Semileptonic decays of *D* and *B* mesons from unquenched lattice QCD

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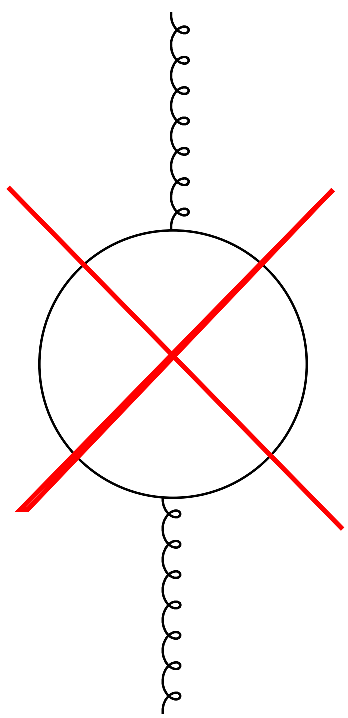
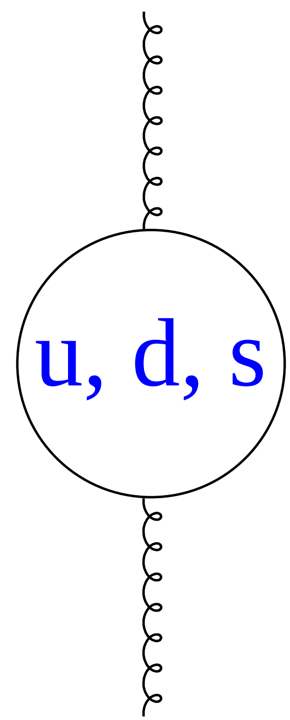
for Fermilab+MILC collaboration

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Status of Lattice QCD

Past $\Longrightarrow$ Now

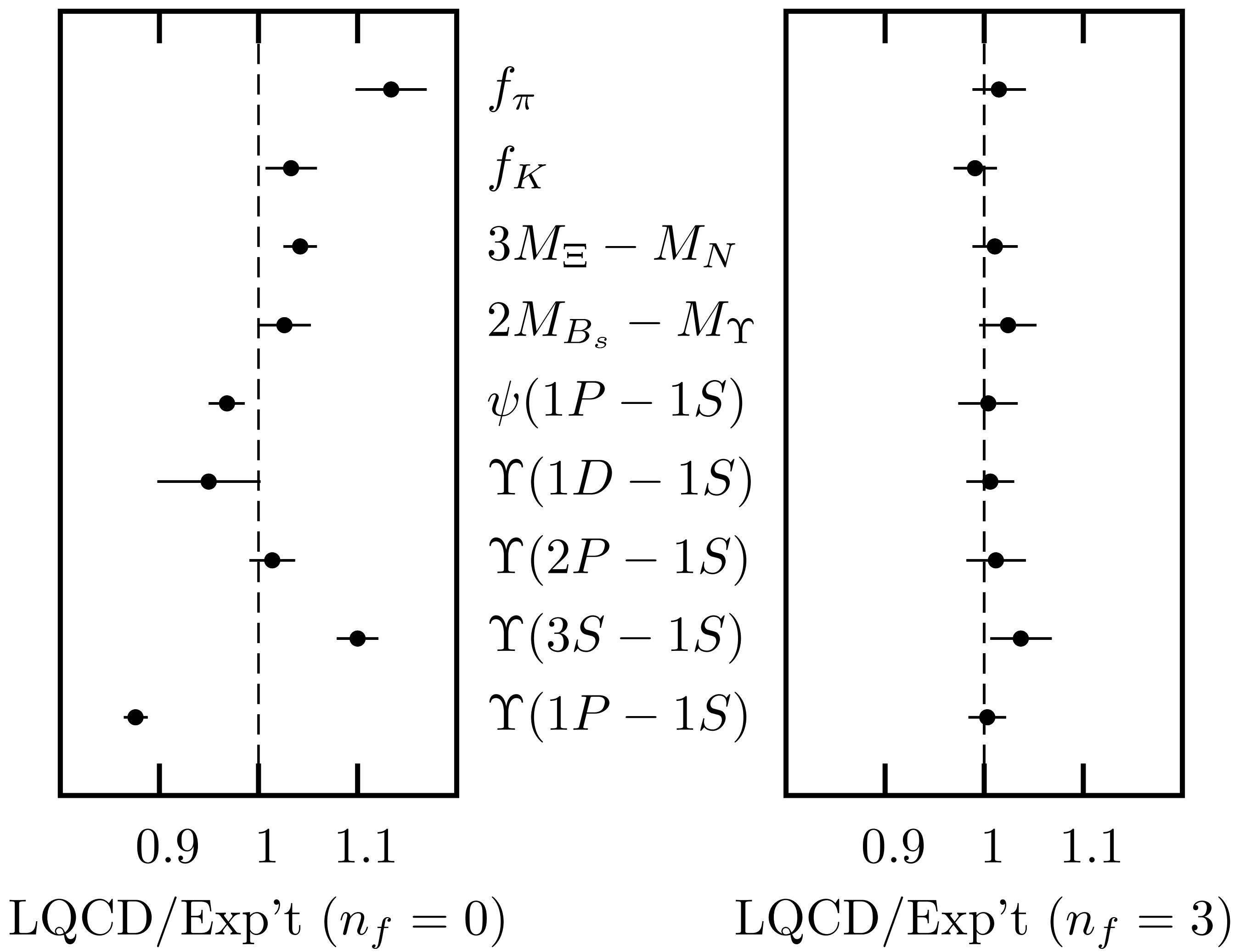




$n_f = 0$, "Quenched"

$n_f = 3$, "Unquenched"

(HPQCD+UKQCD+MILC+Fermilab Collab., PRL92,2004)



$n_f = 0$: $\approx$ 10% deviations from Exp't

$\Longrightarrow$ $n_f = 3$: agree to Exp't within 2-3%.

Next step :

Determination/Check of $\mathbf{V}_{\mathrm{CKM}}$ with $n_f = 3$ Lattice QCD

$$\begin{pmatrix} \mathbf{V_{ud}} & \mathbf{V_{us}} & \mathbf{V_{ub}} \\ \pi \to l\nu & K \to \pi l\nu & B \to \pi l\nu \\ \mathbf{V_{cd}} & \mathbf{V_{cs}} & \mathbf{V_{cb}} \\ D \to \pi l\nu & D \to K l\nu & B \to D l\nu \\ D \to l\nu & D_s \to l\nu & \\ \mathbf{V_{td}} & \mathbf{V_{ts}} & \mathbf{V_{tb}} \\ \langle B_d|\overline{B}_d\rangle & \langle B_s|\overline{B}_s\rangle & \end{pmatrix}$$

Most $\mathbf{V}_{\mathrm{CKM}}$ can be determined from Lattice QCD + Experiment.

