Lattice calculation of the lowest-order hadronic contribution to the muon anomalous magnetic moment

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DPF2004 (UC Riverside)

T. Blum, Phys. Rev. Lett. 91:052001, 2003
T. Blum, hep-lat/0310064
M. Gockeler, *et al.*, (QCDSF Collaboration) Nucl. Phys. B688,135 (2004)

OUTLINE of the talk

1. Introduction

2. Lattice calculation of the vacuum polarization

3. Discussion/Outlook

I. Introduction to muon g-2...

The magnetic moment μ is proportional to its spin

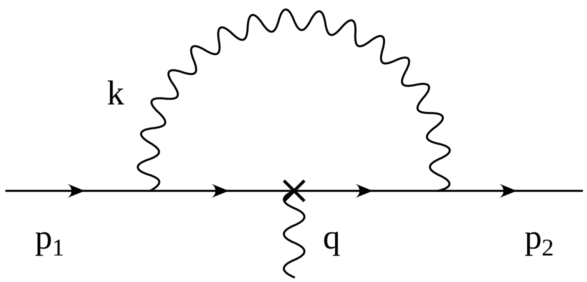
$$\vec{\mu} = g \left(\frac{e}{2m}\right) \vec{S}$$

The Landé g-factor is predicted from the Dirac eq. to be

$$g = 2$$

for elementary fermions

In the quantum (field) theory this will change.



$$\gamma^\mu \;\to\; \Gamma^\mu(q) = \left(\gamma^\mu F_1(q^2) + \frac{i\,\sigma^{\mu\nu}\,q_\nu}{2m} F_2(q^2)\right)$$

which results from Lorentz invariance and current-conservation (Ward-Takahashi identity) when the muon is on-shell.

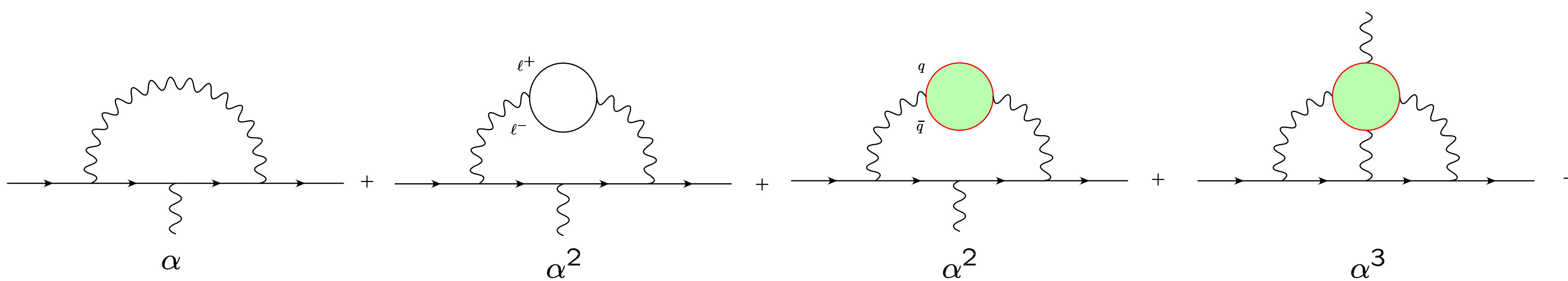
It is straightforward to show

$$\begin{aligned} g &= 2(F_1(0) + F_2(0)) \\ &= 2(1 - F_2(0)) \\ F_2(0) &= \frac{g-2}{2} \equiv a_\mu \end{aligned}$$

