

CP-violation in  $B^0 \rightarrow (\rho\pi)^0 \rightarrow \pi^+\pi^-\pi^0$  using  
a time-dependent Dalitz plot analysis

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DPF, 2004,

Riverside, CA

# The CKM matrix and Unitarity Triangle

The CKM mixing matrix relates the quarks in the weak interaction basis ( $d', s', b'$ ), and the quark mass eigenstates ( $d, s, b$ ):  $(d', s', b') = V_{CKM} (d, s, b)$

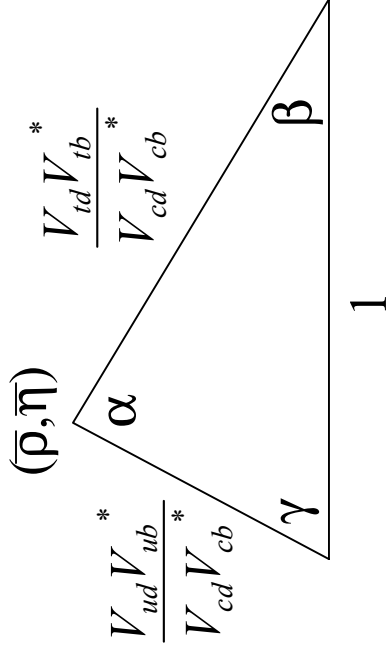
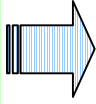
$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Wolfenstein parametrization:

$$V_{CKM} \approx \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda(1 + iA^2\lambda^4\eta) & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2(1 + i\lambda^2\eta) & 1 \end{pmatrix}$$

Unitarity leads to:

$$V_{ud}^* V_{ub}^* + V_{cd}^* V_{cb}^* + V_{td}^* V_{tb}^* = 0$$



$\lambda = \sin\theta_C$  (Cabibbo angle)

Independent parameters:  $\lambda, \rho, \eta, A$

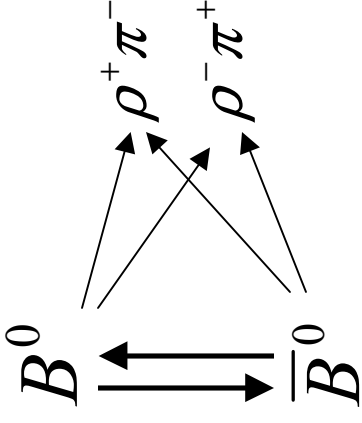
The angles of the triangle are related to CP-violation.

Measure  $\alpha$  from CP-asymmetries in  $b \rightarrow u\bar{u}d$  processes.

$$B \rightarrow \pi\pi, B \rightarrow \rho\pi, B \rightarrow \rho\rho$$

# Previous $B^0 \rightarrow \rho^+ \pi^-$ Results (non-Dalitz approach)

$\rho\pi$  is not a CP eigenstate:



Time dependent decay rate:

$$\Gamma(B^0 \rightarrow \rho^\pm \pi^\mp) \propto (1 \pm A_{CP}) (1 - (S \pm \Delta S) \sin(\Delta m \Delta t) + (C \pm \Delta C) \cos(\Delta m \Delta t))$$

C: direct CPV

S: CPV in interference between mixing and decay

$\Delta C$ ,  $\Delta S$ : parameters related to  $\rho\pi$  not being a CP eigenstate.

$A_{CP}$ : asymmetry between  $\rho^+ \pi^-$  and  $\rho^- \pi^+$

Babar results from 113 fb<sup>-1</sup> with a quasi-2-body approach. (Select the  $\rho$  bands on the Dalitz plot, cut away interference regions)

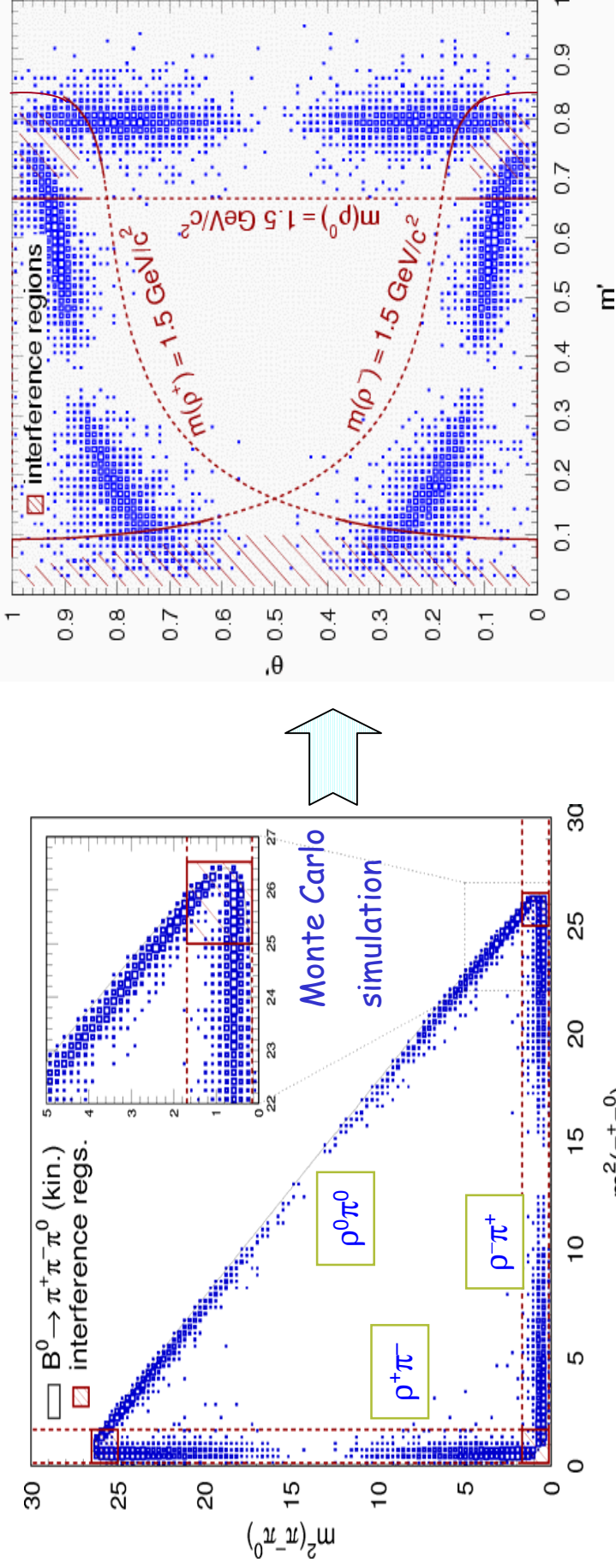
$$S = -0.13 \pm 0.18 \pm 0.04, \quad C = 0.35 \pm 0.13 \pm 0.05$$