Next step :

Determination/Check of $\mathbf{V}_{\mathrm{CKM}}$ with $n_f = 3$ Lattice QCD

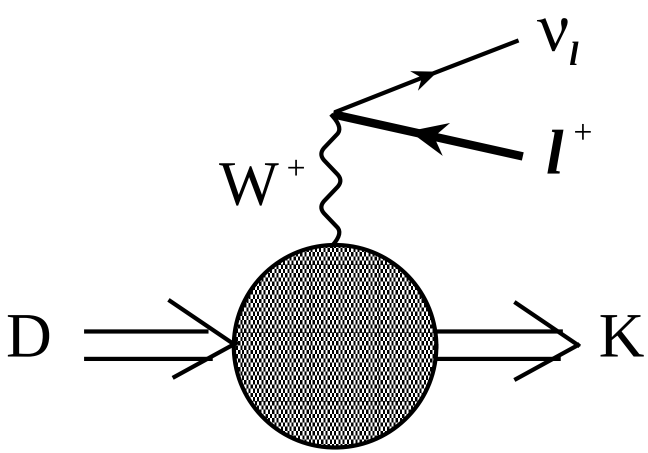
$$
\begin{pmatrix}
\mathbf{V_{ud}} & \mathbf{V_{us}} & \mathbf{V_{ub}} \\
\pi \to l\nu & K \to \pi l\nu & B \to \pi l\nu \\
\mathbf{V_{cd}} & \mathbf{V_{cs}} & \mathbf{V_{cb}} \\
D \to \pi l\nu & D \to K l\nu & B \to D l\nu \\
D \to l\nu & D_s \to l\nu & \\
\mathbf{V_{td}} & \mathbf{V_{ts}} & \mathbf{V_{tb}} \\
\langle B_d | \overline{B}_d \rangle & \langle B_s | \overline{B}_s \rangle &
\end{pmatrix}
$$

5 $\mathbf{V}_{\mathrm{CKM}}$ elements can be determined from semileptonic decays.

One example :

$\mathbf{V_{cs}}$ from semileptonic decay $D \to K l \nu$

Experiment



$$\equiv \Gamma(D \to K l \nu) \propto \int dq^2 \, |f_+(q^2)|^2 |\mathbf{V_{cs}}|^2$$

$$(q = p_D - p_\pi)$$

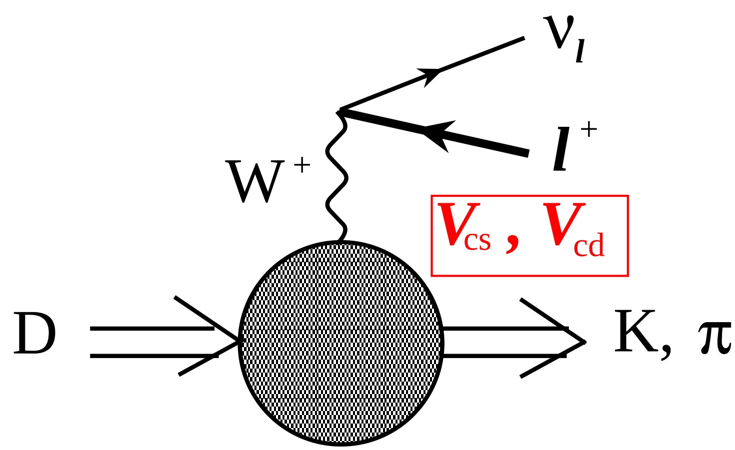
Lattice

$$\langle \pi(p_\pi)|V^\mu|D(p_D)\rangle = f_+(q^2)\left[p_D + p_\pi - \frac{m_D^2 - m_\pi^2}{q^2} q\right]^\mu + f_0(q^2)\,\frac{m_D^2 - m_\pi^2}{q^2}\, q^\mu$$

This talk

First complete $n_f = 3$ LQCD calculations for semileptonic decays of D mesons

$D \to Kl\nu$, $D \to \pi l\nu$



and similarly for B mesons

$B \to \pi l\nu$, $B \to Dl\nu$

$\Longrightarrow$ $|\mathbf{V_{cs}}|, |\mathbf{V_{cd}}|, |\mathbf{V_{ub}}|, |\mathbf{V_{cb}}|$ (**4** CKM from $n_f = 3$ LQCD)

What's new ?

Two dominant errors are successfully reduced:
(quenching, $m_l \to m_{ud}$ extrap.)

- first $n_f = 3$ calculations for semileptonic D decays.

  previous($n_f = 0$) error: $10 - 20\%$ $\Longrightarrow$ Now($n_f = 3$): $< 3\%$

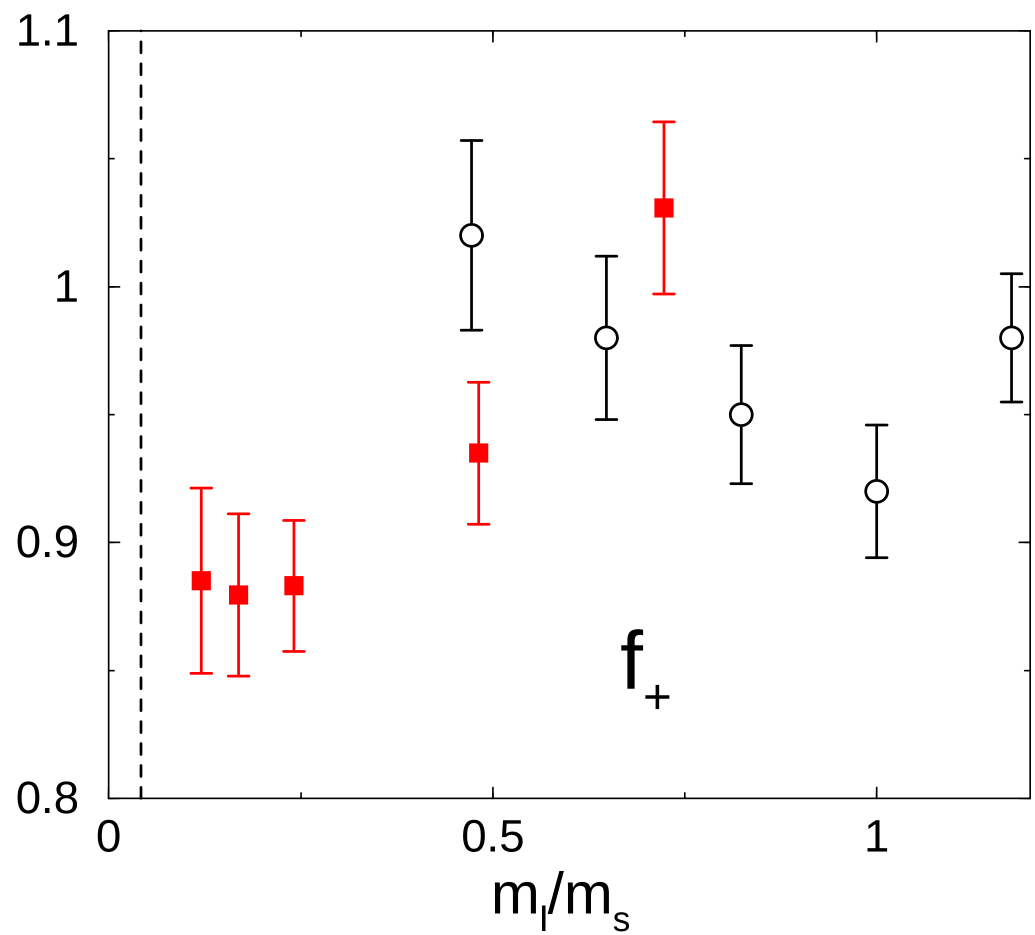
- use of "fast" fermion (*1-spin staggered*-type action) for light quarks

