The Stability of Branonium

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Introduction

- Gravitational potential φ outside of a point source in $(3+1)$ dimensions is determined by a Laplace equation

$$\nabla^2\varphi = \left(\partial_r^2 + \frac{2}{r}\partial_r\right)\varphi = 0$$

- BPS brane solutions are determined by a similar Laplace equation on the conformally flat $(D-d)$-dimensional space transverse to the worldvolume

$$\nabla^2\varphi = \left(\partial_r^2 + \frac{(D-d-1)}{r}\partial_r\right)\varphi = 0$$

- If $D-d=3$ the solution scales as for a point charge in three spatial dimensions, regardless of the extended nature of the brane $\longrightarrow$ **Branonium.**

p-brane solutions and Branonium

$$S_{\rm Bulk} = \frac{1}{2\kappa^2}\int d^Dx\sqrt{-g}\left[R(g) - \frac{1}{2}\nabla_A\Phi\nabla^A\Phi - \frac{1}{2(d+1)!}e^{a\Phi}F^2_{[d+1]}\right]$$

Splitting the spacetime coordinates as $(x^A) = (x^\mu, y^m)$, where $\mu = 0, .., p$ and $m = d, .., D-1$, an "electric" p-brane solution to the above action is

$$\begin{aligned} ds^2 &= e^{2\nu_0}h^{-\tilde{\gamma}}\eta_{\mu\nu}dx^\mu dx^\nu + e^{2\beta_0}h^{\gamma}\delta_{mn}dy^m dy^n \\ e^\Phi &= e^{\phi_0}h^\sigma \\ A_{\mu_1\ldots\mu_d} &= \epsilon_{\mu_1\ldots\mu_d}\, e^{d\nu_0 - \frac{1}{2}a\phi_0}\zeta h^{-1} \end{aligned}$$

where ν_0, β_0, ϕ_0 are constants, $\tilde{d} = D - d - 2$ and

$$\gamma = \frac{4d}{\Delta(D-2)}, \quad \tilde{\gamma} = \frac{4\tilde{d}}{\Delta(D-2)}, \quad \sigma = \frac{2a}{\Delta}, \quad \zeta = \frac{2}{\sqrt{\Delta}}, \quad \Delta = a^2 + \frac{2d\tilde{d}}{D-2}$$

Antibrane Worldvolume Action

Potential experienced is controlled by the harmonic function h

$$h = 1 + \frac{k_0}{r^{\tilde{d}}}\, e^{-\frac{1}{2}a\phi_0 - \tilde{d}\beta_0}$$

The worldvolume action for a probe antibrane moving in the BPS background is

$$S_{\rm Probe} = -\mathcal{M} \int d^d x \left[h^{-\eta} \sqrt{1 + h^{\omega} \sum\nolimits_n (\partial Y^n)^2} - q\zeta h^{-1} \right]$$

where $\mathcal{M}$ is the ADM mass density of the antibrane and

$$\eta = \frac{2[a(\tilde{d} - d) + \tilde{d}d]}{\Delta(D-2)}, \qquad \omega = \frac{4}{\Delta}, \qquad q = \pm 1, \qquad \mathcal{M} = T e^{-\frac{1}{2}a\phi_0}$$

Moduli

- Promote constants ν_0, β_0, ϕ_0 to functions $\nu(x), \beta(x), \phi(x)$

- Reduce *total* action $S = S_{\rm Bulk} + S_{\rm Probe}$ on the p-brane ansatz

$$
\begin{aligned}
ds^2 &= e^{2\nu} h^{-\tilde{\gamma}} \tilde{g}_{\mu\nu}(x) dx^\mu dx^\nu + e^{2\beta} h^{\gamma} \delta_{mn} dy^m dy^n \\
e^{\Phi} &= e^{\phi} h^{\sigma} \\
A_{\mu_1 \ldots \mu_d} &= \epsilon_{\mu_1 \ldots \mu d}\, e^{d\nu - \frac{1}{2} a\phi} \zeta h^{-1} \ .
\end{aligned}
$$

- Arrive at an effective $(p+1)$-dimensional action describing **coupled** behaviour of the bulk and brane moduli