$$\Delta S = 0.33 \pm 0.18 \pm 0.03, \quad \Delta C = 0.20 \pm 0.13 \pm 0.05$$

$$A_{CP} = -0.11 \pm 0.06 \pm 0.03$$

**PRL 91, 201802 (2003)**

# $B^0 \rightarrow (\rho\pi)^0 \rightarrow \pi^+\pi^-\pi^0$ Dalitz plot



Square Dalitz plot: more uniform repartition of events. Better parametrization.

$$m' \equiv \frac{1}{\pi} \arccos \left( 2 \frac{m_{+-} - m_{+-}[\min]}{m_{+-}[\max] - m_{+-}[\min]} - 1 \right),$$

$$\theta' \equiv \frac{1}{\pi} \theta_{+-}$$

$$m_{+-}[\max] = m_{B^0} - m_{\pi^0}$$

$$m_{+-}[\min] = 2m_{\pi^+}$$

$\theta_{+-}$  is the angle between the pos. track and neg.  $B^0$  momentum in the  $\{+-\}$  rest frame

# $B^0 \rightarrow (\rho\pi)^0$ decay rates

Snyder-Quinn approach, PRD 48, 2139 (1993)

Extract  $\alpha$  and the strong phases using the interference between the  $B^0 \rightarrow \pi^+\pi^-\pi^0$  amplitudes.

$$A_{3\pi} = f_+ A^{+-} + f_- A^{-+} + f_0 A^{00}$$

$$\bar{A}_{3\pi} = f_+ \bar{A}^{+-} + f_- \bar{A}^{-+} + f_0 \bar{A}^{00}$$

$f_{+,-,0}$  are relativistic Breit-Wigner functions, that depend on the Dalitz plot variables.  $A^{+,-,0,0}$  are decay amplitudes to  $\rho^+\pi^-$ ,  $\rho^-\pi^+$  and  $\rho^0\pi^0$ .

$$\Gamma(\Delta t) \propto \left( |A_{3\pi}|^2 + |\bar{A}_{3\pi}|^2 \right) \frac{e^{-|\Delta t|/\tau}}{4\tau}$$

$Q_{\text{tag}} = +1$  for  $B^0$  and  $-1$  for  $\bar{B}^0$

$$\left( 1 + 2Q_{\text{tag}} \frac{\text{Im} \left[ \bar{A}_{3\pi} A_{3\pi}^* \right]}{|A_{3\pi}|^2 + |\bar{A}_{3\pi}|^2} \sin(\Delta m_d \Delta t) - Q_{\text{tag}} \frac{|A_{3\pi}|^2 - |\bar{A}_{3\pi}|^2}{|A_{3\pi}|^2 + |\bar{A}_{3\pi}|^2} \cos(\Delta m_d \Delta t) \right)$$

# Fit Strategy

Expand  $A_{3\pi}$  as the sum of the form factors,  $f_{+,-,0}$ , multiplied by observable coefficients, U and I. **PRD 62, 054002 (2000)**

$$|A_{3\pi}|^2 \pm |\bar{A}_{3\pi}|^2 = \sum_{\kappa \in \{+,0,-\}} |f_\kappa|^2 U_\kappa^\pm + 2 \sum_{\sigma < \kappa \in \{+,0,-\}} \left( \text{Re} [f_\kappa f_\sigma^*] U_\kappa^\pm - \text{Im} [f_\kappa f_\sigma^*] U_{\kappa\sigma}^{\pm, \text{Im}} \right)$$

$$\text{Im}(\bar{A}_{3\pi} A_{3\pi}^*) = \sum_{\kappa \in \{+,0,-\}} |f_\kappa|^2 I_\kappa + \sum_{\sigma < \kappa \in \{+,0,-\}} \left( \text{Re} [f_\kappa f_\sigma^*] I_{\kappa\sigma}^{\text{Im}} + \text{Im} [f_\kappa f_\sigma^*] I_{\kappa\sigma}^{\text{Re}} \right)$$

Assume that the amplitude is dominated by  $\rho^{+,-,0}$  and the radial excitations  $\rho(1450)$ ,  $\rho(1700)$

U's and I's are determined in the likelihood fit. 27 observables.

