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Quark Confinement Theory Based On Solution of Dirac Equation and Color Gauge Field Equations

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Outline

- Six Questions to be Answered
- Framework and Solution Approach
- Theoretical Results
- Calculations of Mass Spectra
- Summary

Six Questions A Quark Contiuum Theory Must Answer

- How is the Chiral Symmetry Spontaneously Broken?
- What Makes Quarks and Gluons Stick together?
- How do Massless Quarks and Gluons acquire Mass?
- Why do Quarks appear in Six Flavors?
- Why Can't Quarks be Ejected from Baryons ?
- Can Hadron Mass Spectra be Accurately Calculated?

Baryon Equation Framework

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - iq_i \left[A_\mu, A_\nu \right]$$

$$\partial^\mu F_{\mu\nu} - iq_i \left[A_\mu, F_{\mu\nu} \right] = j_\nu$$

$$\sum_{i,g=1}^{3,8} \left[c\sigma_i \bullet \left(\boldsymbol{p}_{Bi} + q_i \boldsymbol{A}_g \right) \right] \varphi\left(\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_3 \right) = \sum_{i=1}^{3} \left[\mathrm{E}_i - q_i V_g \right] \chi\left(\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_3 \right)$$

$$\sum_{i,g=1}^{3,8} \left[c\sigma_i \bullet \left(\boldsymbol{p}_{Bi} + q_i \boldsymbol{A}_g \right) \right] \chi\left(\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_3 \right) = \sum_{i=1}^{3} \left[\mathrm{E}_i - q_i V_g \right] \varphi\left(\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_3 \right)$$

Approach for Solving Equations

- **Express Color Gauge Fields as Multipole Expansion**
  - Dipole Color Magnetic Field Dominates in Confinement Region
- **Define Dynamical Relationship for Chiral Symmetry Breaking**
  - Long Range Fields Canceled
  - Quarks Trapped in Color Magnetic Field
- **Calculate Phase of Quark Spinor Wave Function**
  - Leads to Color Magnetic Flux Quantization
  - Gluons Trapped by Encirculating Quarks
- **Use Hartree Approximation for Quark Spinor Wave Functions**
  - One Multi-Particle Problem becomes Three Single-Particle Problems
  - Quark Interactions expressed as Self-Consistent Color Fields
- **Use Iterative Born Approximation Method to Solve Dirac Equation**
  - Spherical Harmonics Functions Describe Angular Behavior
  - Closed-Form Solutions Obtain for Radial Differential Equation

Chiral Symmetry Breaking (CSB)

- **Dynamical Relationship that Synchronizes Quark and Gluon Energy Flow**

$$\mathbf{v} = \frac{(\boldsymbol{E} \times \boldsymbol{B})}{(\boldsymbol{B} \bullet \boldsymbol{B})}$$

- **Dynamical Relationship Yields to Net Color Magnetic Field in Lorentz Force Equation**

$$\boldsymbol{F} = q(\mathbf{v} \times \boldsymbol{B} + \boldsymbol{E}) = q\left(\frac{(\boldsymbol{E}\bullet\boldsymbol{B})\boldsymbol{B} - (\boldsymbol{B}\bullet\boldsymbol{B})\boldsymbol{E}}{\boldsymbol{B}\bullet\boldsymbol{B}}\right) + q\boldsymbol{E} = q\left(\frac{\boldsymbol{E}\bullet\boldsymbol{B}}{\boldsymbol{B}\bullet\boldsymbol{B}}\right)\boldsymbol{B}$$

What Makes Quarks Stick to Gluons?

- **Color Magnetic Dipole Field Confines Quark**
  - Gluons Trap Quarks
- **Phase-Locking of Quarks Leads to Color Flux Quantization**
  - Quarks Trap Gluons
  - Winding Number, n_i , serves as new Quantum Number

$$\delta_{\mathrm{i}} = \frac{q_{\mathrm{i}}}{\hbar} \oint A_\mu dx_\mu = n_i (2\pi)$$

$$q_i \Phi_{\mathrm{i}} = q_i \oint \mathbf{A}_{\boldsymbol{i}} \cdot \mathrm{d}\mathbf{x}_{\boldsymbol{i}} = q_i \oiint \mathbf{B}_{\boldsymbol{i}} \bullet \mathrm{d}\mathbf{S}_{\boldsymbol{i}} = n_i (2\pi\hbar)$$

How Do Quarks Gain Rest Mass?