  $\Longrightarrow$ simulated mass m_l is much closer to real u, d mass m_{ud}

  $\Longrightarrow$ much smaller error from $m_l \to m_{ud}$ extrapolation

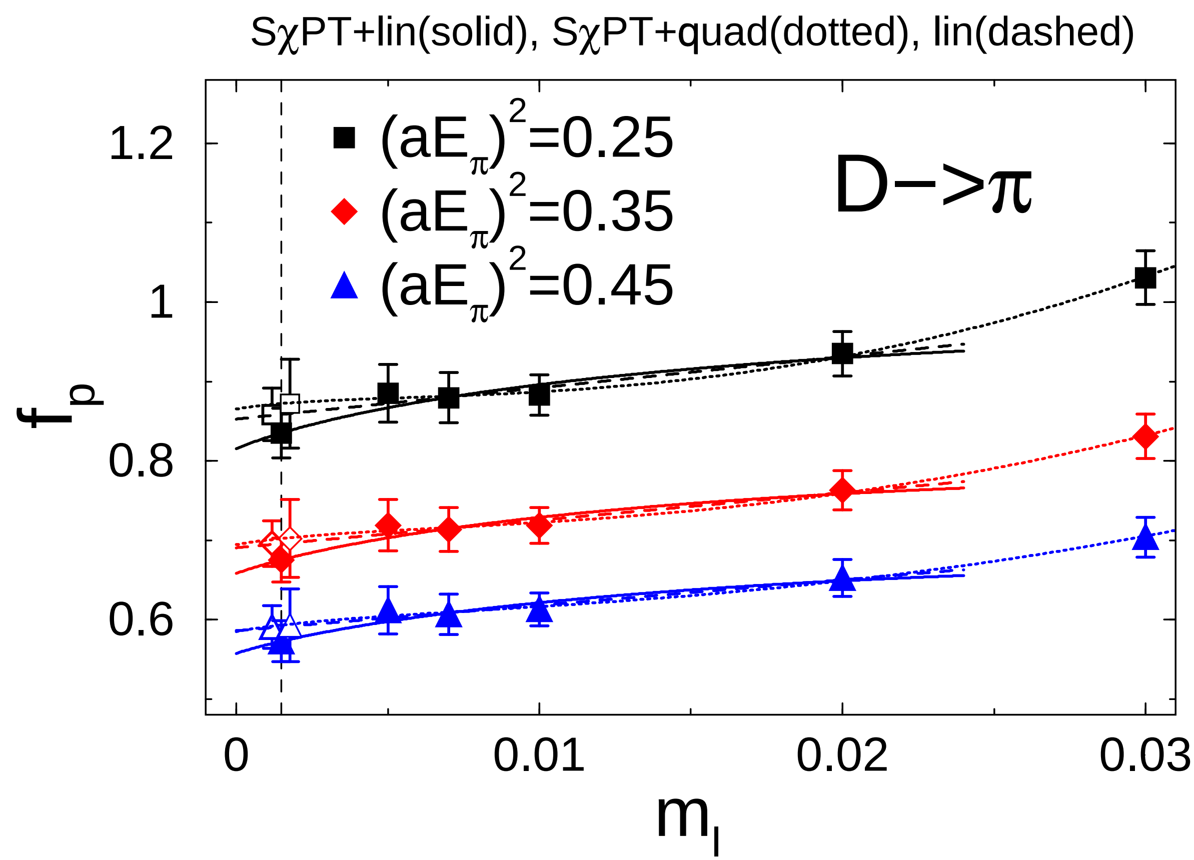
  previous(Wilson) error: $10 - 20\%$ $\Longrightarrow$ Now(staggered): $\approx 3\%$

How close can we reach to real m_{ud} quark mass ?



Wilson-type light	[FNAL'01]	$m_l/m_s \geq 1/2$	error $\approx$ 10%
staggered-type light	[this work]	$m_l/m_s < 1/2$	error $\approx$ 3%

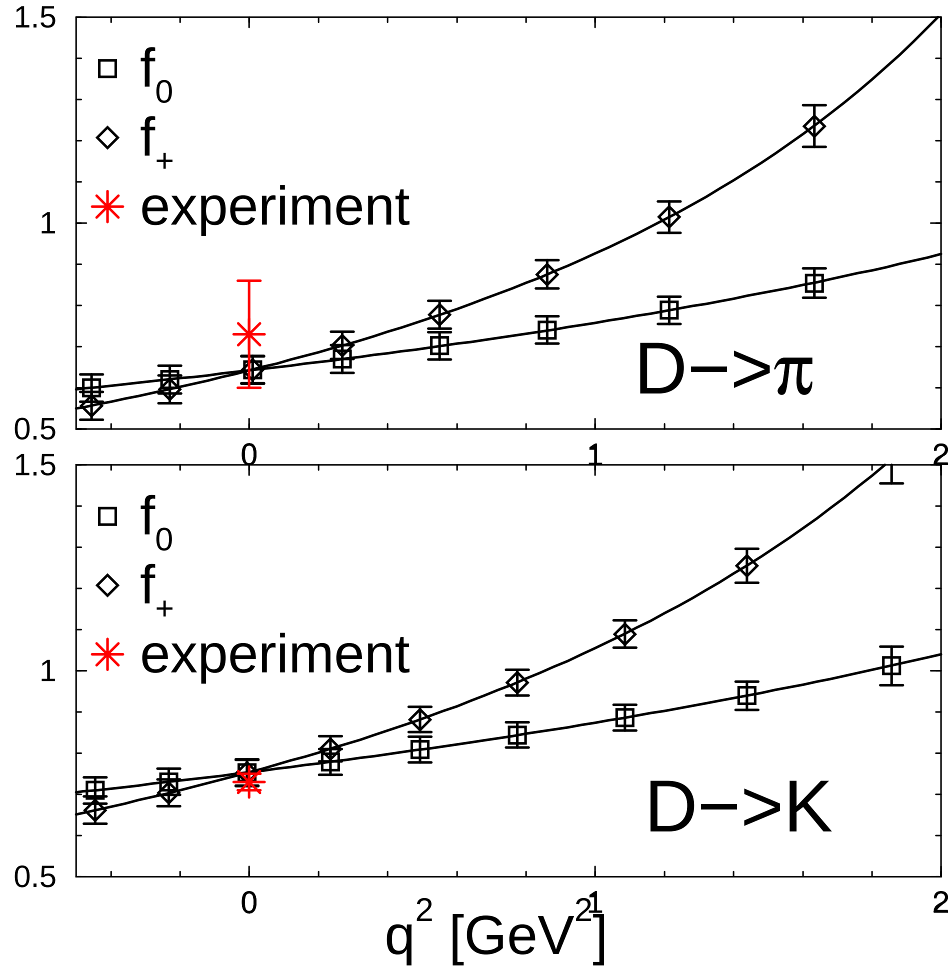
Exptrapolation of $m_l \to m_{ud}$ (“chiral fit”)



