High precision measurements and the minimal scale

Constraining models with Large eXtra Dimensions through high precision experiments

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Publications

- *This talk:* S. Hossenfelder, M. Bleicher, S. Hofmann, J. Ruppert, S. Scherer, H. Stöcker: Phys.Lett. B575 (2003) 85-99; U. Harbach, S. Hossenfelder, M. Bleicher, H. Stöcker: Phys.Lett.B584:109-113,2004
- *LXD model:* N. Arkani-Hamed, S. Dimopoulos, G.R. Dvali: Phys.Lett.B429:263-272,1998
- *Minimal scale:* A. Kempf, G. Mangano, R.B. Mann: Phys.Rev.D52:1108-1118,1995; A. Smailagic, E. Spallucci, T. Padmanabhan: hep-th/0308122
- *g-2 data:* M. Davier, S. Eidelman, A. Hocker, Z. Zhang: Eur.Phys.J.C31:503-510,2003

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Large eXtra Dimensions

Motivation

The hierarchy problem: Gravity is much weaker than all other forces. Why?

$$F_{grav} = G m_1 m_2 \frac{1}{r^2} = \frac{m_1}{m_{Planck}} \frac{m_1}{m_{Planck}} \frac{1}{r^2}$$

$\frac{m}{m_Planck} \ll 1$ for all elementary particles.

Motivation

Idea for a solution:

- Spacetime has more than 3+1 dimensions
- Gravity penetrates all dimensions, SM fields are bound to our submanifold
- Since we don’t see them, Extra dimensions are compactified, but with a larger radius

The way it works

On distances below the compactification radius R, the higher-dimensional gravitational potential follows from Gauss' law:

$$G(r) \sim \frac{1}{M_f^{d+2} r^d} \frac{m_2}{r^2} \quad (r \ll R)$$

For distances above, the graviational force cannot further propagate into the extra dimensions, and the usual Newtonian potential is recovered:

$$G(r) \sim \frac{1}{m_{Planck}^2} \frac{m_2}{r^2} \quad (r \gg R)$$

The way it works

From the continuity condition at the compactification radius

$$\frac{1}{M_f^{d+2} R^d} \frac{m_2}{R^2} = \frac{1}{m_{Planck}^2} \frac{m_2}{R^2}$$

follows the relation between the fundamental and the Planck scale:

$$m_{Planck}^2 = M_f^{2+d}$$

Features

The LXD model's most remarkable feature is:

- The Planck scale is a derived, not a fundamental scale. The new fundamental scale M_f is significantly smaller.

The Minimal Scale

Motivation

FACT: Spacetime distance resolution has a lower limit!

This emerges from both

- String theory T-duality and
- combination of quantum theory and general relativity

Motivation

Imagine a particle probed by another particle.

- To resolve a spacetime distance a, the probing particle must have wave number (and momentum) $p = k = 1/a$.
- As its energy increases, it disturbs the spacetime region it is supposed to probe.
- One obtains a lower limit, typically of order $l_{Planck} = 1/m_{Planck}$.

Minimal Scale in LXDs

In models with Large eXtra Dimensions, the minimal scale is significantly higher!

$$L_f \sim 1/M_f$$

This increased minimal length scale leads to observable phenomena.

Quantum theory with minimal scale

Inclusion into quantum theory

Full theory including both quantum theory and gravitation does not (yet) exist.

$\Rrightarrow$ To achieve experimental prediction, the notion of a minimal length scale must be included manually into quantum theory.

Inclusion into quantum theory

How to do this?
Answer: Simply restrict allowed wavelengths to be greater than L_f, and thus:

$$k < \frac{1}{L_f}$$

Since momentum should not have an upper bound, this leads to a modification of the de Broglie-relations.