$\rho^0\pi^0$  is small, (set some params. to zero)  $\rightarrow$  16 observables

Other resonances such as  $\mathbf{f}_0$  are included in the background model.

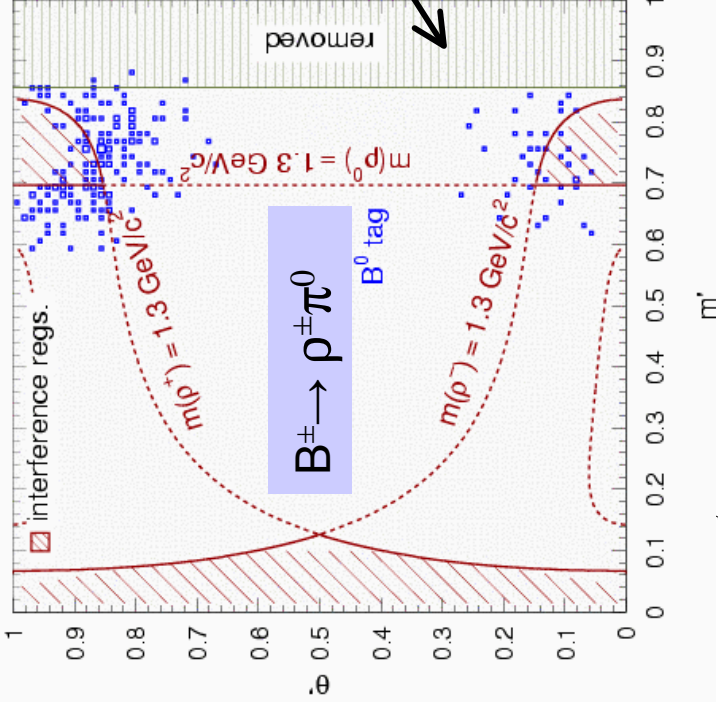
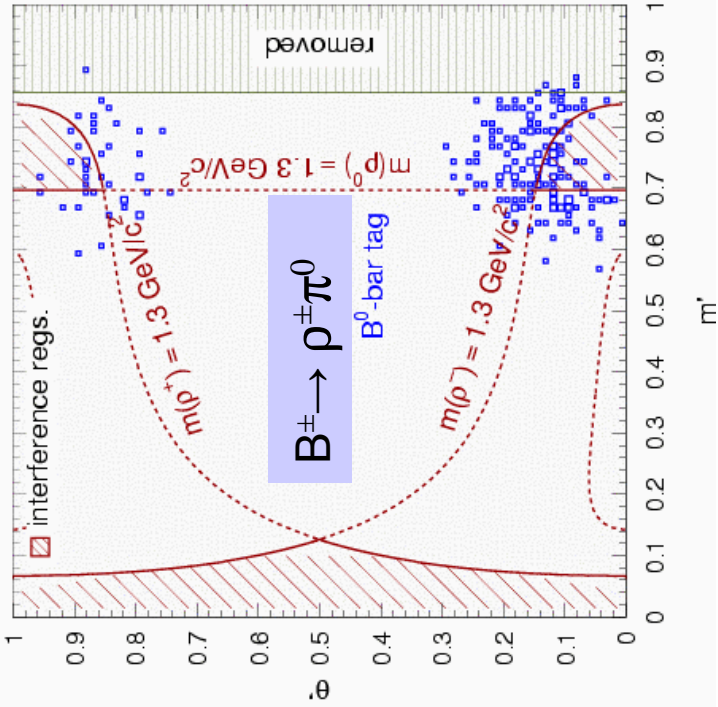
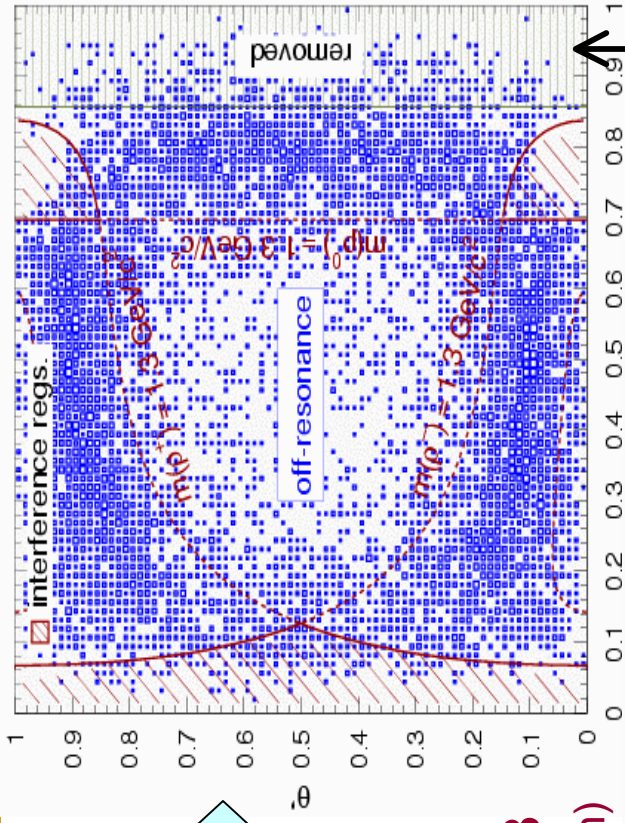
A possible  $\sigma$  resonance is not considered. (expected to be insignificant in neutral B decays.)