χPT+lin fit (solid)	$f = A(1+\delta f^{\chi PT}) + Bm_l$	central value
χPT+quad fit (dotted)	$f = A(1+\delta f^{\chi PT}) + Bm_l + Cm_l^2$	+(0-3)%
linear fit (dashed)	$f = A + Bm_l$	+3%

Main results

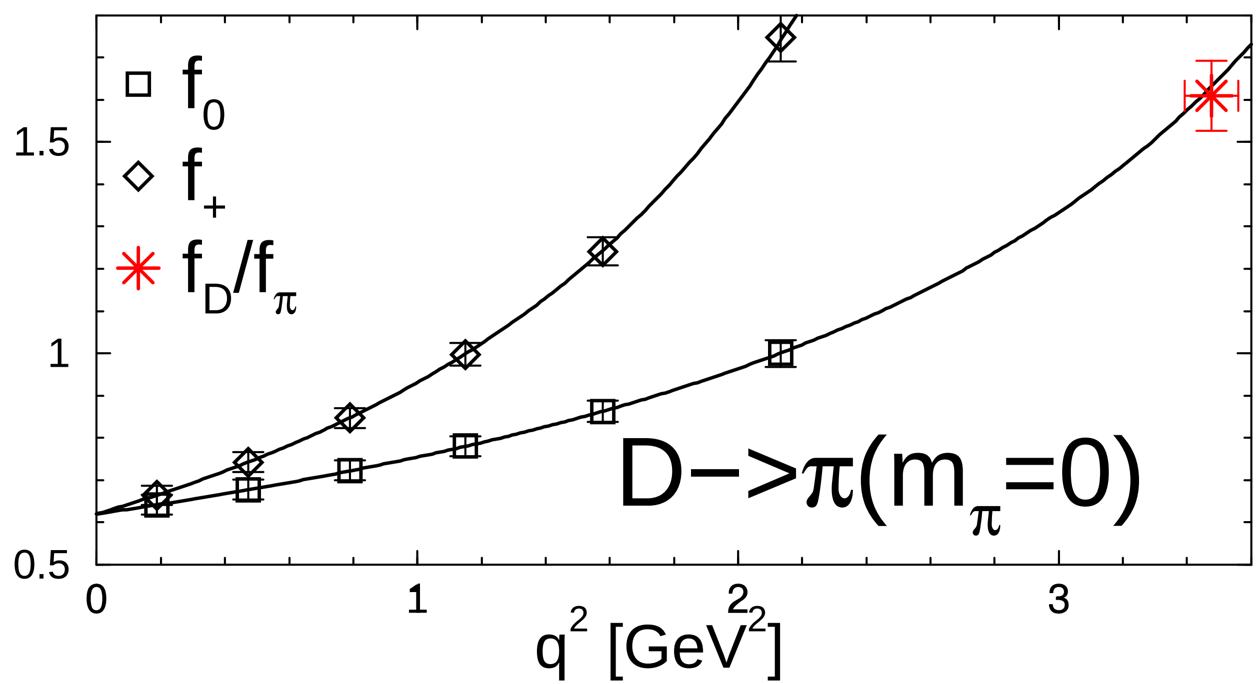
$$\langle \pi | V^{\mu} | D \rangle \quad = \quad f_+(q^2) \left[p_D + p_\pi - \frac{m_D^2 - m_\pi^2}{q^2} q \right]^{\mu} + f_0(q^2) \frac{m_D^2 - m_\pi^2}{q^2} q^{\mu}$$



$$f_+^{D\to\pi}(0) = 0.64(3)(6) \quad , \quad f_+^{D\to K}(0) = 0.73(3)(7)$$

Check for $D \to \pi$ decay:

$f_0^{D\to\pi}(q^2_{\max}) = f_D / f_\pi$ ("Soft pion relation")



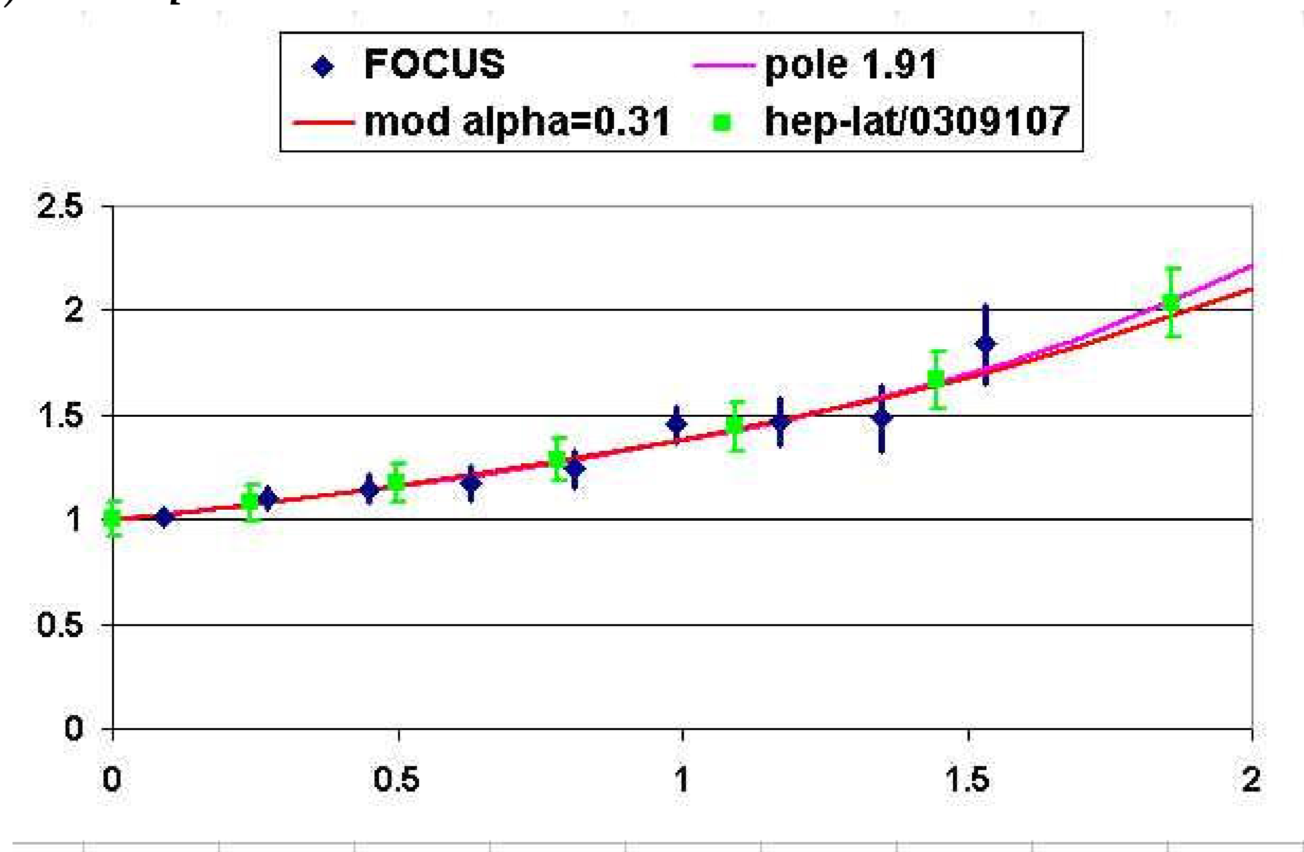
f_D: preliminary result by J. Simone *et.al*