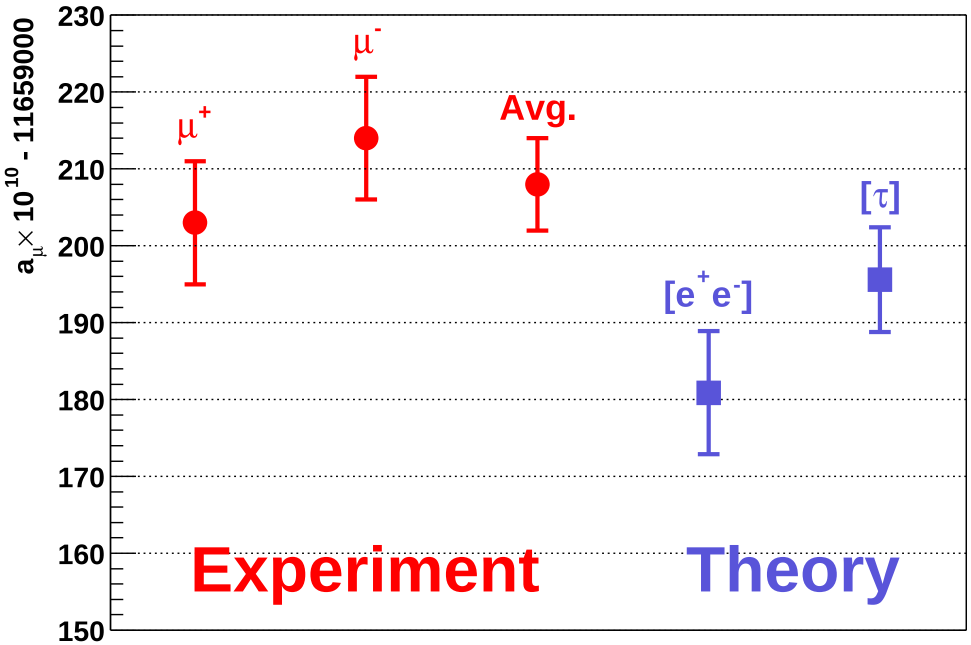
is the anomalous magnetic moment. Corrections start at $\mathcal{O}(\alpha)$.

Compute these corrections order-by-order in perturbation theory by expanding $\Gamma^{\mu}(q^2)$ in QED coupling constant

$$\alpha = \frac{e^2}{4\pi} = \frac{1}{137} + \dots$$



Status of the experimental measurement (Muon $(g-2)$ Collaboration, BNL-E821) and Theoretical calculation of a_μ.



- $a_\mu(\text{exp}) = 11\,659\,208(6) \times 10^{-10}$ $(0.5\,ppm)$

- 2.7 σ discrepancy with theory (SM) (e^+e^- annihilation)

- 1.4 σ discrepancy with theory (SM) (τ decay)

G.W. Bennett, *et al.* (Muon $(g-2)$ Collaboration), Phys. Rev. Lett. 92 (2004) 161802

The precise measurement at Brookhaven (E821) allows a precision test of the Standard Model

- Hadronic contribution dominates theory error

- e^+e^-: 2.7 σ deviation with experiment

- τ decay: 1.4 σ deviation with experiment

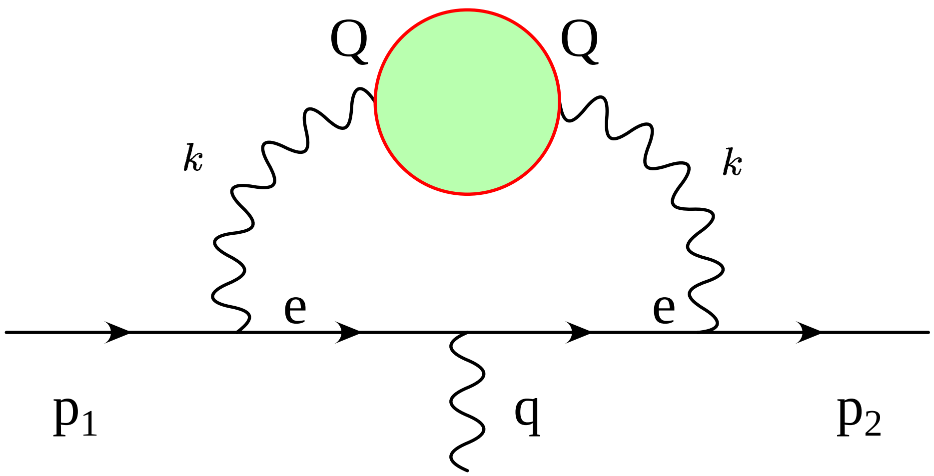
Taking the difference of the two theory calculations, the uncertainty in hadronic contribution is a few %, perhaps more.