- **Flux Quantization Equation Describes a Phase Vortex**
  - Results from Topological Defect
  - Vortex strength characterized by Winding Number
- **Core Energy of Vortex**
  - Quark & Gluon Free Energies converted to Condensation Energy
  - Core Energy expressed as Quark-Gluon Composite Rest Mass

How Does Rest Mass Energy Show up in Dirac Hamiltonian?

- **Color Vector Potential Serves Two Roles with respect to CSB**
  - Color Field Interacts with Quark Spinor Amplitude
  - Color Field Adjusts Phase to Maintain Local Gauge Invariance

$$q_i \mathbf{A}_{\boldsymbol{i}} = q_i \boldsymbol{A}_{\boldsymbol{Bi}} + \nabla \psi_i$$

  - Where

$$\nabla \psi_i = \mathrm{M}_{0i} c$$

- **Gauge Choice for** ψ_i **Introduces Rest Mass Term**

How Do Quarks Gain Rest Mass?

- **Dirac Hamiltonian *Before* Chiral Symmetry Breaking**

$$H^2 = c^2(\boldsymbol{p}_{Bi})^2 + c^2(q_i \boldsymbol{A}_{Bi})^2 \;\; + 2q_i c^2(\boldsymbol{L}_i + \boldsymbol{s}_i) \bullet \boldsymbol{B}_{Bi}$$

- **Dirac Hamiltonian *After* Chiral Symmetry Breaking**

$$\mathrm{H}^2 = c^2(\mathbf{p}_i)^2 + c^2(q_i \mathbf{A}_i)^2 \;\; + \mathrm{M}_{0i}^2 c^4 + 2q_i c^2(\mathbf{L}_i + \mathbf{s}_i) \bullet \mathbf{B}_i$$

Why Do Quarks Appear in Flavors?

- **Quark-Gluon Composite Behaves as single particle**
  - $$\mathbf{P}_i = \gamma_i \mathrm{M}_{0i} \beta_i c = \mathrm{M}_i \beta_i c$$
  - $$\mathbf{P}_i^2 = \mathbf{p}_i^2 + (q_i \mathbf{A}_i)^2$$
  - $$\mathrm{F}_i = \frac{\mathrm{M}_\mathrm{i}^2 \beta_\mathrm{i}^3 c^3}{n_\mathrm{i} \hbar} = \frac{\mathrm{M}_\mathrm{j}^2 \beta_\mathrm{j}^3 c^3}{n_\mathrm{j} \hbar} = \frac{\mathrm{M}_\mathrm{k}^2 \beta_\mathrm{k}^3 c^3}{n_\mathrm{k} \hbar} = 690.75\mathrm{Mev/fm}$$
- **Quantized Flux Dynamics produces Constant Stabilizing Force**
  - Phase Vortex provides Transition between Degenerate Ground States
- **Flavor Mass States Calculated with Force Eq. and Winding No.**

Why Aren't Quarks Ejected from Baryons?

- **Color Superconductivity Leads to Strong Rigidity in Quark Spinor Wave Functions**
  - Analogous to ordinary Superconductivity state
  - High Energy Collision produces Impulsive Reactive Force
- **Least path of Resistance for Quark is to Move from One Flavor state to another at Constant Force**
- **Quarks Imprisoned Unless Injected Energy exceeds threshold for Phase-Locked Condition to be Sustained**

Calculating Baryon Spectra
Chart 1 of 3

- **Dirac Equation for Radial Spinor Wave Function**

$$\left[\frac{d^2}{dr^2}+\frac{2}{r}\frac{d}{dr}+\left(k_i^{\,2}-\frac{l(l+1)}{r^2}\right)\right]f_{El}(r)=0$$

$$\hbar k_i=\mathrm{P}_i=\sqrt{\mathrm{E}_{iq}^2/\mathrm{c}^2-\mathrm{M}_{0i}^2\mathrm{c}^2-\mathrm{M}_\mathrm{i}\mathrm{c}^2(\mu_i\bullet\mathbf{B}_i)}$$