Effective Action

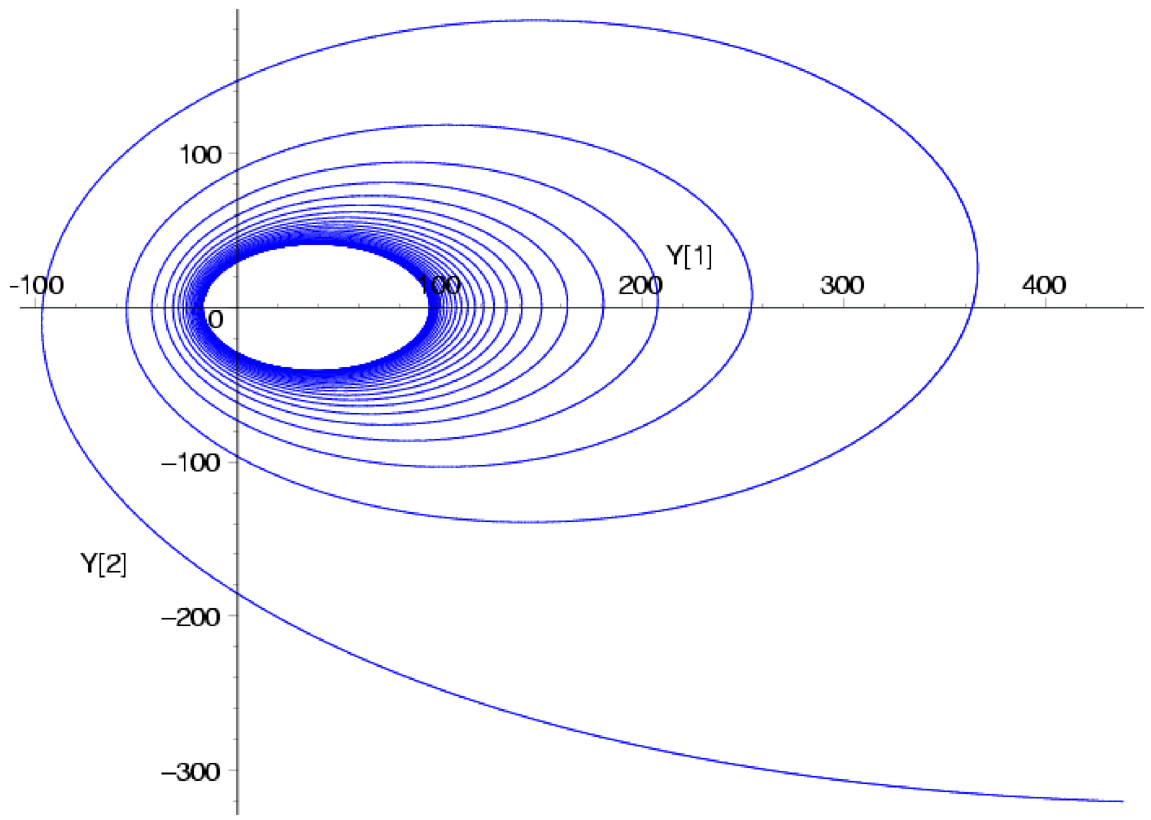
$$S = \frac{V}{2\kappa^2}\int d^dx\sqrt{-\tilde{g}}\left\{\tilde{R} - \frac{1}{2}(\partial\phi)^2 - \frac{(\tilde{d}+2)(\tilde{d}+d)}{(d-2)}(\partial\beta)^2 - \frac{2\kappa^2 T}{V}\left[\frac{1}{2}h^{\omega-\eta}e^{-\frac{1}{2}a\phi-\tilde{d}\beta}\sum\nolimits_n(\partial Y^n)^2 + e^{-\frac{1}{2}a\phi-\frac{d(\tilde{d}+2)}{(d-2)}\beta}\left(h^{-\eta} - q\zeta h^{-1}\right)\right]\right\}$$

- We should take $V \longrightarrow \infty$ since the transverse space is **noncompact**
- The antibrane terms then **decouple** and have no effect on the bulk

- Can we keep V **finite** so the brane and bulk are **coupled**?
- Yes, if we embed into the brane-orientifold background