f_π: experimental value

Comparison to FOCUS experiment for $D \to K$ decay

FOCUS result is preliminary.

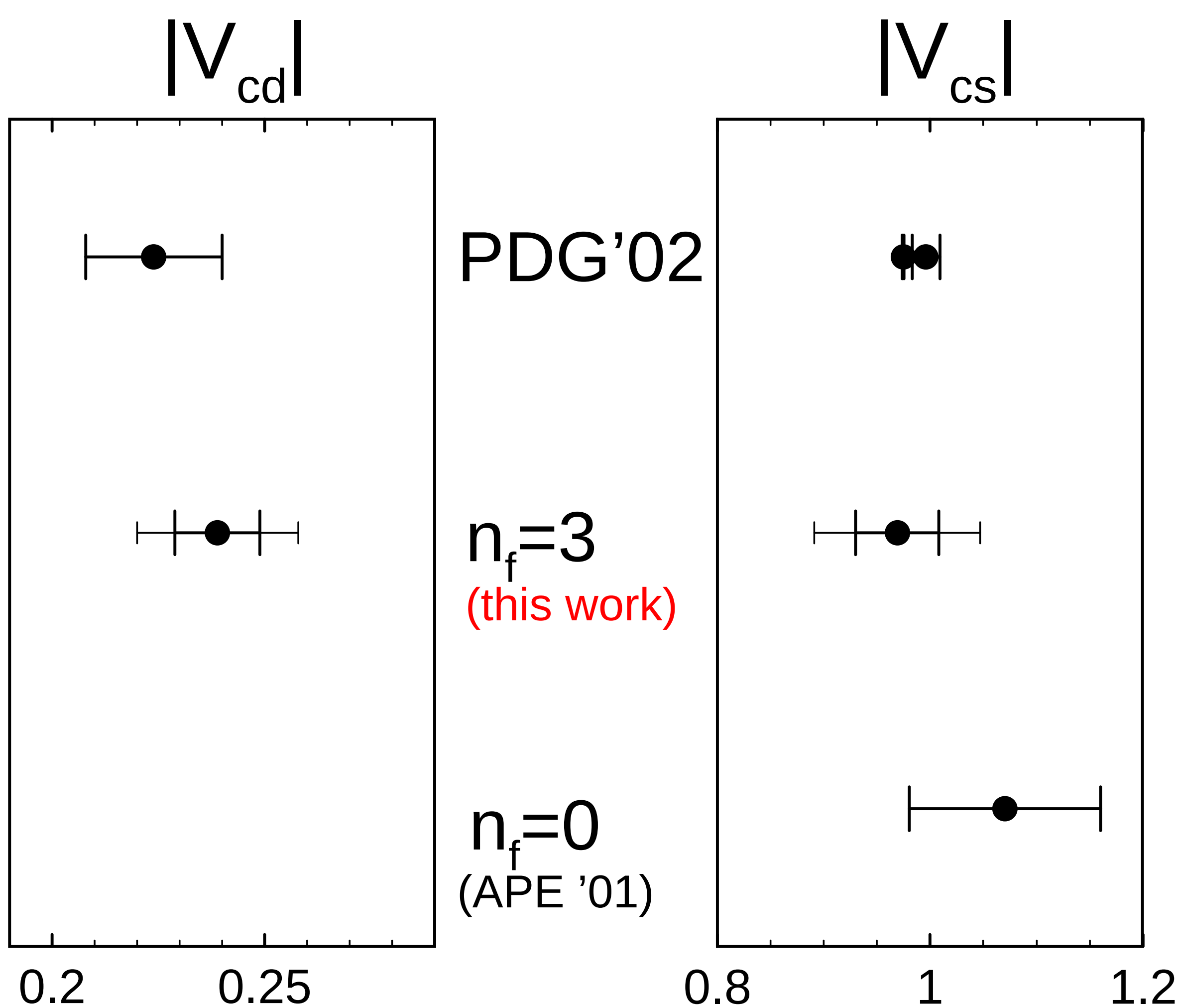
$f_+(q^2)/f_+(0)$ vs q^2:



(provided by FOCUS collab.)

CKM matrix

$$|V_{cq}|^2 = \frac{\Gamma_{\rm exp}}{(\Gamma/|V_{cq}|^2)_{\rm lat}} \quad , \quad (\Gamma/|V_{cq}|^2)_{\rm lat} \propto \int dq^2 |f_+(q^2)|^2 .$$



$$|V_{cd}| = 0.239(10)(24)(20) \quad , \quad |V_{cs}| = 0.969(39)(94)(24)$$

Systematic errors

- $m_l \to m_{ud}$ extrap. : 3 fits [SχPT fits, linear] differ by 3%.