# Backgrounds

Dominant:  $q\bar{q}$  continuum: 

Model 38 B-background modes, including  $B \rightarrow$  charm decays.

Use different fixed Dalitz-plot background shapes for different B tags. (asymmetry from misreconstruction)



Cut removes most of  $K_S^0 \pi^0$  decays

# Analysis Features

Build a maximum likelihood using PDFs for 5 discriminating variables:

Energy difference:  $\Delta E = E_B^* - E_{\text{beam}}^*$   
transformed range  $(-1, 1)$

Energy substituted

$$m_{\text{ES}} = \sqrt{E_{\text{beam}}^{*2} - p_B^2}$$

mass:

B-decay: isotropic  
Continuum qq: jet-like

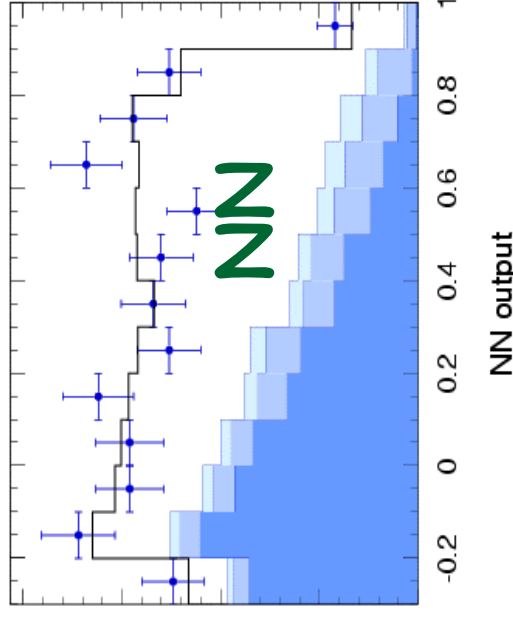
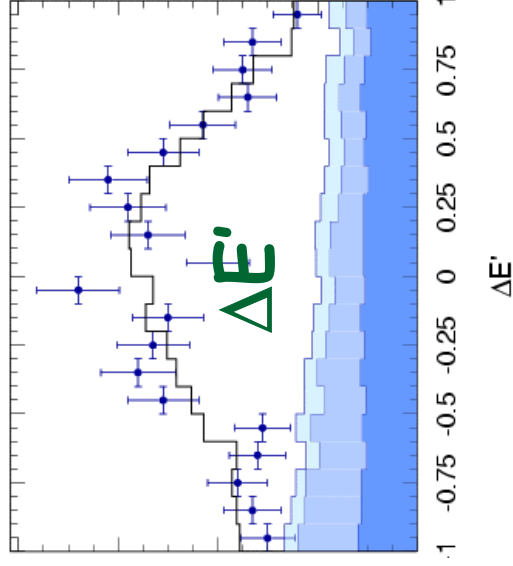
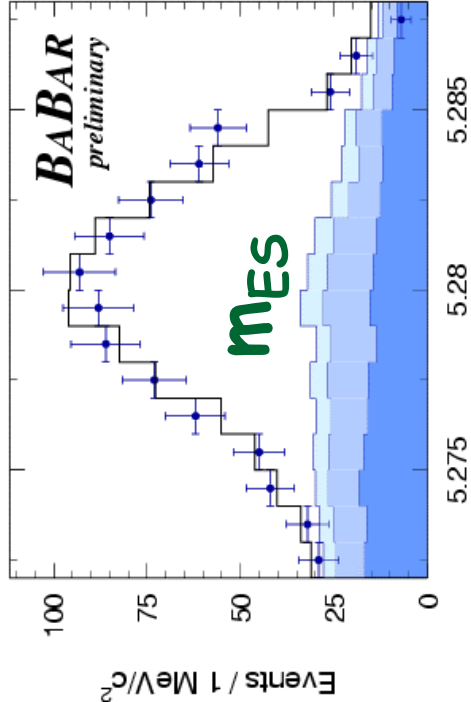
} Combine event shape variables into a Neural Network (NN).

$$\text{Likelihood} \propto \text{Pdf}(m_{\text{ES}}) \times \text{Pdf}(\Delta E) \times \text{Pdf}(\text{NN}) \times \text{Pdf}(\Delta t) \times \text{Pdf}(\text{DP})$$

- Model signal efficiency variation across the Dalitz Plot. 24% average.
- Variation of signal misreconstruction rate across the Dalitz Plot. 19% on average, mostly from low momentum neutrals.
- Take into account the correlations: DP-NN, DP- $\Delta E$

# Data Projections

Use **192 fb<sup>-1</sup>** of data. **17031** events selected, **80%** are continuum background

