D-Strings and the Warped Deformed Conifold

Christopher Herzog

KITP, UCSB

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Motivation

To learn about strongly coupled gauge theories from geometry

- The warped deformed conifold solution (Klebanov, Strassler) or KS solution is a gravitational model of a confining $\mathcal{N} = 1$ supersymmetric $SU(N)$ gauge theory.

To learn about cosmic strings

- The KS solution embedded in a compact Calabi-Yau manifold is a cosmological model where D-strings and fundamental strings at the bottom of the throat can be interpreted as cosmic strings (KKLT, Copeland, Jackson, Jones, Myers, Polchinski).

Mystery

What is the D-string in the KS solution?

1. The low energy theory on a D3-brane is a 3+1 dimensional gauge theory
2. D3-brane + f-string = D3-brane w/ electric flux
3. D3-brane + D-string = D3-brane w/ magnetic flux

If f-strings are tubes of confined electric flux (Klebanov, Strassler), how can D-strings be tubes of confined magnetic flux?
S. Gubser, C. H., I. Klebanov, "Symmetry Breaking and Axionic Strings in the Warped Deformed Conifold", hep-th/0405282.

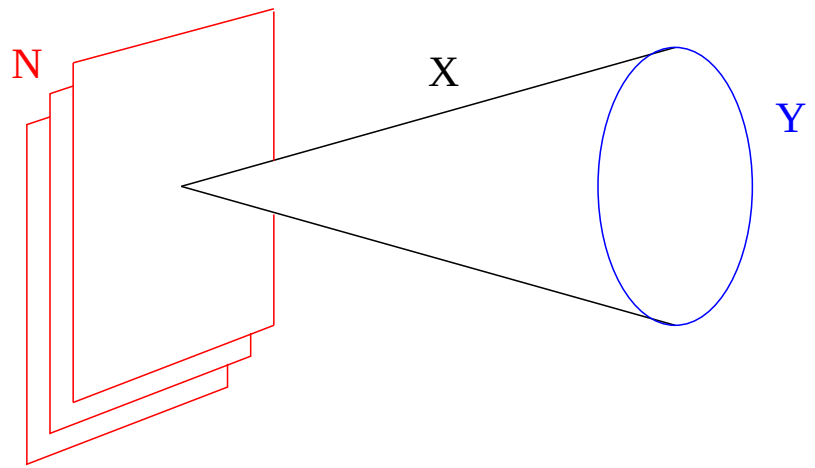
Main Point

- The KS solution has a global $U(1)_B$ baryon number symmetry.
- This symmetry can be broken spontaneously by giving expectation values to certain baryonic operators.
- The corresponding Goldstone boson is a pseudoscalar, and the D-strings create a monodromy for this massless field.

(Still hope to recover a pure $\mathcal{N} = 1$ $SU(N)$ theory in the $g_{YM}^2 N \to 0$ limit.)

Constructing KS

Place a stack of N D3-branes at the tip of a six dimensional Calabi-Yau cone X



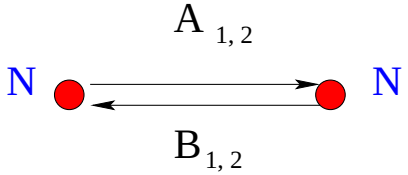
- $X = \mathbb{C}^3 \rightarrow$ original AdS/CFT correspondence
- $X =$ conifold $\rightarrow$ KS solution (Klebanov, Witten, . . .)

The Conifold

- $z_1^2 + z_2^2 + z_3^2 + z_4^2 = 0$ where $z_i \in \mathbb{C}$
- The conifold is a cone; $X = \mathbb{R}^+ \times T^{1,1}$
- $T^{1,1}$ topologically is $S^2 \times S^3$

The Conifold Gauge Theory

$\mathcal{N} = 1$ $SU(N) \times SU(N)$ with bifundamental matter



$$W = \epsilon^{ij}\epsilon_{kl}\mathrm{Tr}A_iB_kA_jB_l$$

- $U(1)_R$: $A_i \to e^{i\alpha}A_i$; $B_j \to e^{i\alpha}B_j$; $z_k \to e^{2i\alpha}z_k$
- $U(1)_B$: $A_i \to e^{i\beta}A_i$; $B_j \to e^{-i\beta}B_j$; $z_k \to z_k$