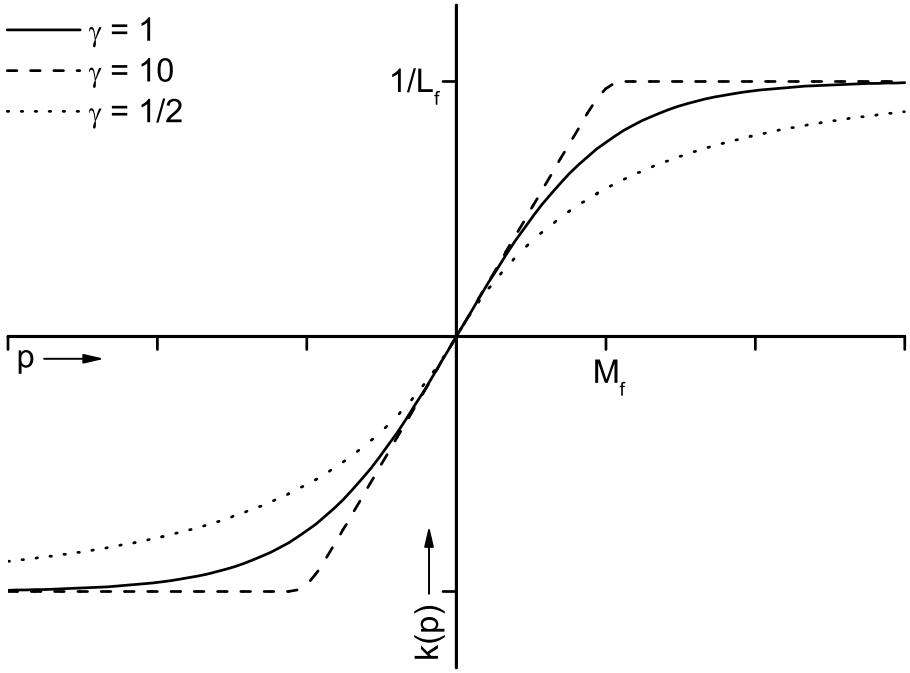
Inclusion into quantum theory

What do we expect from such a relation $k(p)$?

- upper limit $1/L_f$
- uneven (because of parity)
- linear behaviour for small k

Inclusion into quantum theory

A possible choice:



$$L_f k^\mu(p^\mu) = \tanh^{\frac{1}{\gamma}}\left[\left(\frac{p^\mu}{M_f}\right)^\gamma\right]$$

Inclusion into quantum theory

The new relation changes the basics of quantum theory:

Operators

$$
\begin{aligned}
\langle x|\hat{x}^\mu|x\rangle &= x^\mu \\
\langle x|\hat{k}^\mu|x\rangle &= -\,\mathrm{i}\,\frac{\partial}{\partial x^\mu} \\
\langle x|\hat{p}^\mu|x\rangle &= p\left(-\,\mathrm{i}\,\frac{\partial}{\partial x^\mu}\right)
\end{aligned}
$$

Inclusion into quantum theory

Commutators

$$\begin{aligned}\langle x|\left[\hat{x}^\mu, \hat{k}^\nu\right]|x\rangle &= \mathrm{i}\,\delta^{\mu\nu} \\ \langle x|\left[\hat{x}^\mu, \hat{p}^\nu\right]|x\rangle &= \mathrm{i}\,\delta^{\mu\nu}\frac{\partial p^\mu}{\partial k^\mu}\end{aligned}$$

Normalisation

$$\langle p'|p\rangle = \langle k(p')|k(p)\rangle = \delta\big(k(p') - k(p)\big) = \frac{\partial p}{\partial k}\delta(p' - p)$$

Momentum space measure

$$dp \to \frac{\partial k}{\partial p}dp$$

Inclusion into quantum theory

Does it make sense?

Let's look at some features one would expect from a theory with a minimal length:

Inclusion into quantum theory

Does it make sense?

Let's look at some features one would expect from a theory with a minimal length:

The 'natural' smooth cut-off

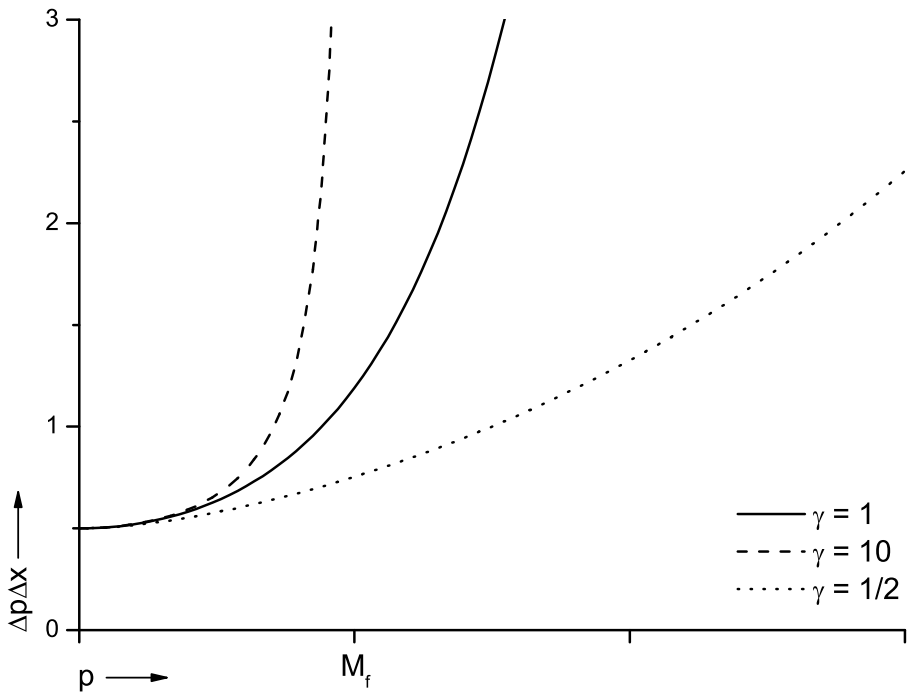
$$\frac{\partial k}{\partial p} = \frac{1}{L_f}\frac{1}{\cosh^2(\frac{p}{M_f})} = \frac{4}{L_f}\frac{1}{\left(\exp(\frac{p}{M_f}) + \exp(-\frac{p}{M_f})\right)^2}$$