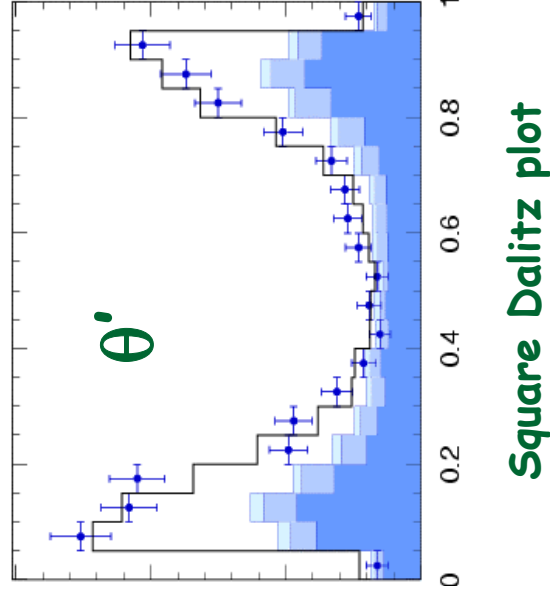
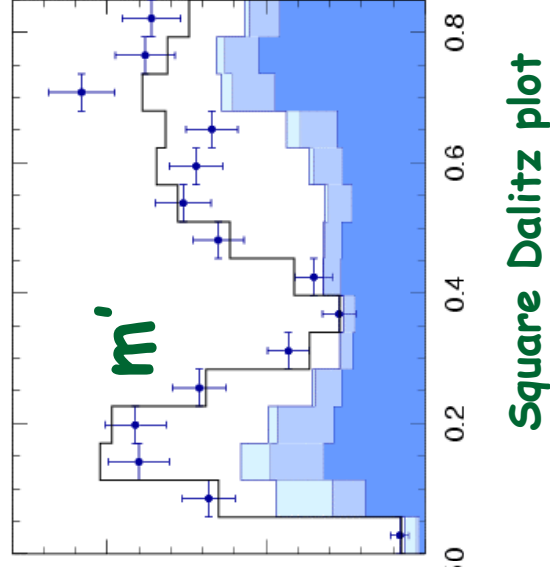
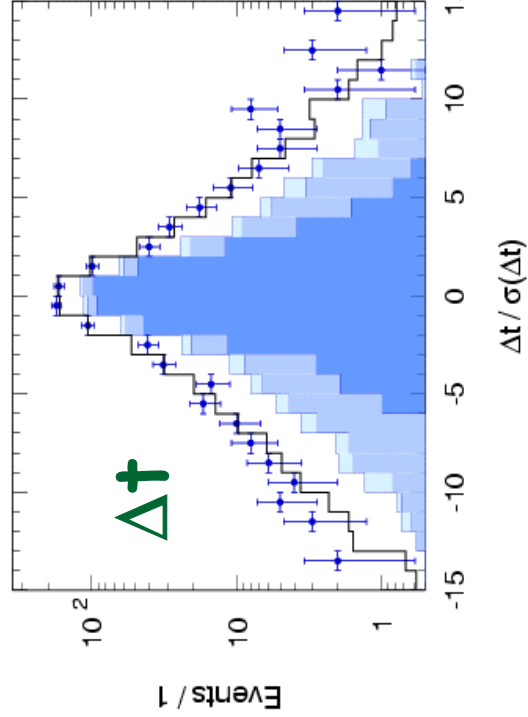


$m_{ES}$  (GeV/c<sup>2</sup>)

$\Delta E'$

NN output

continuum, B-background, misreconstructed-signal



$\Delta t / \sigma(\Delta t)$

Square Dalitz plot

Square Dalitz plot

# Results

$B^0 \rightarrow \pi^+ \pi^- \pi^0$  event yield:  $1184 \pm 58$  (stat.)

Extract the physics parameters from the fitted U's and I's

The trigonometrical functions of  $\alpha$  are “hidden” in the U's and I's

$$C = \frac{1}{2} \left( \frac{U_+^-}{U_+^+} + \frac{U_-^-}{U_-^+} \right) \quad S = \frac{I_+^+}{U_+^+} + \frac{I_-^-}{U_-^+} \quad A_{CP} = \frac{U_+^+ - U_-^+}{U_+^+ + U_-^+}$$

$$\Delta C = \frac{1}{2} \left( \frac{U_+^-}{U_+^+} - \frac{U_-^-}{U_-^+} \right) \quad \Delta S = \frac{I_+^+}{U_+^+} - \frac{I_-^-}{U_-^+}$$

	Q2B(113 fb <sup>-1</sup> ), LP03	Dalitz(192 fb <sup>-1</sup> ) <b>New!</b>
$A_{\rho\pi}$	$-0.114 \pm 0.062 \pm 0.027$	$-0.088 \pm 0.049 \pm 0.013$
<b>C</b>	<b><math>0.35 \pm 0.14 \pm 0.05</math></b>	<b><math>0.34 \pm 0.11 \pm 0.05</math></b>
$\Delta C$	$0.20 \pm 0.14 \pm 0.05$	$0.15 \pm 0.11 \pm 0.03$
<b>S</b>	<b><math>-0.13 \pm 0.18 \pm 0.04</math></b>	<b><math>-0.10 \pm 0.14 \pm 0.04</math></b>
$\Delta S$	$0.33 \pm 0.18 \pm 0.03$	$0.22 \pm 0.15 \pm 0.03$