Breaking Conformal Invariance

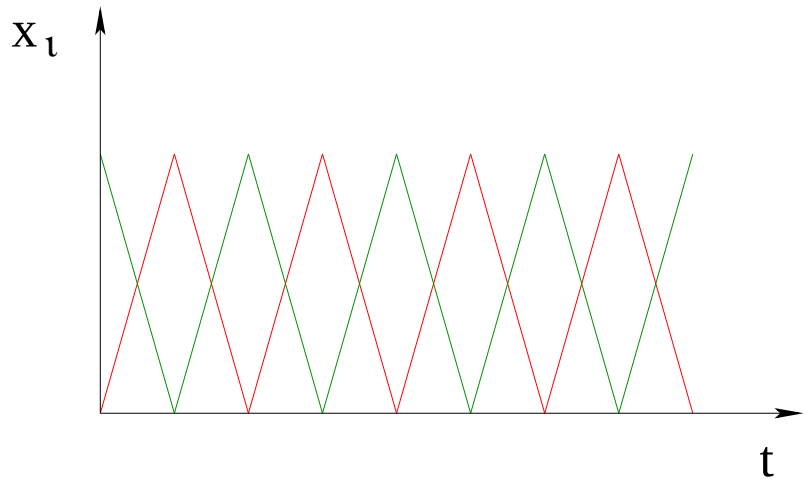
We add M D5-branes wrapped on the S^2 (Klebanov, Nekrasov, Tseytlin)

$$SU(N) \times SU(N) \rightarrow SU(N+M) \times SU(N)$$

The β functions are no longer zero

$$\beta_1 = 3M \; ; \quad \beta_2 = -3M$$

The KS Cascade



$x_i = 8\pi^2/g_i^2$ and $t = \ln(\mu/\Lambda)$

Seiberg Duality

$$\begin{array}{ccccc} SU(N) & \rightarrow & SU(N) & ; & \beta_1 \rightarrow 3M \\ SU(N+M) & \rightarrow & SU(N-M) & ; & \beta_2 \rightarrow -3M \end{array}$$

Uncertainties in field theory analysis alleviated by KS supergravity solution.
Cascade steps small for large $g_s M$.

Confinement and Chiral Symmetry Breaking

- D5-branes wrapped on S^2 imply F_3 flux through the transverse S^3
- Deforming the conifold $\sum_i z_i^2 = \epsilon$ gives the S^3 a finite size at the tip of the cone
- $U(1)_R$ which sends $z_i \to e^{2i\alpha} z_i$ is broken to $\mathbb{Z}_2$

Strings have finite tension, confinement

Starting with $SU(nM) \times SU((n+1)M)$ in the UV leads to $SU(M)$ in the IR. However, SUGRA can be trusted at $g_s M >> 1$ where the cascade steps are very small. Extra degrees of freedom from theories in the cascade with more colors are easily accessible.

Thinking about $U(1)_B$

D-strings are solitonic strings which create a monodromy of a massless pseudoscalar Goldstone boson.

- Recall $U(1)_B$ acts as $A_i \to e^{i\beta} A_i$ and $B_j \to e^{-i\beta} B_j$
- Possible to form gauge invariant, baryon-like operators out of antisymmetric products of the A's and B's, e.g. take $SU(2M) \times SU(M)$ theory

$$\begin{aligned}
\mathcal{B} &\sim \epsilon_{\alpha_1 \alpha_2 \ldots \alpha_{2M}} (A_1)_1^{\alpha_1} (A_1)_2^{\alpha_2} \ldots (A_1)_M^{\alpha_M} \\
&\qquad (A_2)_1^{\alpha_{M+1}} (A_2)_2^{\alpha_{M+2}} \ldots (A_2)_M^{\alpha_{2M}} \ , \\
\bar{\mathcal{B}} &\sim \epsilon^{\alpha_1 \alpha_2 \ldots \alpha_{2M}} (B_1)_{\alpha_1}^1 (B_1)_{\alpha_2}^2 \ldots (B_1)_{\alpha_M}^M \\
&\qquad (B_2)_{\alpha_{M+1}}^1 (B_2)_{\alpha_{M+2}}^2 \ldots (B_2)_{\alpha_{2M}}^M \ .
\end{aligned}$$

Spontaneously Breaking $U(1)_B$

Superpotential for $SU(2M) \times SU(M)$ (Seiberg)

$$W \sim M^2 + \lambda(\det M - \mathcal{B}\bar{\mathcal{B}} - (\Lambda_{2M})^{4M})$$

where $M \sim A_i B_j$ and λ is a Lagrange multiplier.