$\longrightarrow$ attempt first principles lattice calculation of the hadronic contributions.

Exactly the kind of calculation the lattice is supposed to handle

2. Lattice calculation of the vacuum polarization

Contribution we are after: $\mathcal{O}(\alpha^2)$ hadronic vacuum polarization ($\Pi(k^2)$)



Calculate the blob on the lattice, stick it into the one-loop intregral (`T. Blum, Phys. Rev. Lett. 91:052001,2003`)

In the continuum the vacuum polarization for a single fermion is defined as

$$
\begin{aligned}
\Pi^{\mu\nu}(q) &= \int \mathrm{d}^4x\, \mathrm{e}^{i\,q\,(x-y)} \langle J^\mu(x) J^\nu(y)\rangle \qquad\qquad (J^\mu(x) = \bar\psi\gamma_\mu\psi(x)) \\
&= (q^2 g^{\mu\nu} - q^\mu q^\nu)\Pi(q^2)
\end{aligned}
$$

and satisfies the Ward-Takahashi identity (charge conservation)

$$
q_\mu\, \Pi^{\mu\nu}(q) = 0 \qquad\qquad\qquad (\partial_\mu\, J^\mu = 0)
$$

On the lattice, these go over to

$$
\begin{aligned}
\Pi^{\mu\nu}(\widehat q) &= (\widehat q^2 \delta^{\mu\nu} - \widehat q^\mu \widehat q^\nu)\Pi(\widehat q^2) \\
\widehat q &= \frac{1}{2a}\sin aq^\mu \quad (q^\mu = 2\pi\, n_\mu/L,\ n^\mu = 0,1,\ldots,L-1) \\
J^\mu(x) &= \frac{1}{2}\Big(\bar\psi(x+\widehat\mu)U^\dagger(x)(1+\gamma^\mu)\psi(x) - \bar\psi(x)U(x)(1-\gamma^\mu)\psi(x+\widehat\mu)\Big)
\end{aligned}
$$