- finite a errors ($a = 1/8$ fm)

  staggered u, d, s quark: $O(\alpha_s a^2 \Lambda^2) \approx$ 2%, $O(\alpha_s a^2 \mathbf{p}^2) \approx$ 5%

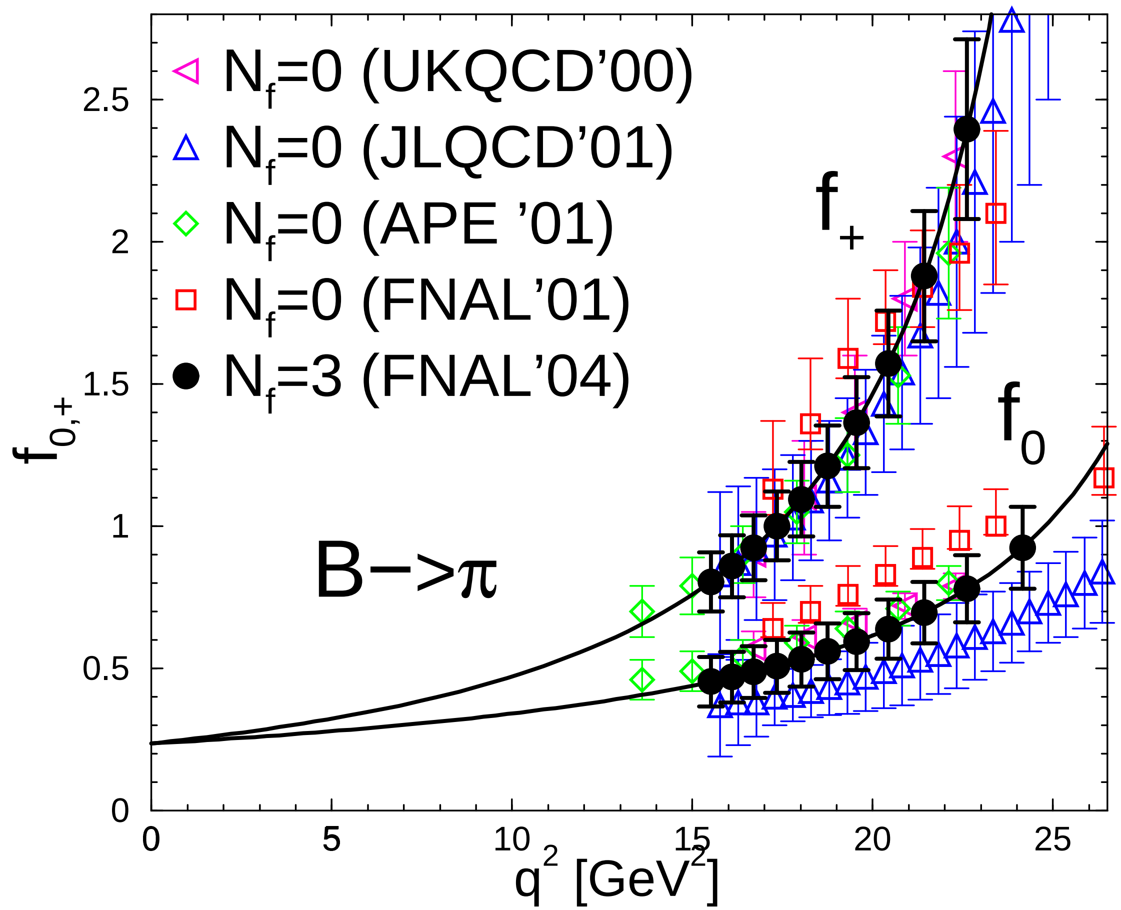
  Wilson c, b quark: $O(\alpha_s a \Lambda, a^2 \Lambda^2) \approx$ 7%

- other errors : 3-pt fitting (3%), a^{-1}(1%)

TOTAL systematic error $\approx$ (3 + 2 + 5 + 7 + 3 + 1)% $\approx$ 10%

$$|V_{cd}| = 0.239(10)(24)(20) \quad , \quad |V_{cs}| = 0.969(39)(94)(24)$$

$B \to \pi$ results (preliminary)



Using CLEO's branching ratio

$$|V_{ub}| \times 10^3 = 2.88(36)(29)(59) \quad \text{with } \mathcal{B}(q^2 \geq 16\ \mathrm{GeV}^2)$$

$$|V_{ub}| \times 10^3 = 3.60(52)(36)(30) \quad \text{with } \mathcal{B}(\text{all } q^2)$$

$B \to Dl\nu$ decay

$$\langle D|V^\mu|B\rangle = \sqrt{m_B m_D} \times [h_+(w)(v_B + v_D)^\mu + h_-(w)(v_B - v_D)^\mu],$$

where $w = v_B \cdot v_D$.

$$\frac{d\Gamma(B \to Dl\nu)}{dw} \propto |\mathcal{F}_{B\to D}(w)|^2 |\mathbf{V_{cb}}|^2$$

$$\mathcal{F}_{B\to D}(w) = h_+(w) - \frac{m_B - m_D}{m_B + m_D} h_-(w).$$

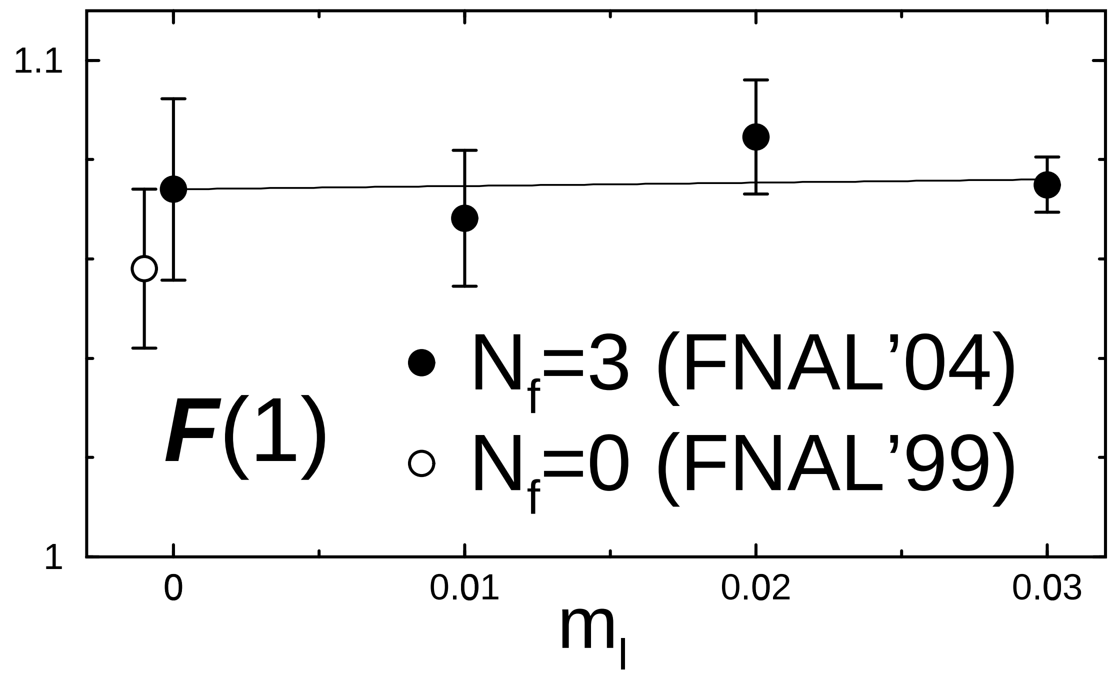
We focus on the zero recoil limit ($w = 1$).

