Non-abelian dynamics on giant gravitons from their AdS/CFT dual realization.

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Work in progress with V. Balasubramanian, B. Feng, M Huang.

Motivation

The AdS/CFT gives a dual description of string theory in negatively curved space time.

The most celebrated and studied case is the $\mathcal{N} = 4$ SYM.

The duality is a strong-weak coupling duality for the 't Hooft coupling. It is hard to make computations.

- We know a lot about closed strings in the CFT dual of AdS.
  - They are described by traces.
  - They seem to give rise to an integrable structure.
  - Well known limits: BMN, semiclassical strings, supergravity (BPS multiplets).

What do we know about D-branes in AdS/CFT?

- Some BPS states.
- Low energy fluctuations for single D-brane states.
- Add infinite D-branes (defect CFT, flavor)

We don't know very much about the dynamics of branes on AdS.

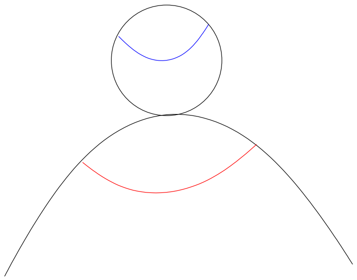
Even qualitative aspects are missing!

- Operators that describe branes at angles?
- Gauge symmetry on compact D-branes?
- DBI effective action?

Plan for the rest of talk

- How to describe BPS compact D-branes (giant gravitons).
- Matrix model description and Young tableaux.
- Fluctuations on a single giant.
- Multiple giants.
- Gauss law and gauge invariance.

How to describe BPS compact D-branes (giant gravitons).



Two giant gravitons: one along AdS and one along S.

both are half BPS: same quantum numbers as a graviton of large angular momentum.

Dual description of giants

They have R-charge: $J = \Delta$.

We can check in the free field theory limit which operators have this property. There is one complex scalar field Z with this property. All others have $\Delta > J$.

The operator must be made out only of Z.

Via operator state correspondence for the CFT, this has to be built out of the S-wave of the scalar field Z on S^3 only.

One can reduce the problem to a one matrix model for this mode on S^3.

The one matrix model is just the large N gauged harmonic oscillator.

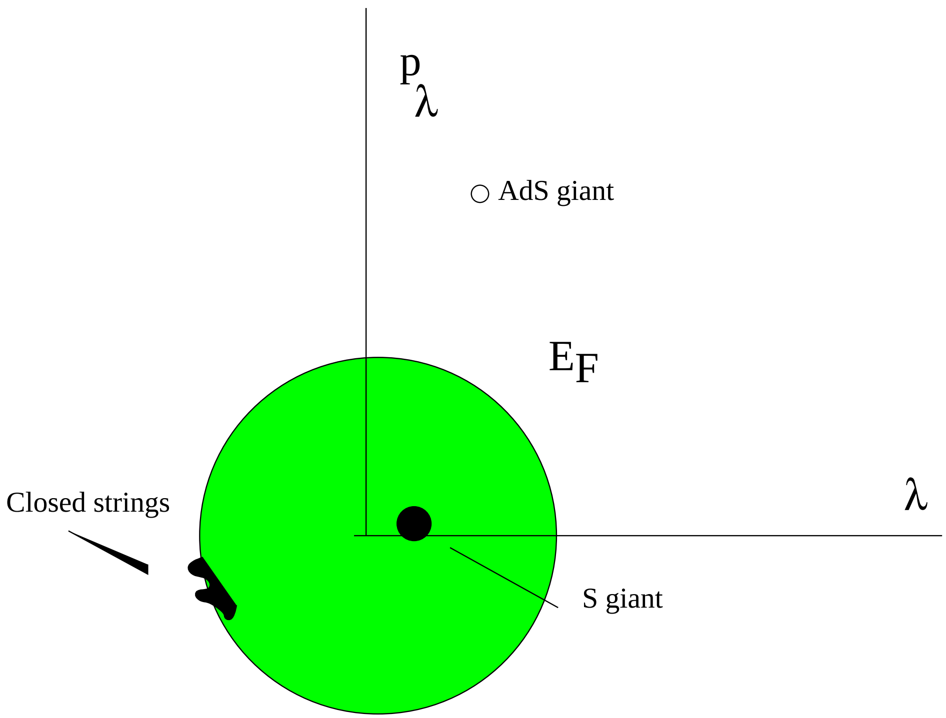
One can describe the excitations in terms of free fermions in the Harmonic oscillator potential, these are the eigenvalues of Z. One can connect this to the quantum hall effect (on the lowest landau level for free fermions) and the c=1 matrix model.

The two types of giant gravitons are holes and particles in the quantum hall system. Traces describe the edge collective excitations.

The wave functions are easy to describe in terms of Slater determinants of single particle wave functions in the harmonic oscillator.

The giants along AdS are described by a particle. This is, an eigenvalue of Z. These have no cap on their energy.

The giants along S are described by a hole. These have a maximum energy. This is because the Fermi sea has finite depth.

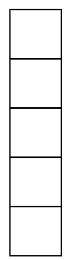


These states can also be described by characters associated to different young tableaux.