Quenched Results

New domain wall fermion results (preliminary), $m_{val} \sim m_s/2 - m_s$

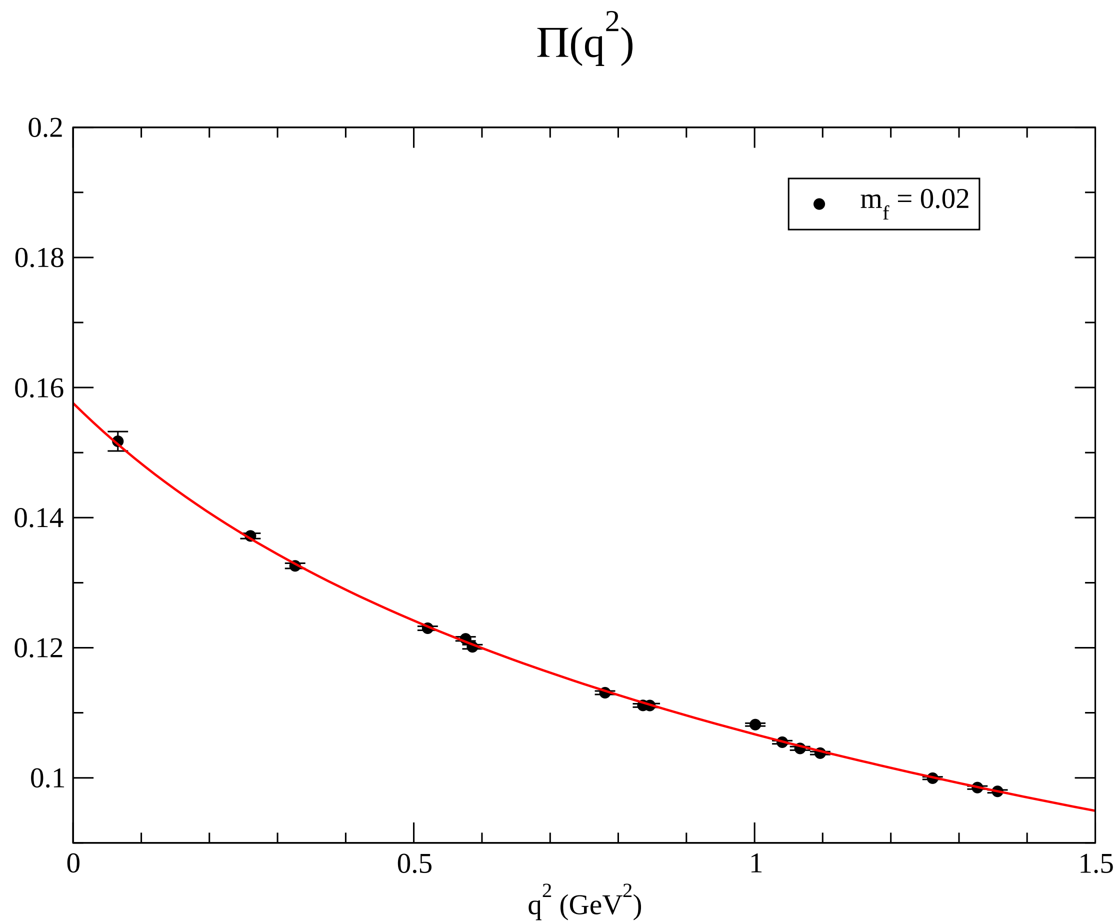
Quenched dwf $a^{-1} = 1.31$

Dispersion relation gives

$$\Pi(q^2) \sim \frac{f_V^2}{q^2 + M_V^2} + \log\left(a^2(q^2 + \mu^2)\right)$$

(Shifman, *et al.*, QCDSF)

$\chi^2 \sim 4.7/\text{dof} \to 0.6$ if outlier is dropped



a_μ for a *single* flavor:

a^{-1}	m_{val}	$a_\mu^{\text{had}}(\alpha^2)$ $(Q = 1)$
1.3	0.02	750(35) x10^{-10}
1.3	0.04	669(23) x10^{-10}

After summing u, d, and s quark contributions, consistent with previous results (Blum, QCDSF). Statistical accuracy improved to about 5%

Quenched results: $\gtrsim$ 2/3 $\times$ dispersive result. This may not be surprising: 72% came from the ρ(770) resonance (in the e^+e^- cross-section).

2+1 flavor QCD calculation

(in progress)

using the MILC Collaboration's lattices (essential):

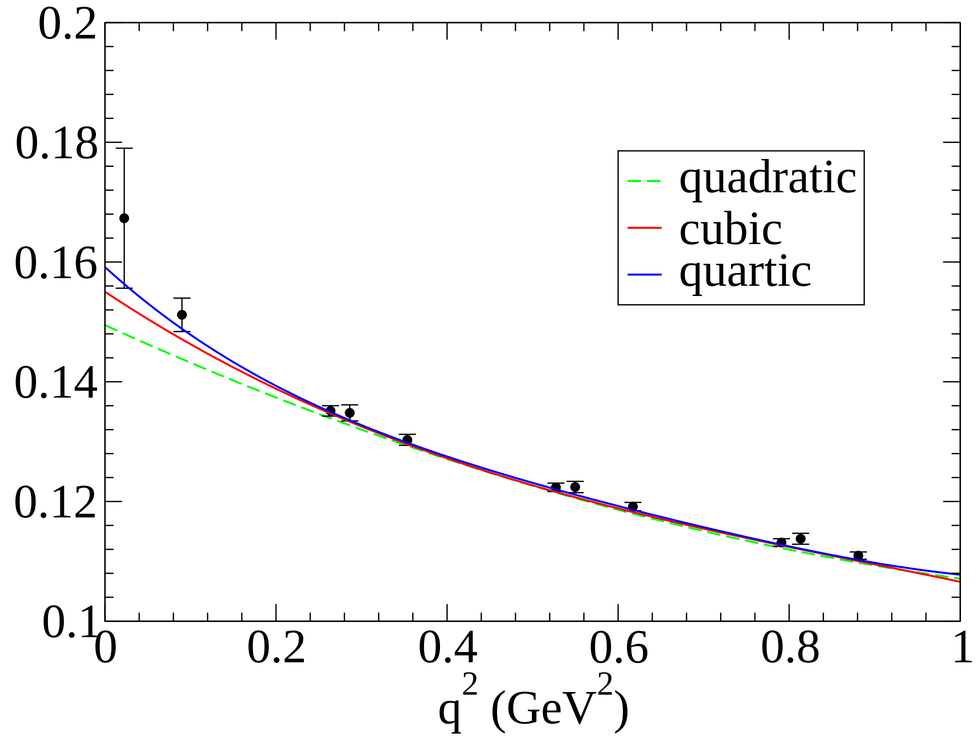
a (fm)	size	m_l	m_s	m_{val}	# configs
0.121(3)	$20^3 \times 64$	0.01	0.05	0.05	57
0.121(3)	$20^3 \times 64$	0.01	0.05	0.01	439
0.120(3)	$24^3 \times 64$	0.005	0.05	0.005	143
0.086(2)	$28^3 \times 96$	0.0062	0.031	0.031	41
0.086(2)	$28^3 \times 96$	0.0062	0.031	0.0062	248
0.086(2)	$40^3 \times 96$	0.0031	0.031	0.0031	114
0.085(2)	$28^3 \times 96$	quenched		0.031	29
0.085(2)	$28^3 \times 96$	quenched		0.0062	31