Parameters related to direct CP

# Direct CP results

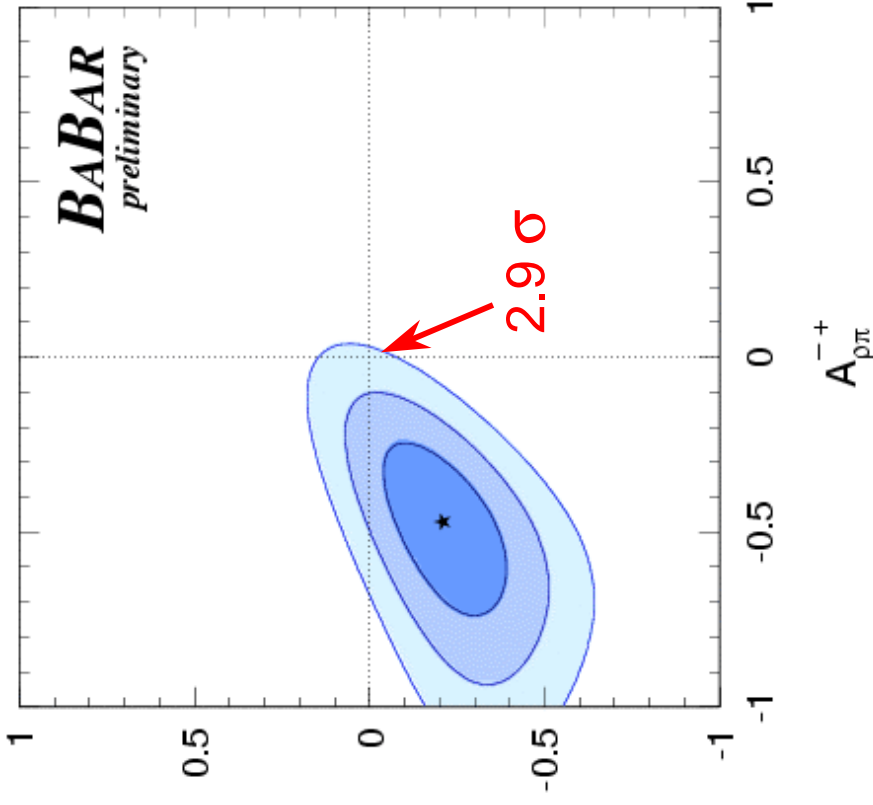
Define physically more intuitive quantities:

$$A_{\rho\pi}^{+-} \propto N(B^0 \rightarrow \rho^+ \pi^-) - N(\bar{B}^0 \rightarrow \rho^- \pi^+)$$

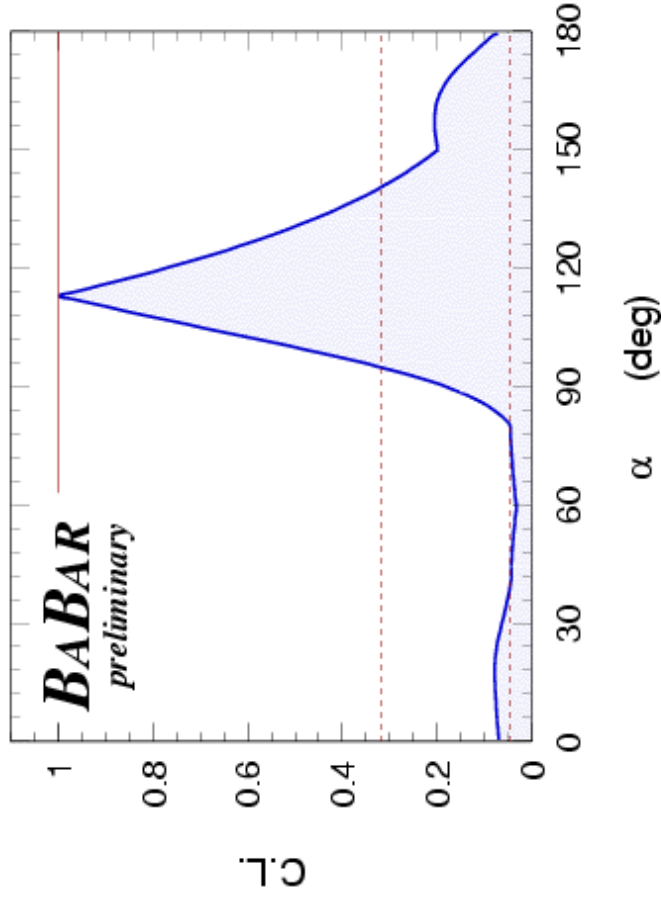
$$A_{\rho\pi}^{-+} \propto N(B^0 \rightarrow \rho^- \pi^+) - N(\bar{B}^0 \rightarrow \rho^+ \pi^-)$$

$$\begin{aligned} A_{\rho\pi}^{+-} &\equiv \frac{A_{\rho\pi} - C - A_{\rho\pi} \Delta C}{1 - C - A_{\rho\pi} \Delta C} \\ &= -0.47^{+0.14}_{-0.15} \pm 0.06 \end{aligned}$$

$$\begin{aligned} A_{\rho\pi}^{-+} &\equiv \frac{A_{\rho\pi} + C + A_{\rho\pi} \Delta C}{1 + \Delta C + A_{\rho\pi} C} \\ &= -0.21 \pm 0.11 \pm 0.04 \end{aligned}$$

