

# Transverse Momentum Fluctuations at RHIC

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## **I. Fluctuations – nonequilibrium two-body correlations**

- ▶  $p_t$  fluctuation observables.
- ▶ approach to equilibrium.
- ▶ initial and near-equilibrium fluctuations.
- ▶ Effects of Jets & Flow.

# Systematic Case for Thermalization

**clues:** thermalized parton matter at RHIC?

- ▶ jets, energy loss – interacting partonic system
- ▶ flow – pressure due to thermalized partons
- ▶ particle ratios – local equilibrium in central collisions

**wanted:** more info on **phase space correlations**

- ▶ long range properties of correlation functions.
- ▶ fluctuations: simple way to characterize correlations.

# Dynamic Fluctuations

**dynamic = variance – thermal contribution** [Pruneau, Voloshin and Sean Gain](#)

multiplicity  $N$

$$R_{AA} = \frac{\langle N^2 \rangle - \langle N \rangle^2 - \langle N \rangle}{\langle N \rangle^2}$$

mean  $p_t$

$$\langle \delta p_{t1} \delta p_{t2} \rangle \equiv \frac{1}{\langle N_{\text{pairs}} \rangle} \left\langle \sum_{\text{pairs } i \neq j} \delta p_{ti} \delta p_{tj} \right\rangle$$

$$\delta p_t \equiv p_t - \langle p_t \rangle$$

**probe two-body correlations:**

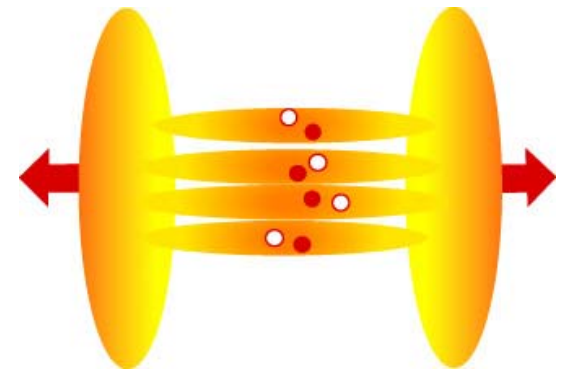
$$r(p_1, p_2) = n(p_1, p_2) - n(p_1)n(p_2)$$

$$R_{AA} \propto \iint r(p_1, p_2)$$

$$\langle \delta p_{t1} \delta p_{t2} \rangle \propto \iint \delta p_{t1} \delta p_{t2} r(p_1, p_2)$$