$$\mu_\mathrm{i}\bullet\mathbf{B}_\mathrm{i}=\tfrac{2}{3}\alpha_i\left(\frac{\hbar}{\mathrm{c}}\frac{\mathbf{j}_\mathrm{i}\bullet\mathbf{j}_\mathrm{j}}{\mathrm{M}_\mathrm{i}\mathrm{M}_\mathrm{j}}\frac{1}{r_\mathrm{i}^{\,3}}+\frac{\hbar}{\mathrm{c}}\frac{\mathbf{j}_\mathrm{i}\bullet\mathbf{j}_\mathrm{k}}{\mathrm{M}_\mathrm{i}\mathrm{M}_\mathrm{k}}\frac{1}{r_\mathrm{i}^{\,3}}\right)$$

Calculating Baryon Spectra
Chart 2 of 3

- **Zeroth Order Born Approximation Solution**

$$\varphi_i^{(0)} = j_l(k_i^{(0)} r) \mathrm{Y}_{l,m}(\theta, \phi) \Sigma_{s,m_s}$$

$$k_i^{(0)} = \frac{1}{\hbar c}\sqrt{E_i^2 - M_{0i}^2 c^4}$$

Calculating Baryon Spectra
Chart 3 of 3

- **Obtain 1st Order Born Approximation Solution by including the *j * j* coupling term and approximation for *r***

$$r_i = \frac{n_i \hbar}{\mathrm{M}_i \beta_i c}$$

$$\left(\mu_i \bullet \mathbf{B}_i\right)^{(0)} = \left(\frac{\beta_i^3}{n_i^3}\right)\left(\frac{n_i}{n_j/\beta_j + n_k/\beta_k}\right)\left(\frac{\mathrm{M}_i}{\mathrm{M}_j}\frac{\mathbf{j}_i \bullet \mathbf{j}_j}{\hbar^2} + \frac{\mathrm{M}_i}{\mathrm{M}_k}\frac{\mathbf{j}_i \bullet \mathbf{j}_k}{\hbar^2}\right)$$

$$\varphi_i^{(1)} = j_l(k_i^{(1)} r)\mathrm{Y}_{l,m}(\theta,\phi)\Sigma_{s,m_s}$$

$$k_i^{(1)} = \frac{1}{\hbar c}\sqrt{E_i^2 - M_{0i}^2 c^4 - \left(\mu_i \bullet \mathbf{B}_i\right)^{(0)}}$$

Quark Energy Eigenvalues (1st Born Approximation)

- **Total Winding Number is:** $n_i = f_i + l_i$

$$k_i^{(1)} = \frac{1}{\hbar c}\sqrt{E_i^2 - M_{0i}^2 c^4 - (\mu_i \bullet \mathbf{B}_i)^{(0)}}$$

$$\frac{\mathbf{j}_i \bullet \mathbf{j}_j}{\hbar^2} = \frac{1}{2}\left[j_{pair}(j_{pair}+1) - j_i(j_i+1) - j_j(j_j+1) \right]$$

$$\frac{\mathbf{j}_k \bullet \mathbf{j}_i}{\hbar^2} = \frac{\mathbf{j}_k \bullet \mathbf{j}_j}{\hbar^2} = \frac{1}{4}\left[j_{total}(j_{total}+1) - j_{pair}(j_{pair}+1) - j_k(j_k+1) \right]$$

$$\mathbf{j}_{pair} = \mathbf{j}_i + \mathbf{j}_j$$

$$\mathbf{j}_{total} = \mathbf{j}_{pair} + \mathbf{j}_k$$

Quark Flavor Rest Masses For Baryons

Flavor	Rest Mass (Mev)	Winding Number
up	4	1
down	7	1
strange	100	2
charm	1510	2
bottom	4882	3
top	175,000	3

Baryon Mass Spectrum (MEV)
Inter-quark Force = 690.75
(Page 1 of 2)