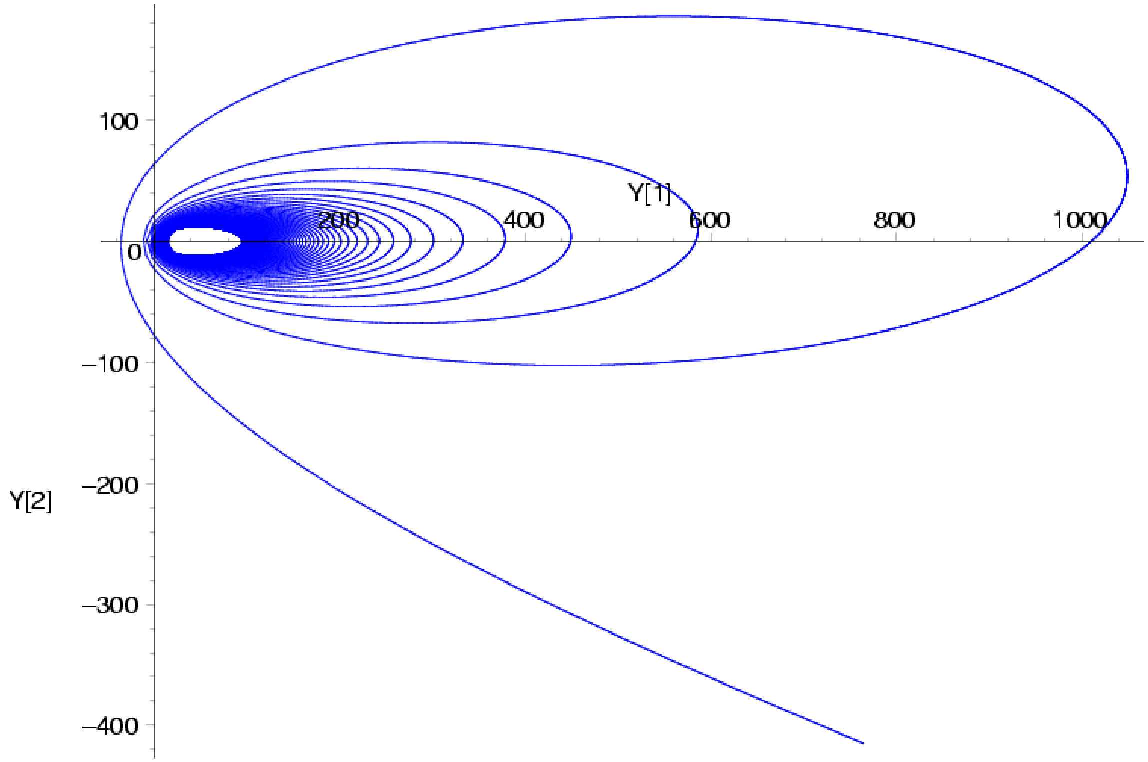
$$h = 1 + \epsilon\left(\frac{1}{r^{\tilde{d}}} - c_1\right) - \epsilon\left(\frac{1}{|\mathbf{r}-\mathbf{r}_0|^{\tilde{d}}} - c_2\right) + \text{images}$$