Ratio method (S. Hashimoto *et.al.* '99)

$$\frac{C^{DV_0B}(t)C^{BV_0D}(t)}{C^{DV_0D}(t)C^{BV_0B}(t)} \to \frac{\langle D|V_0|B\rangle\langle B|V_0|D\rangle}{\langle D|V_0|D\rangle\langle B|V_0|B\rangle} = |h_+^{B\to D}(1)|^2$$

$\mathcal{F}(1) = 1$ in $B = D$ limit.

$B \to D$ results (preliminary)



$$\mathcal{F}_{B\to D}^{n_f=3}(1) = 1.074(18)(15)$$

Using Belle's branching ratio

$$|V_{cb}| \times 10^2 = 3.83(07)(06)(64)$$

Summary

First complete $n_f = 3$ LQCD results for D, B decays.

Two errors (quenching, $m_l \to m_{ud}$ extrap) are successfully reduced.

$\mathbf{D \to K l \nu}$, $\mathbf{D \to \pi l \nu}$

- $f_+^{D\to\pi}(0) = 0.64(3)(6)$ $(\Longrightarrow \mathbf{V_{cd}})$
- $f_+^{D\to K}(0) = 0.73(3)(7)$ $(\Longrightarrow \mathbf{V_{cs}})$

$\mathbf{B \to \pi l \nu}$, $\mathbf{B \to D l \nu}$ (preliminary)

- $f_+^{B\to\pi}(0) = 0.24(3)(2)$ $(\Longrightarrow \mathbf{V_{ub}})$
- $\mathcal{F}_{B\to D}(1) = 1.07(2)(2)$ $(\Longrightarrow \mathbf{V_{cb}})$

“Our” CKM matrix with $n_f = 3$ LQCD
(from semileptonic decays)

$$
\begin{pmatrix}
\mathbf{V_{ud}} & \mathbf{V_{us}} & \mathbf{V_{ub}} \\
\text{N/A} & \text{N/A} & 3.6(5)(4)(3)\times10^{-3} \\
\mathbf{V_{cd}} & \mathbf{V_{cs}} & \mathbf{V_{cb}} \\
0.24(1)(2)(2) & 0.97(4)(9)(2) & 3.8(1)(1)(6)\times10^{-2} \\
\mathbf{V_{td}} & \mathbf{V_{ts}} & \mathbf{V_{tb}} \\
\text{N/A} & \text{N/A} & \text{N/A}
\end{pmatrix}
$$

value(stat)(syst)(expt)

4/9 being determined with $n_f = 3$ LQCD.

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\mathrm{N/A} & \mathrm{N/A} & \mathrm{N/A}
\end{pmatrix}
$$

value(stat)(syst)(expt)

4/9 being determined with $n_f = 3$ LQCD.

LQCD unitarity check!

$$(|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2)^{1/2} = 1.00(4)(9)(2)$$

Future Plans

- Reduce finite a error by taking $a \to 0$ limit with *two* lattices:
  $a = 1/8$ fm (this work) and $a = 1/12$ fm (underway)

- Compare with CLEO-c exp't for CKM-independent ratio

$$\frac{d\Gamma(D \to Kl\nu)/dq^2}{\Gamma(D_s \to l\nu)} \propto \frac{|f_+(q^2)|^2}{|f_{D_s}|^2} \cdot \frac{|\not{V}_{cs}|^2}{|\not{V}_{cs}|^2}$$

  ($\Longrightarrow$ stringent check for LQCD)

- $K \to \pi l \nu$ decay ($\Longrightarrow$ $\mathbf{V_{us}}$)

Extra slides

CPU time $\propto 1/m_q$. (very slow for u, d quarks)

(3) “1-spin” Naive fermion

Reduce CPU time by “spin diagonalization”

$$\psi(x) = \Omega(x)\chi(x)\ ,\ \bar{\psi}(x) = \bar{\chi}(x)\Omega^{\dagger}(x) \tag{1}$$

$$\Omega(x) = \gamma_0^{x_0}\gamma_1^{x_1}\gamma_2^{x_2}\gamma_3^{x_3}$$

$$\bar{\psi}(x)(\gamma\cdot\Delta + m)\psi(x) = \bar{\chi}_{\alpha}(x)(\eta(x)\cdot\Delta + m)\delta_{\alpha\beta}\chi_{\beta}(x)$$

Then, drop spin index

$$\chi_{\alpha}\ \ (\alpha = 1-4)\ \Longrightarrow\ \chi$$

1-spin χ is much faster than 4-spin ψ in simulation

Oscillating state

Naive quark action has "doubling" symmetry under

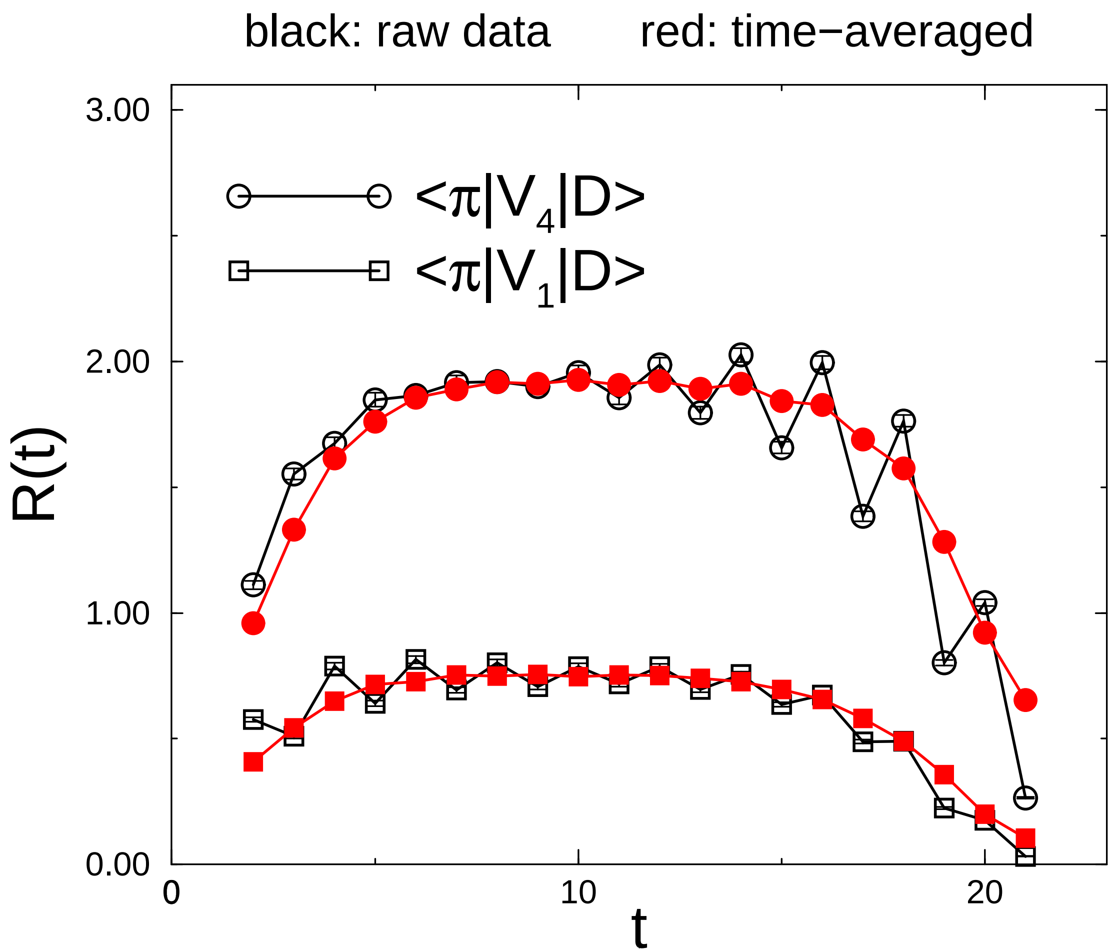
$$\psi(x) \to i\gamma_5\gamma_0(-1)^t\psi(x)$$

