, b, \mu)\,, \qquad \phi_+(k^+, b, \mu) = R(b, \mu, \nu)\phi_+(k^+, b, \mu = 1/b)\,.$$

where

$$S(k^+, b, \nu) = \exp\left\{-\int_{1/b}^{k^+} \frac{d\bar{\mu}}{\bar{\mu}}\left[\ln\frac{k^+}{\bar{\mu}}A(\alpha_s(\bar{\mu})) + B(\nu, \alpha_s(\bar{\mu}))\right]\right\},$$

$$R(b, \mu, \nu) = \exp\left[-\int_{1/b}^{\mu} \frac{d\bar{\mu}}{\bar{\mu}}\gamma(\alpha_s(\bar{\mu}))\right].$$

with the 1-loop anomalous dimensions

$$A = \frac{\alpha_s}{\pi}C_F\,,$$

$$B = \frac{\alpha_s}{2\pi}C_F \ln\left(\nu^2 e^{2\gamma_E - 1}\right),$$

$$\gamma = -\frac{\alpha_s}{4\pi}C_F\left(5 + 2\ln\nu^2\right).$$

The normalization of the B meson wave function is defined by

$$\int_0^\infty dk^+ \lim_{b\to 1/k^+} \Phi_+(k^+, b, \mu) = \int_0^\infty dk^+ \phi_+(k^+, b = 1/k^+, \mu)\,.$$

The normalizability is not spoiled by the RG evolution effect!!

Summary and Conclusion

❖ Review:

- Coll. factorization theorem and kt factorization theorem.
- The B-LCDA in collinear factorization ruins the normalizability.

❖ Our Achievements:

- NLO corrections different from those in coll. factorization;
- a more appropriate definition of B-WF confirmed;
- the treatment of different types of logarithms summarized;
- the normalizability of the B-WF confirmed all right, contrary to the observation derived in the collinear factorization theorem.