		State	F1	F2	F3	L1	L2	L3	J total	J pair	Jk	Calc. Mass	% Error
1	N (938)	P11	1	1	1	0	0	0	0.5	1.0	0.5	938	0.0%
2	Delta (1232)	P33	1	1	1	0	0	0	1.5	1	0.5	1221	-0.9%
3	Lambda (1115)	P01	1	2	1	0	0	0	0.5	1,0	0.5	1115	0.0%
4	Sigma (1192)	P11	1	1	2	0	0	0	0.5	1	0.5	1183	-0.8%
5	Sigma (1385)	P33	1	1	2	0	0	0	1.5	1	0.5	1376	-0.7%
6	Xi (1311)	P11	1	2	2	0	0	0	0.5	1	0.5	1328	0.8%
7	Xi (1534)	P33	1	2	2	0	0	0	1.5	1	0.5	1531	-0.1%
8	Omega (1672)	P33	2	2	2	0	0	0	1.5	1	0.5	1679	0.4%
9	Lambda-c (2285)	P01	1	2	1	0	0	0	0.5	1,0	0.5	2296	0.5%
10	Sigma-c (2420)	P11	1	1	2	0	0	0	0.5	1	0.5	2416	-0.2%
11	Xi-c (2565)	P13	1	2	2	0	0	0	0.5	1	0.5	2560	-0.2%
12	Omega-c (2704)	P01	2	2	2	0	0	0	0.5	1	0.5	2704	0.0%
13	Lambda-b (5624)	P01	1	3	1	0	0	0	0.5	1,0	0.5	5636	0.2%

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Baryon Mass Spectrum (MEV)

Inter-quark Force = 690.75

		State	F1	F2	F3	L1	L2	L3	J total	J pair	Jk	Calc. Mass	% Error
14	N(1440)	P11	1	1	1	1	1	1	0.5	1	0.5	1442	0.2%
15	N(1520)	D13	1	1	1	1	1	0	1.5	1	0.5	1503	-1.1%
16	N(1535)	S11	1	1	1	1	2	1	0.5	1	0.5	1554	1.2%
17	N(1680)	F15	1	1	1	3	1	1	2.5	1	1.5	1675	-0.3%
18	N(1720)	P13	1	1	1	3	3	1	1.5	0	1.5	1722	0.1%
19	N(2250)	G19	1	1	1	4	1	1	4.5	5	0.5	2264	0.6%
20	Delta(1700)	D33	1	1	1	2	1	1	1.5	2	0.5	1680	-1.2%
21	Delta(1905)	F35	1	1	1	3	1	1	2.5	3	0.5	1902	-0.2%
22	Lambda(1405)	S01	1	2	1	0	1	1	0.5	1	0.5	1416	0.8%
23	Lambda(1670)	S01	1	2	1	2	2	0	0.5	0	0.5	1689	1.1%
24	Lambda(1690)	D03	1	2	1	1	1	0	1.5	0	0.5	1710	1.2%
25	Lambda(1820)	F05	1	2	1	4	0	1	2.5	1	1.5	1805	-0.8%
26	Sigma (1670)	D13	1	1	2	1	1	0	1.5	1	0.5	1652	-1.1%
27	Xi (1690)	P11	1	2	2	1	1	1	0.5	0	0.5	1673	-1.0%
28	Xi (1820)	D13	1	2	2	1	0	1	1.5	2	0.5	1835	0.8%
29	Omega (2250)	D31	2	2	2	2	2	2	1.5	2	0.5	2241	-0.4%
											RMS Error		0.7%

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Meson Mass Spectrum (MEV)

Inter-quark Force = 610 Mev/fm

	F1	L1	F2	L2	J Tot.	J 1	J 2	Calc. Mass	% Error
1 K* (892)	1	0	2	0	1	0.5	0.5	887	-0.5%
2 rho (770)	1	0	1	0	1	0.5	0.5	775	0.7%
3 rho (1691)	1	4	1	6	3	2.5	0.5	1704	0.7%
4 Phi (1020)	2	0	2	0	1	0.5	0.5	1024	0.4%
5 Phi (1680)	2	4	2	4	1	3.5	2.5	1671	-0.5%
6 Phi (1850)	2	7	2	4	3	6.5	3.5	1842	-0.4%

Meson Mass Spectrum (MEV)
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Charmonian (MEV)
Inter-quark Force for Charmonium is 4131 Mev/fm