SUSY vacuum for

$$M = 0\,, \quad \lambda = 0\,, \quad \mathcal{B}\bar{\mathcal{B}} = -(\Lambda_{2M})^{4M}$$

$$\mathcal{B} = i\xi(\Lambda_{2M})^{2M}\,; \quad \bar{\mathcal{B}} = \frac{i}{\xi}(\Lambda_{2M})^{2M}$$

The pseudoscalar corresponds to the phase of ξ.

Pseudoscalar as SUGRA perturbation

- Asymptotic form matches baryon current for $AdS_5 \times T^{1,1}$ (Klebanov, Witten, Ceresole, Dall'Agata, D'Auria, Ferrara)

$$\delta F_3 \neq 0 \; ; \quad \delta F_5 \neq 0$$

- Satisfies all SUGRA equations of motion to first order
- Normalizable perturbation

Supersymmetry means the pseudoscalar belongs to a full chiral $\mathcal{N} = 1$ multiplet with a **complex** scalar.

We think we have found the scalar partner of this Goldstone boson in supergravity

$$\delta G_{\mu\nu} \neq 0 \; ; \quad \delta B_2 \neq 0$$

Recovering pure $\mathcal{N} = 1$ $SU(N)$ Yang-Mills

- In the limit $g_s M \to 0$, $\Lambda_{2M} >> \Lambda_M$
- "Pion" decay constant $f_\pi \sim \Lambda_{2M} >> \Lambda_M$
- Goldstone boson interactions suppressed by $\Lambda_M / f_\pi << 1$

One tentatively concludes the pseudoscalar should decouple at small $g_s M$.

Cosmic Strings

- If we embed the KS solution in a compact Calabi-Yau, the $U(1)_B$ should become gauged (gauge coupling $\sim 1/V_6$).
- We expect the spontaneous breaking should become the Higgs mechanism.
- The D-strings become Abrikosov-Nielsen-Olesen vortices of an Abelian-Higgs model in this context.

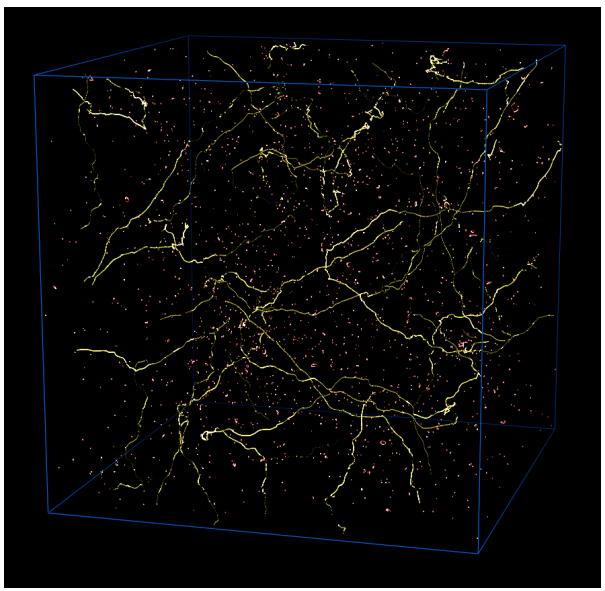


Figure: Computer simulation of the cosmic string network in the matter dominated era (Allen, Shellard).

Directions for Future Research

1. We worked out the Goldstone boson to first order in the supergravity equations of motion. Can the full one complex parameter set of solutions be found? (Papadopoulos, Tseytlin)
2. There is another global symmetry $SO(4)$ associated to the KS solution. Is there a Goldstone boson associated with breaking this symmetry? Halmagyi and Corrado suggest that these SUGRA solutions might interpolate between the Pilch-Warner solution and the KS solution.
3. The supersymmetry of the KS solution can be softly broken by giving the gaugino a mass. Does this process introduce tachyons? (Sonnenschein, Schvellinger)