Evolution 1



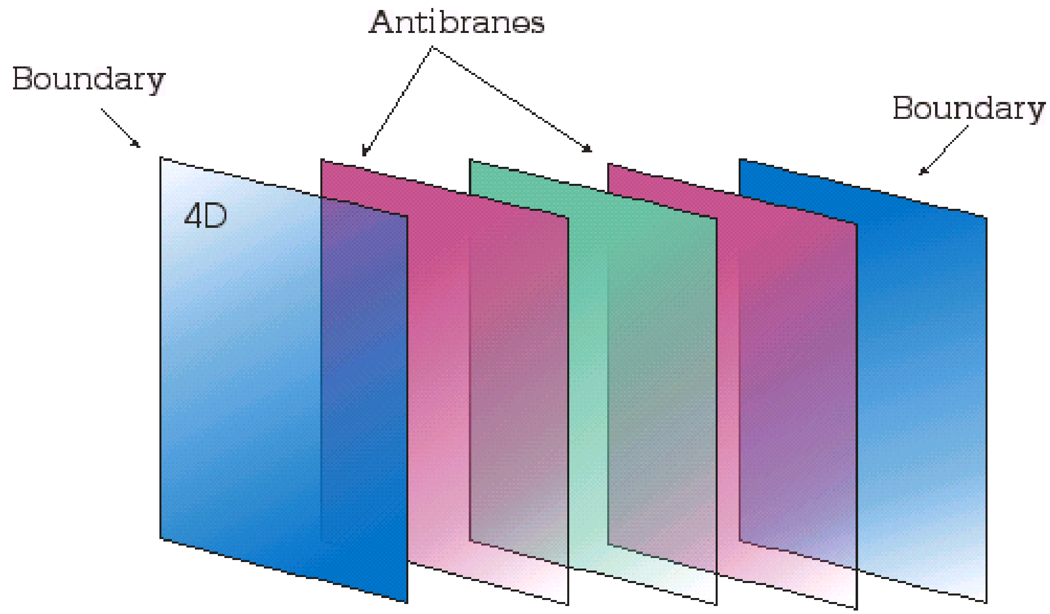
The (Y^1, Y^2) *phase-plane for* $V = 10^9$ *and* $\dot{\beta}_0 = \dot{\phi}_0 = 0$.

Evolution 2



Same initial conditions as before, but with k increased ten-fold.

Comparison to Heterotic M-Theory



Look for similar BPS brane solution along 1D transverse space z, assuming the ansatz

$$ds^2 = e^{2\nu(z)}\eta_{\mu\nu}dx^\mu dx^\nu + e^{2\beta(z)}(\pi\rho dz)^2 \quad , \quad \phi \equiv \phi(z)$$

Multi domain wall solution

One finds the BPS array solution

$$
\begin{aligned}
e^{\nu} &= e^{-\frac{\beta_4}{2}} H_k^{1/2} \quad , \qquad H_k = 1 - \frac{2}{3} e^{\beta_4 - \phi_4} h_k \\
e^{\beta} &= e^{\beta_4} H_k^2 \qquad , \qquad h_k = \sum_{r=0}^{k-1} q_r (|z| - z_r) - \frac{1}{2} \sum_{r=0}^{N_B+1} q_r (1 - z_r)^2 \\
e^{\phi} &= e^{\phi_4} H_k^3
\end{aligned}
$$

This leads to an associated four-dimensional effective theory

$$
\begin{aligned}
S = -\frac{1}{2\kappa_4^2} \int_{M_4} d^4x \sqrt{-\bar{g}} \Bigg\{ & \frac{1}{2}\overline{R} + \frac{1}{4}(\partial\phi_4)^2 + \frac{3}{4}(\partial\beta_4)^2 + \sum_{k=1}^{N_B} \frac{1}{2} q_k e^{\beta_4 - \phi_4} (\partial z_k)^2 \\
& + \frac{1}{2} |\widetilde{q}| e^{\beta_4 - \phi_4} (\partial \widetilde{z})^2 + \widetilde{U} \Bigg\}
\end{aligned}
$$

Heterotic Inflation

In the interval $z_{n-1} \leq |\widetilde{z}_k| \leq z_n$ the antibrane potential takes the form

$$\widetilde{U} = \frac{(|\widetilde{q}_k| - \widetilde{q}_k)e^{-2\beta_4 - \phi_4}}{1 - \frac{2}{3}e^{\beta_4 - \phi_4}[\sum_{r=0}^{n-1} q_r(|\widetilde{z}_k| - z_r) - \frac{1}{2}\sum_{r=0}^{N_B+1}(1 - z_r)^2]}$$

For static moduli this leads to a typical number of e-folds

$$N_{\mathrm{e}} \sim \frac{\tilde{q}}{q_r} \ll 1$$

- E-folds **negligible** due to light nature of antibrane
- This actually reflects the compact nature of the 1D transverse space
- **Ubiquitous problem** of D-brane inflationary models
- Cyclical motion would avoid this?

Conclusions

- Any time evolution of the bulk moduli will spoil the anti D6 elliptical orbits
- This evolution is only avoided for static initial conditions **and** a noncompact transverse space
- The evolution of bulk moduli is **unavoidable** if the transverse space is finite in extent
- Nonetheless, cyclic realisation of inflation is interesting
- Possible M-Theory analogs of this orbital construction?