and improved Kogut-Susskind fermions

$m_l \equiv \frac{m_u + m_d}{2} \approx \frac{1}{5} m_s$, Volume $= 28^3 \times 96$ $((2.5\ \mathrm{fm})^3)$

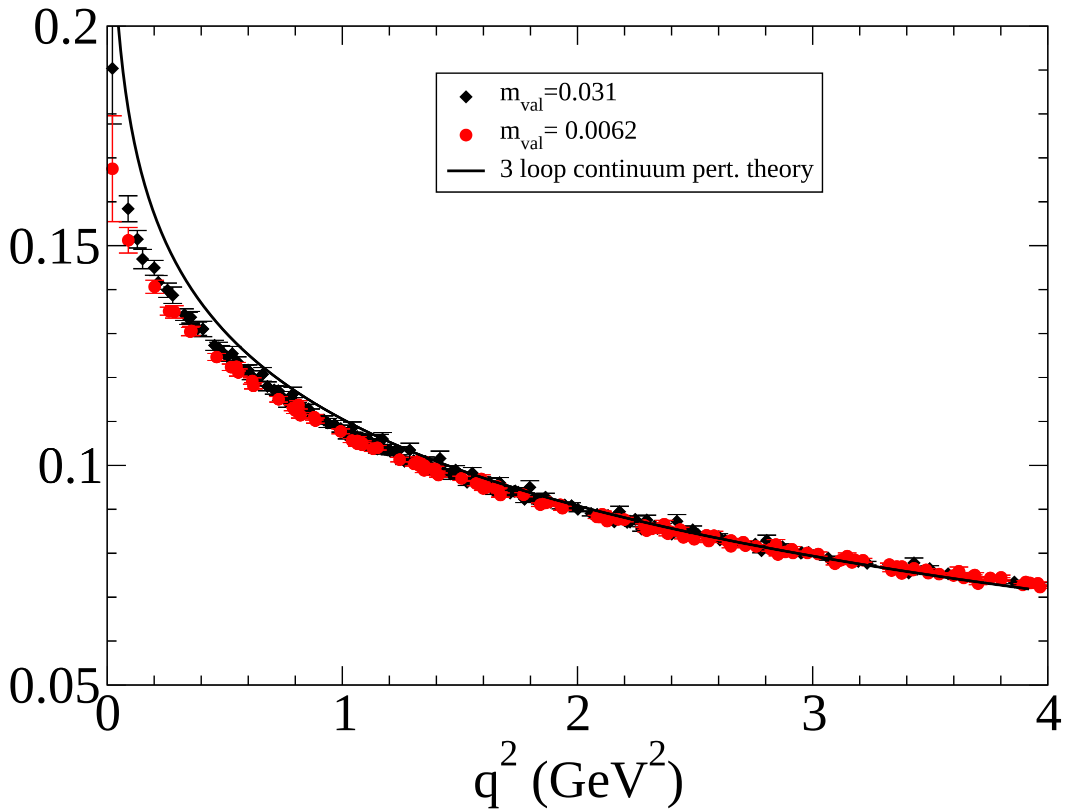
Polynomial in q^2

- $\chi^2/\mathrm{dof} = 4.9, 1.6, 1.2$
- underestimates data
- low q^2 region dominates a_μ contribution
- need better fit function and/or statistics