State	J	Quark f_1	Quark l_1	Quark n_1	Antiquark f_2	Antiquark l_2	Antiquark n_2	Calc. Mass	Meas. Mass	Variance
Charmonium										
η_c(1S)	0	2	0	2	2	0	2	2991	2980	0.4%
J/ψ (1S)	1	2	0	2	2	0	2	3110	3097	0.4%
χ_{c0} (1P)	0	2	1	3	2	1	3	3401	3417	-0.5%
χ_{c1} (1P)	1	2	1	3	2	1	3	3521	3510	0.3%
χ_{c2} (1P)	2	2	1	3	2	1	3	3556	3556	0.0%
ψ (2S)	0	2	1	3	2	2	4	3653	3686	-0.9%
ψ (3770)	1	2	1	3	2	2	4	3756	3770	-0.4%
ψ (4040)	1	2	2	4	2	3	5	4057	4040	0.4%
ψ (4160)	1	2	2	4	2	4	6	4184	4159	0.6%
ψ (4415)	1	2	3	5	2	4	6	4410	4415	-0.1%

Meson Mass Spectrum (MEV)

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Bottomonium (MEV)
Inter-quark Force for Bottomonium is 8266 Mev/fm

State	J	Quark f_1	Quark l_1	Quark n_1	Antiquark f_2	Antiquark l_2	Antiquark n_2	Calc. Mass	Meas. Mass	Variance
Bottomonium										
Υ (1S)	1	3	0	3	3	0	3	9460	9460	0.0%
χ_{b0}(1P)	0	3	1	4	3	1	4	9817	9860	-0.4%
χ_{b1}(1P)	1	3	1	4	3	1	4	9850	9892	-0.4%
χ_{b2}(1P)	2	3	1	4	3	1	4	9877	9913	-0.4%
Υ (2S)	0	3	1	4	3	2	5	10044	10023	0.2%
χ_{b0}(2P)	0	3	2	5	3	2	5	10215	10232	-0.2%
χ_{b1}(2P)	1	3	2	5	3	2	5	10237	10255	-0.2%
χ_{b2}(2P)	2	3	2	5	3	2	5	10254	10286	-0.3%
Υ (3S)	1	3	2	5	3	3	6	10346	10355	-0.1%
Υ (4S)	1	3	3	6	3	4	7	10538	10580	-0.4%
Υ (10860)	1	3	4	7	3	5	8	10859	10865	-0.1%
Υ (11020)	1	3	4	7	3	5	8	11946	11019	-0.7%

Summary

- ***Up Quark* is the fundamental particle & flavor**
  - Gains Mass Through Chiral Symmetry Breaking
- ***Down Quark* is Excited State of *Up Quark***
  - Electron and Anti-Neutrino Capture thru Weak Force Interaction
  - *Up & Down Quarks* form 1st Generation with Winding Number = 1
- ***Charm* and *Strange Quarks* are Excited States of 1st Generation**
  - They form 2nd Generation with Winding Number = 2
- ***Top* and *Bottom Quarks* are Excited States of 2nd Generation**
  - They form 3rd Generation with Winding Number = 3
- **Calculated Baryon and Meson Mass Spectra Agree with Experimental Value to within a Fraction of 1%.**

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Abstract

A dynamical relationship (DR) between quarks and color gauge fields is used to solve the Dirac equation. The DR results in: (1) cancellation of the (1/r) color scalar potential field thus breaking the chiral symmetry; (2) quarks that interact only through the color vector potential fields; (3) a coherence condition between quark spinor phase angles leading to quantized color magnetic flux in integer units of Planck's constant; (4) a winding number for quantized flux that serves as a quark flavor quantum number; and (5) a means to calculate the strong force 'running coupling parameter'.

These conditions are analogous to key features of electronic superconductivity. When these five features are coupled with the quark Dirac equations, the relativistic Hamiltonian for quarks and gluons indicates that quarks gain mass from the trapped gluons in the quantized color magnetic flux. Dirac equation solutions for spinors are expressed in spherical Bessel functions and spherical harmonics. The well-defined quantum states (and quantum numbers) correspond to quark energy, momentum, spin angular momentum, orbital angular momentum, total angular momentum, color charge and flavor. The rest mass for each of the quark flavor is 'put in by hand'.

Calculated mass spectra results are presented for 29 baryons (*u, d, s, c, b* combinations) and 28 meson states (u, d, s, c, b combinations). The results compare with experimental values to within a fraction of a percent.