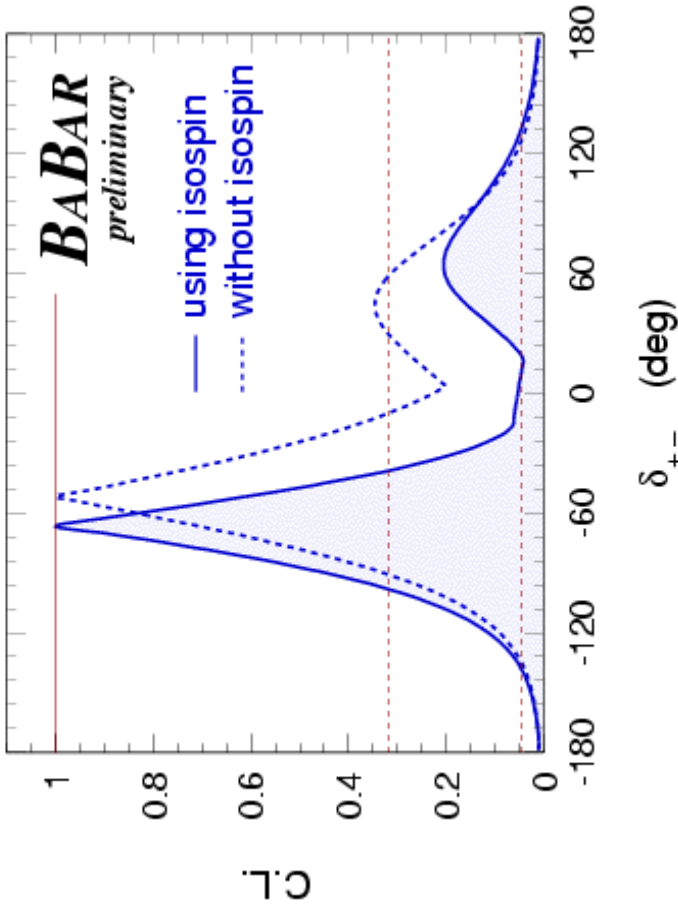


# $\alpha$ and strong phase results



$$\alpha = (113_{-17}^{+27} \pm 6)^\circ$$

*weak constraint at two standard deviations*

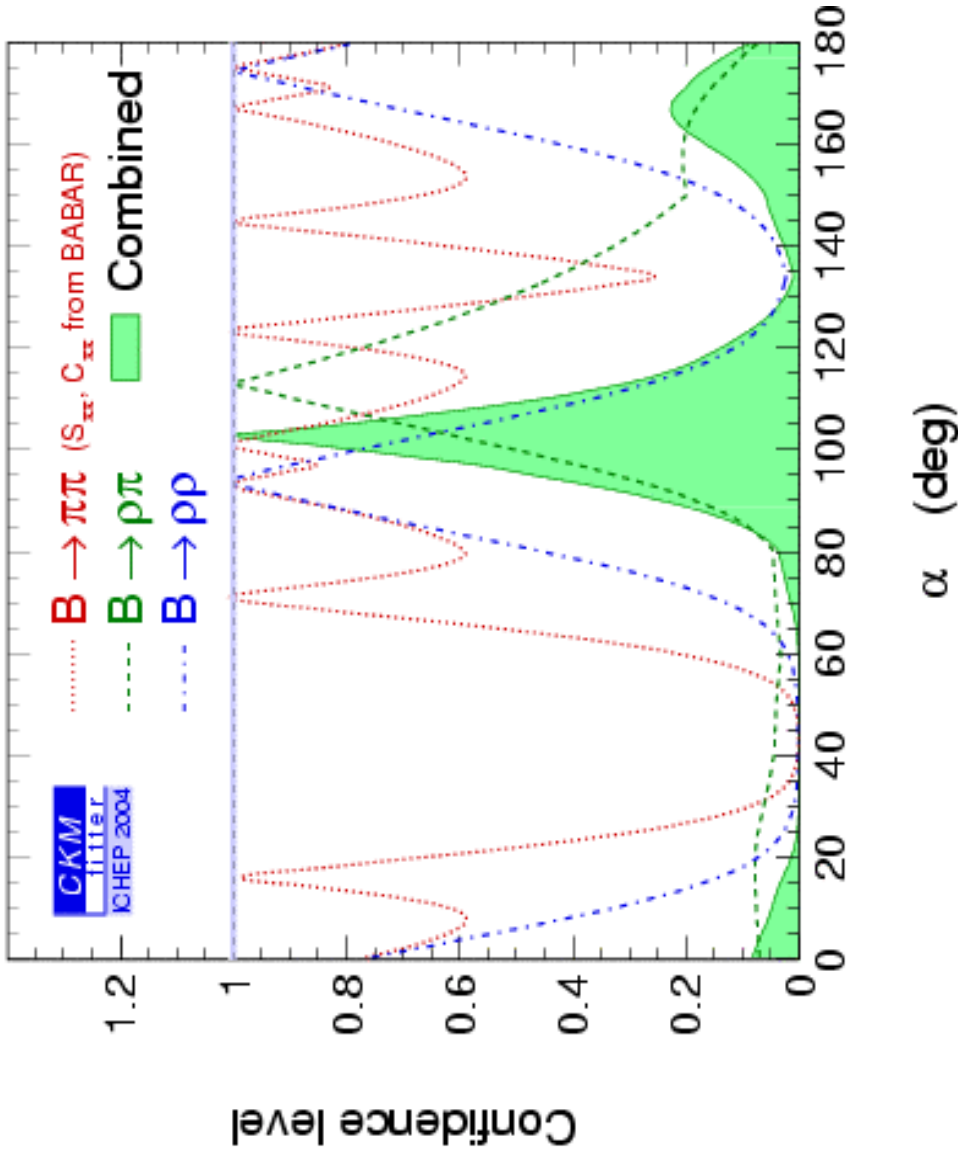


(Isospin relations reduce the number of parameters)