$$\xrightarrow{p\to\infty} \frac{4}{L_f}\frac{1}{\exp^2(\frac{p}{M_f})}$$

Inclusion into quantum theory

The uncertainty relation

$$\Delta p \Delta x \geqq \frac{1}{2}\left|\frac{\partial p}{\partial k}\right| = \frac{1}{2}\frac{1}{1 - L_f^2 k^2(p)}$$



High precision measurements

First-order approximation

Since we are interested in the first effects, we approximate the $p(k)$ relation for small momentum:

$$p^\mu(k^\mu) \approx \left(1 + \frac{1}{3} L_f^2 |k^\mu|^2\right) k^\mu$$

The resulting first-order momentum operator (in one dimension) is:

$$\langle x|\hat{p}|x\rangle \approx -\,\mathrm{i}\left(1 - \frac{1}{3} L_f^2 \frac{\partial^2}{\partial x^2}\right) \frac{\partial}{\partial x}$$

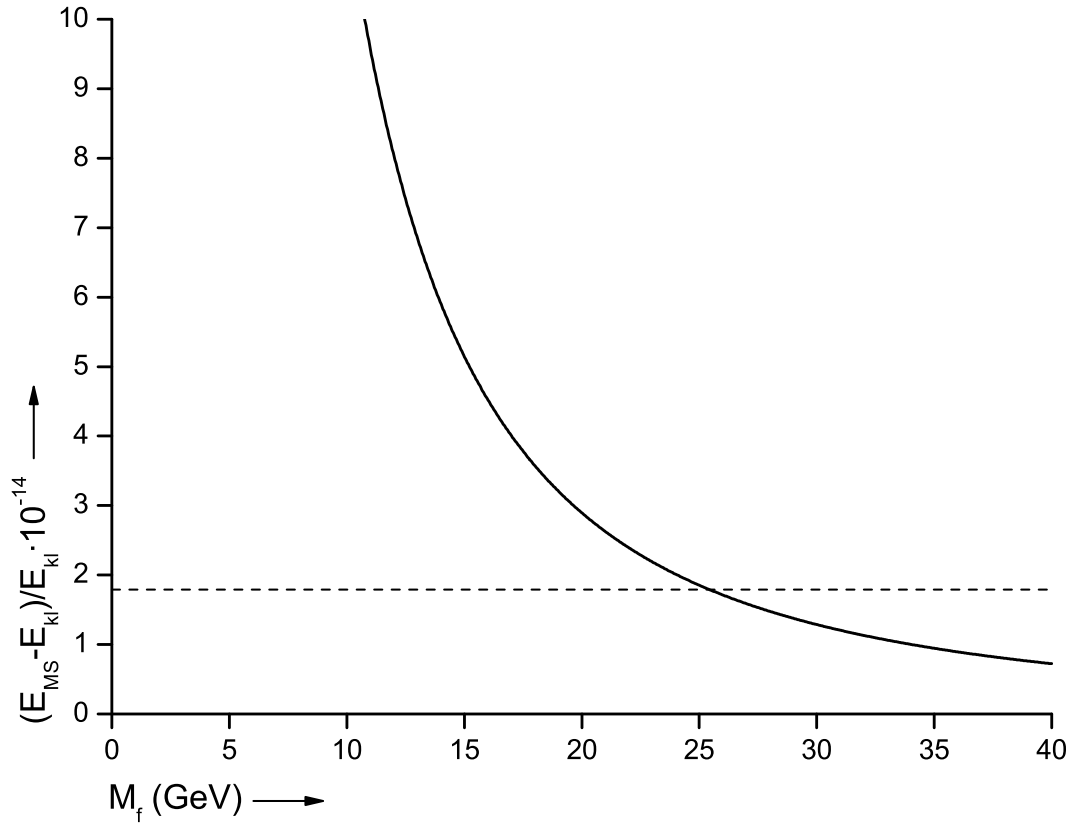
The Hydrogen atom

The Hydrogen S1-S2 transition energy is known to very high precision. The correction of the energy levels due to the minimal length scale is given by:

$$E_n \approx -\frac{e^4 m}{2n^2}\left(1 - \frac{e^4 m^2}{M_f^2 n^2}\right)$$

The Hydrogen atom

A plot of the relative deviation:



The muon gyromagnetic moment

The inclusion of the minimal length modifications into the Dirac equation

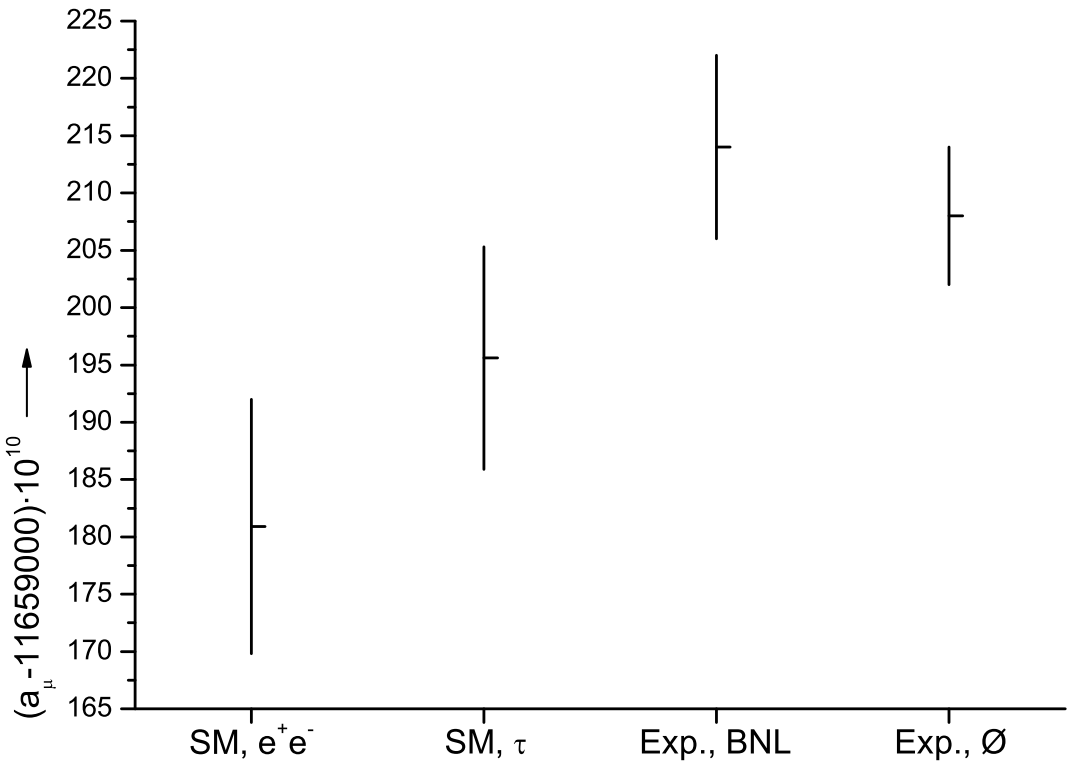
$$\mathrm{i}\,\frac{\partial}{\partial t}\psi(\vec{r}) \approx \left(-\mathrm{i}\left(1-\frac{1}{3}L_f^2 m^2\right)\vec{\alpha}\cdot\vec{\nabla}+\left(1+\frac{1}{3}L_f^2\left(\triangle-m^2\right)\right)\beta m\right)\psi(\vec{r})$$

leads (after some algebra) to a modified gyromagnetic moment:

$$g \approx g_{SM}\left(1-\frac{1}{3}\frac{m^2}{M_f^2}\right)$$

The muon gyromagnetic moment

The Standard Model and experimental data:



The muon gyromagnetic moment

The present data clearly indicates that modifications due to effects beyond the Standard Model (which includes the minimal length scale) have to be smaller than 10^{-8}. This leads to a limit on the fundamental scale:

$$M_f \gtrsim 500\,\mathrm{GeV}$$

Conclusions

- In models with LXDs, the minimal scale increases significantly and thus has to be considered when making experimental predicitions.
- It provides a new mean of constraining those models.
- Present data on the muon gyromagnetic moment already sets a limit on the fundamental scale.