$\Longrightarrow$ heavy-light 2-,3-point functions have oscillating states:

$$\begin{aligned} C_{2,3}(t) &= Ae^{-E\cdot t} + (-1)^t A' e^{-(E+\Delta E)\cdot t} + \cdots, \\ R(t) &\equiv \frac{C_3^{D\to\pi}(t_x,t)}{C_2^{\pi}(t)\, C_2^{D}(t_x-t)} = \langle\pi|V_\mu|D\rangle + (-1)^t e^{-\Delta E\cdot t} \times \cdots \end{aligned}$$

method 1. take average: $\frac{R(t)+R(t+1)}{2} \sim \langle\pi|V_\mu|D\rangle + O(e^{-\Delta E\cdot t}\cdot\Delta E)$.

method 2. make fits to $C_3^{D\to\pi}$, $C_2^{\pi,D}$ separately.



black: raw data
red: time−averaged
<π|V4|D>
<π|V1|D>
R(t)
3.00
2.00
1.00
0.00
0
10
20
t

How to compute $\langle \pi | V_\mu | D \rangle$ (cont'd)

Matching between "lattice" $\leftrightarrow$ "continuum"

$$\begin{aligned} \langle \pi | V_\mu^{\mathrm{cont}} | D \rangle &= Z_{V_\mu}^{hl} \, \langle \pi | V_\mu^{\mathrm{lat}} | D \rangle \\ Z_{V_\mu}^{hl} &\equiv \rho_{V_\mu} \sqrt{Z_V^{hh} Z_V^{ll}} \quad (\rho_{V_\mu} \approx 1) \end{aligned}$$

quasi-non-perturbative determination for $Z_{V_\mu}^{hl}$

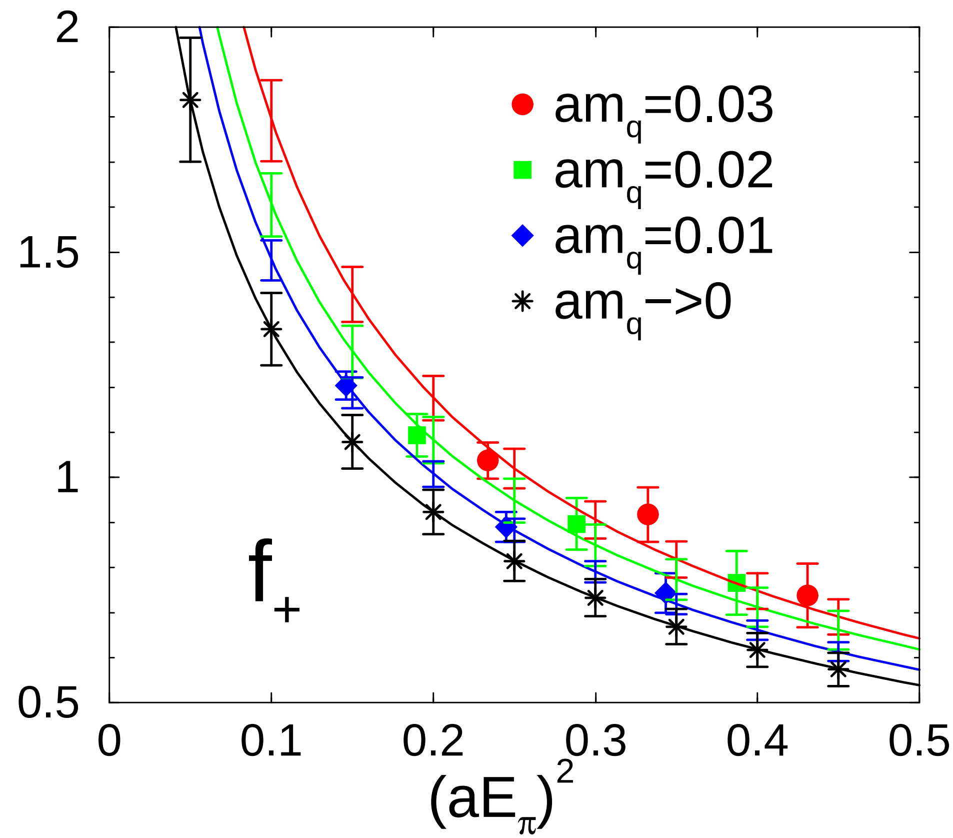
- compute Z_V^{hh}, Z_V^{ll} non-perturbatively from:

$$Z_V \, \langle D | V_4^{qq} | D \rangle = 1 \quad (qq = hh, ll)$$

- compute ρ_{V_μ} perturbatively.
  $O(\alpha_s) \approx 2\%$, $O(\alpha_s^2) \ll 1\%$.
  (1-loop PT by M. Nobes *et.al.*)

Results

$$\langle \pi | V^\mu | D \rangle = f_+(q^2) \left[p_D + p_\pi - \frac{m_D^2 - m_\pi^2}{q^2} q \right]^\mu + f_0(q^2) \frac{m_D^2 - m_\pi^2}{q^2} q^\mu$$



fit form (Becirevic and Kaidalov '99) :

$$f_+(q^2) = \frac{F}{(1 - q^2/M_{D^*})(1 - Aq^2)} \;,\; f_0(q^2) = \frac{F}{(1 - Bq^2)}$$