- (Staggered) Chiral perturbation theory for $\Pi(q^2)$

$m_l \approx \frac{1}{10} m_s$, Volume $= 40^3 \times 96$ ($(3.5\ \mathrm{fm})^3$)



Lighter quark mass result goes in the right direction.
MILC Collaboration is extending this run by a factor of 3

3. Summary/Outlook

- muon g-2 is an important precision test of the Standard Model (*c.f.*, recent measurement at BNL)

- Biggest theoretical uncertainty: hadronic contributions

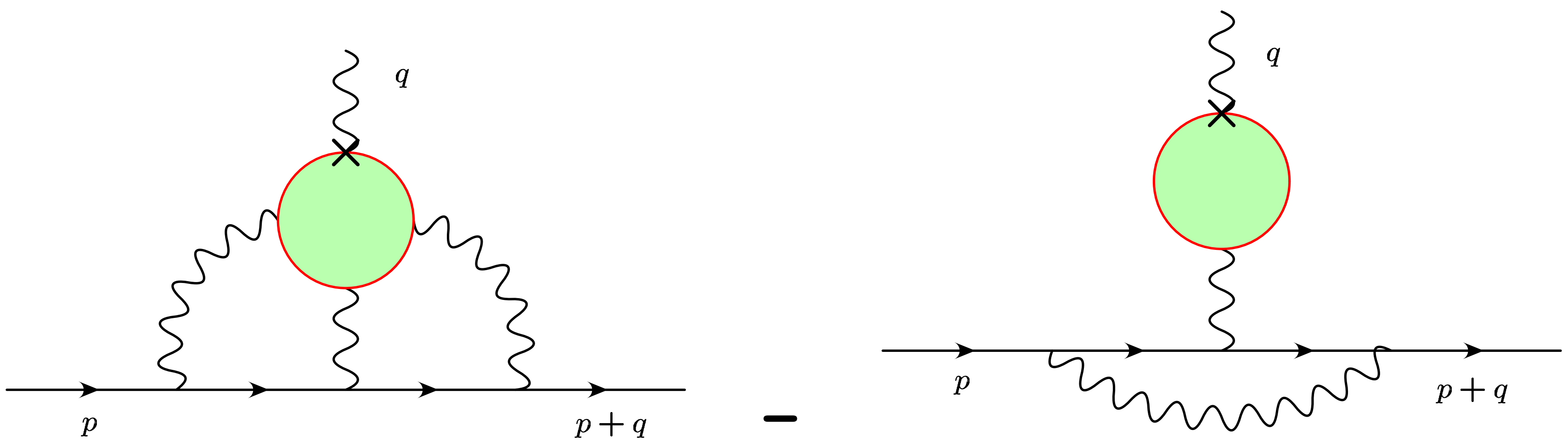
- Dispersive results using e^+e^- and τ data disagree

- First princliples (lattice) calculation is possible, more work to get to few percent uncertainty level and fully leverage experiment (in progress).

- Future:
  - Finish small quark mass/large volume calc
  - Understand lattice spacing errors, mass dependence. (Staggered Chiral Perturbation Theory)
  - Use all 2+1 flavor data to extrapolate to the physical masses, $a \to 0$
  - Dynamical DWF.
  - Calculate disconnected diagram.
  - Hadronic light-by-light (α^3) contribution (more challenging, but required precision not as high)

The hadronic light-by-light contribution (α^3)

Calculating as before likely difficult. Instead, compute completely non-perturbatively, including photons. *i.e.* average over $SU(3) \times U(1)$ gauge fields



But, need to subtract α^2 contribution