Strong phase between  $B^0 \rightarrow \rho^+ \pi^-$  and  $B^0 \rightarrow \rho^- \pi^+$ :

$$\delta_{+-} = \arg(A^{+-*} A^{--}) = (-67_{-31}^{+28} \pm 7)^\circ$$

# $\alpha$ at BaBar



Combined value from all  
3 channels:

$$\alpha = (103^{+10}_{-11})^\circ$$

# Conclusions

- The first time-dependent Dalitz plot analysis of the decay  $B^0 \rightarrow \pi^+ \pi^- \pi^0$  has been carried out at BaBar.  
[hep-ex/0408099, SLAC-PUB-10658](#)
- The new CP parameters are in good agreement with earlier published results.
- Measure direct CP violation at  $2.9\sigma$ .
- Obtain constraints on
  - CKM angle  $\alpha = (113_{-17}^{+27} \pm 6)^\circ$
  - Relative strong phase between  $\rho^+ \pi^-$  and  $\rho^- \pi^+$ :

$$\delta_{+-} = (-67_{-31}^{+28} \pm 7)^\circ$$

# Backup Slides

# Fit Results

$U_+^+$	Coeff. of $ f(p^-) ^2$	$1.19 \pm 0.12 \pm 0.03$
$I_-$	Coeff. of $ f(p^-) ^2 \sin(\Delta m \Delta t)$	$-0.19 \pm 0.11 \pm 0.02$
$I_+$	Coeff. of $ f(p^+) ^2 \sin(\Delta m \Delta t)$	$0.06 \pm 0.11 \pm 0.02$
$U_-^-$	Coeff. of $ f(p^-) ^2 \cos(\Delta m \Delta t)$	$0.22 \pm 0.16 \pm 0.05$
$U_+^-$	Coeff. of $ f(p^+) ^2 \cos(\Delta m \Delta t)$	$0.50 \pm 0.17 \pm 0.05$
$U_{+-}^-, \text{Im}$	Coeff. of $\text{Im}[f(p^+) f(p^-)^*] \cos(\Delta m \Delta t)$	$0.25 \pm 1.4 \pm 0.3$
$U_{+-}^-, \text{Re}$	Coeff. of $\text{Re}[f(p^+) f(p^-)^*] \cos(\Delta m \Delta t)$	$2.0 \pm 1.2 \pm 0.2$
$U_{+-}^+, \text{Im}$	Coeff. of $\text{Im}[f(p^+) f(p^-)^*]$	$0.16 \pm 0.70 \pm 0.14$
$U_{+-}^+, \text{Re}$	Coeff. of $\text{Re}[f(p^+) f(p^-)^*]$	$-0.26 \pm 0.65 \pm 0.17$
$I_{+-}^{\text{Im}}$	Coeff. of $\text{Im}[f(p^+) f(p^-)^*] \sin(\Delta m \Delta t)$	$-5.2 \pm 1.9 \pm 0.7$
$I_{+-}^{\text{Re}}$	Coeff. of $\text{Re}[f(p^+) f(p^-)^*] \sin(\Delta m \Delta t)$	$-0.3 \pm 2.0 \pm 0.5$
$U_0^+$	Coeff. of $ f(p^0) ^2$	$0.16 \pm 0.05 \pm 0.05$
$U_{+0}^+, \text{Im}$	Coeff. of $\text{Im}[f(p^+) f(p^0)^*]$	$0.25 \pm 0.35 \pm 0.18$
$U_{+0}^+, \text{Re}$	Coeff. of $\text{Re}[f(p^+) f(p^0)^*]$	$-0.34 \pm 0.39 \pm 0.15$
$U_{-0}^+, \text{Im}$	Coeff. of $\text{Im}[f(p^-) f(p^0)^*]$	$0.34 \pm 0.43 \pm 0.17$
$U_{-0}^+, \text{Re}$	Coeff. of $\text{Re}[f(p^-) f(p^0)^*]$	$-0.98 \pm 0.44 \pm 0.18$

**Q2B**  
terms  
 $U_+^+ = 1$

**Interfering**  
terms

$\rho^0 \pi^0$   
**related**  
terms

# Systematic Uncertainties

- Parameters that measure the interference effects: Systematic errors dominated by the uncertainty in the signal DP model. ( $\rho$  resonance tail description). Vary the masses and widths of  $\rho(770)$ ,  $\rho(1450)$ , relative amounts, phase and amplitude of  $\rho(1700)$ .

(The phase and amplitude of the  $\rho(1450)$  is floated in the nominal fit.)

The resulting errors from the signal model range from 0.6 on  $I_{+}^{\text{Im}}$  to 0.08 on  $U_{+}^{\text{+Im}}$

- Parameters not sensitive to interference effects: Systematic errors dominated by B-background and fit bias uncertainties.
- Systematic errors from fixing some of the  $\rho^0\pi^0$  parameters is evaluated with Monte Carlo. Largest effect is 0.11, on  $U_{+0}^{\text{+Im}}$