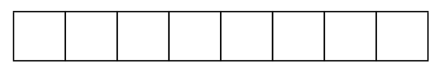
Idea: to describe gauge invariant states in matrix model consider decomposing the upper indices of a product of Z into irreducibles of $SU(N)$. These are characterized by young tableaux.

Lower indices transform as the same young tableaux: one can consider this as an operator from a representation of $SU(N)$ to itself. Take the trace and we get a gauge invariant state.

These are the young tableaux corresponding to the two different giant graviron states:



and



The number of boxes is of order N.

Multiple giants correspond to various long rows or columns.

The first one is a subdeterminant operator:

$$\epsilon\epsilon(Z, Z, \ldots Z, 1, 1, 1, \ldots)$$

Adding an open string

$$\epsilon\epsilon(Z, Z, \ldots Z, 1, 1, 1, \ldots, 1, \text{word})$$

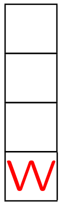
The word should not end or begin in Z, otherwise we are over counting states. This is a combinatorial boundary condition.

Many strings can be obtained with many words replacing the 1's.

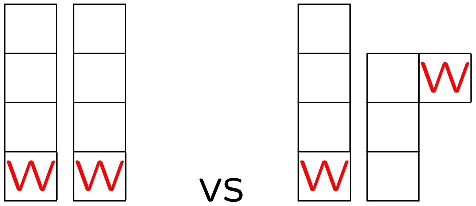
One can prove that these states are in addition to adding closed strings by multiplying by traces, and that they form a Fock space of open strings in the large N limit.

If a word is Y^n for example, then it has an anomalous dimension to one loop which is of order $1/N$, so in the large N limit it's not renormalized. These are the spherical harmonics of the DBI action.

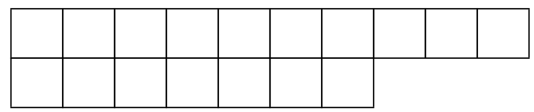
This is the Young tableaux picture:



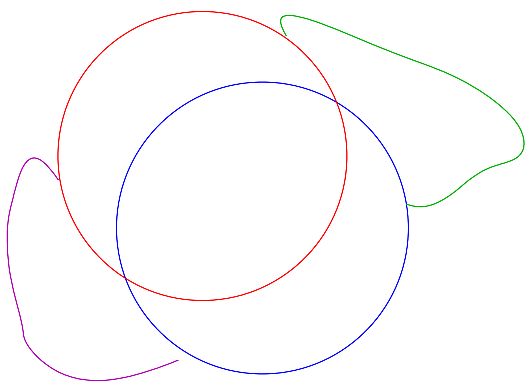
We should keep track of both upper and lower indices: statistics of Z makes upper and lower indices transform the same way, but not for extra words.



Multiple giants:



How do we add strings strectching between giants? And how do we think of Gauss law?



Caricature: two giants with two strings of opposite orientations strecthing between them

Two giants of different sizes. Each corresponds to an eigenvalue/hole having high energy.

For the case of eigenvalues we can treat them semiclassically.

$$Z \sim \begin{pmatrix} z_1 & 0 & \dots & \\ 0 & z_2 & \dots & \\ \vdots & \vdots & 0 & \end{pmatrix}$$

This is just like Higgs mechanism: $U(N)$ is broken to $U(1)^2 \times U(N-2)$.

The phase of z_1 and z_2 are the position of the giant along S^5. If phase and norm coincide then the two rows have equal length, and the symmetry is broken to $U(2) \times U(N-2)$.

A similar semiclassical description for holes is not known.

Gauss law and gauge invariance.

How do we string various strings between giants?

For giants along AdS we can think in terms of the spontaneous symmetry breaking. There are some bifundamentals which under $U(N-2)\times U(1)$, mark them with a different color. Remember that they also carry an upper or lower index with respect to the corresponding $U(1)$ charges (positive or negative charge: orientation)

Open strings stretching between giants will look like states:

$$Y \text{ word } Y$$

acting on a semiclassical background for the eigenvalues.

This has charge under the $U(1)\times U(1)$, so it is not gauge invariant.

We can make it gauge invariant by adding another state

$$Y \text{ word } Y$$

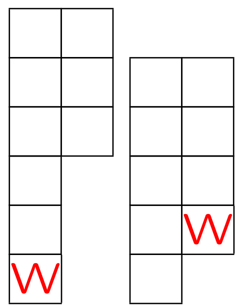
This should be interpreted as Gauss law: the D-brane is compact, so the total incoming string charge hass to be balanced by the total outgoing string charge.

We also see the enhanced gauge symmetry for AdS giants when they come together.

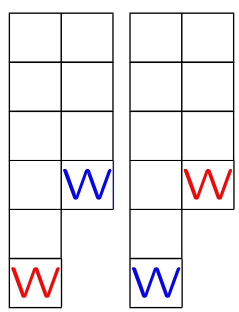
The gauge symmetry of AdS giants is embedded in the $U(N)$ gauge symmetry.

How do we see this for the S giants?

Use double Young tableaux:



This is a string stretching between two giants. Upper and lower indices don't transform the same way. We can not build a gauge invariant state from this. We need to add a second string that goes in the opposite direction:



We see Gauss law appearing again.

One can also see that when giants coincide there are "less states". The states are then in agreement with counting gauge invariant states on a $U(2)$ field theory.