# Time Scales

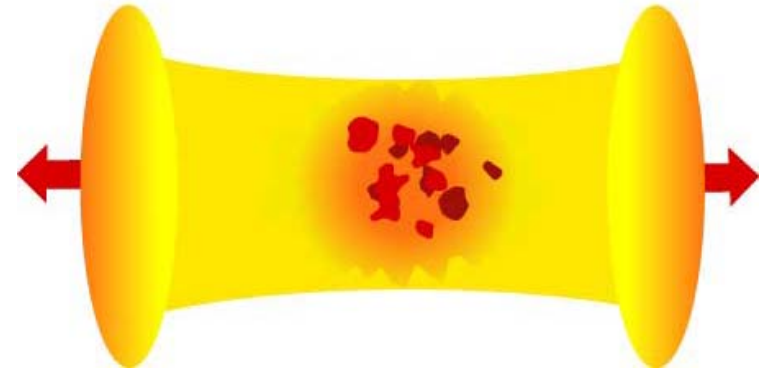
**initially** – string fragmentation



**later** – clumps, size  $\sim \xi$

▶ local thermalization; time  $\sim \nu^{-1}$

scattering rate  $\nu = \langle \sigma v_{rel} \rangle n$

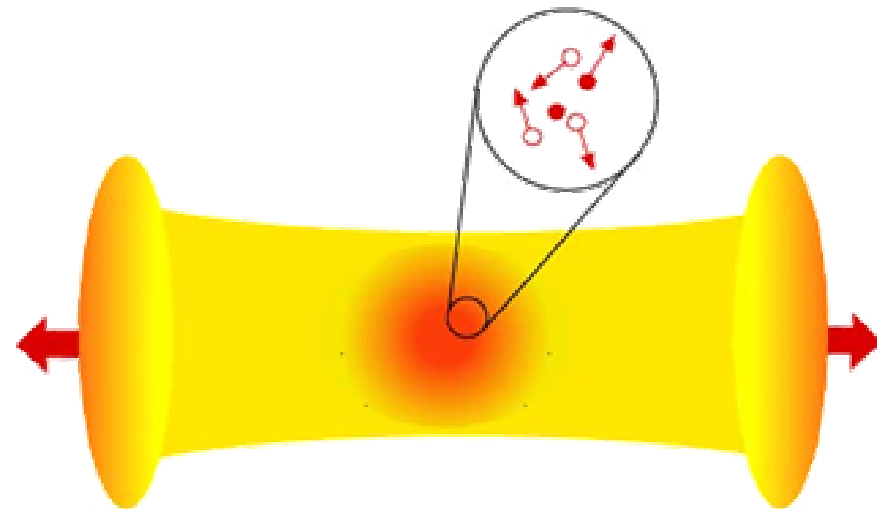


**much later** (if ever)

▶ flow between clumps  $\rightarrow$

homogeneity

▶ diffusion time  $t_{diff} \sim \nu \xi^2$



# Fluctuation Sources

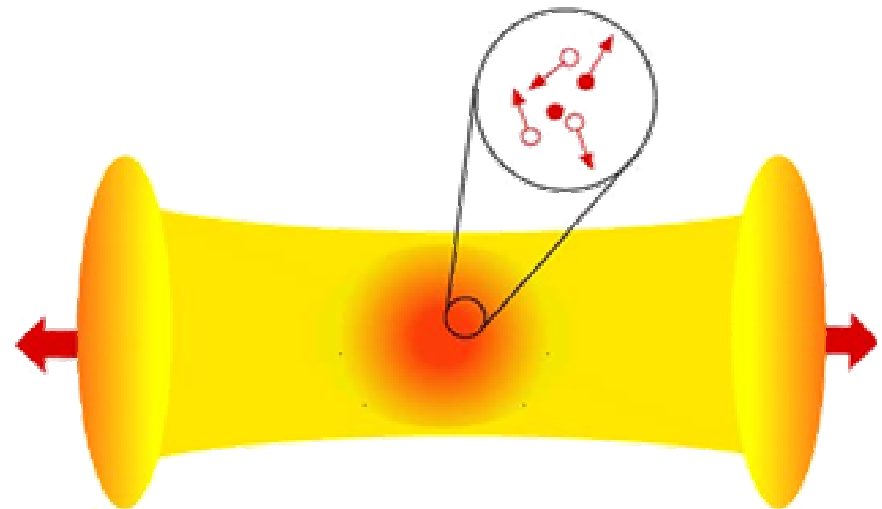
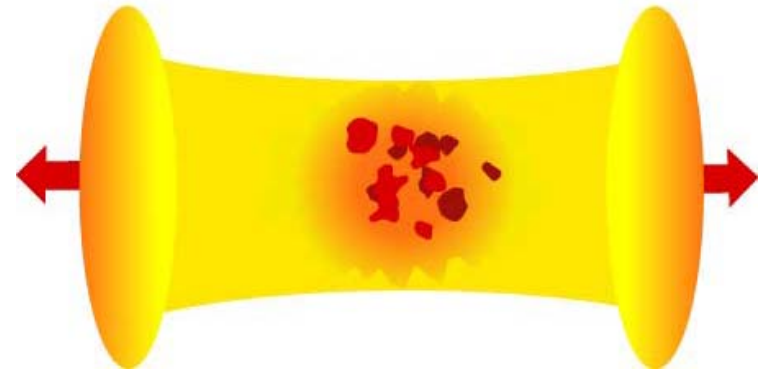
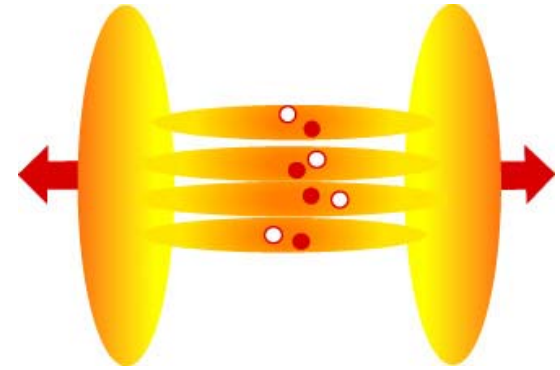
dynamic fluctuations  $\sim (\text{strings})^{-1}$

dynamic fluctuations  $\sim (\text{clumps})^{-1}$

-- larger? depends on clump size

statistical fluctuations  $\sim (\text{particles})^{-1}$

dynamic fluctuations **vanish**



# Approach to Equilibrium

Boltzmann Equation for phase space density  $f(x,p)$

$$\frac{\partial f}{\partial t} + \vec{v}_{\vec{p}} \cdot \vec{\nabla} f = -\nu(f - f^e)$$

approximation: relaxation time  $\nu^{-1}$

Bjorken scaling:  $f = f_0(p'_z, p_t, s)S(t, t_0) + \int_{t_0}^t f^e(p'_z, p_t, s)S(t, s)\nu ds$

**Baym; Gavin; B. Zhang & Gyulassy**

**where  $p'_z = p_z(t/t_0)$**

survival probability –  
chance of no scattering

$$S(t, t_0) = e^{-\int_{t_0}^t \nu ds}$$

$$\langle p_t \rangle = \langle p_t \rangle_0 S + \langle p_t \rangle_e (1 - S)$$

$$\langle p_t \rangle_e \propto \left( \frac{\tau_0}{\tau_F} \right)^\gamma$$

# Partial Thermalization

random walk – n collisions

$$\langle p_t^2 \rangle = \langle p_t^2 \rangle_0 + \kappa(n-1)$$

collision: many walkers bouncing off each other – energy conservation limits  $p_t$  increase

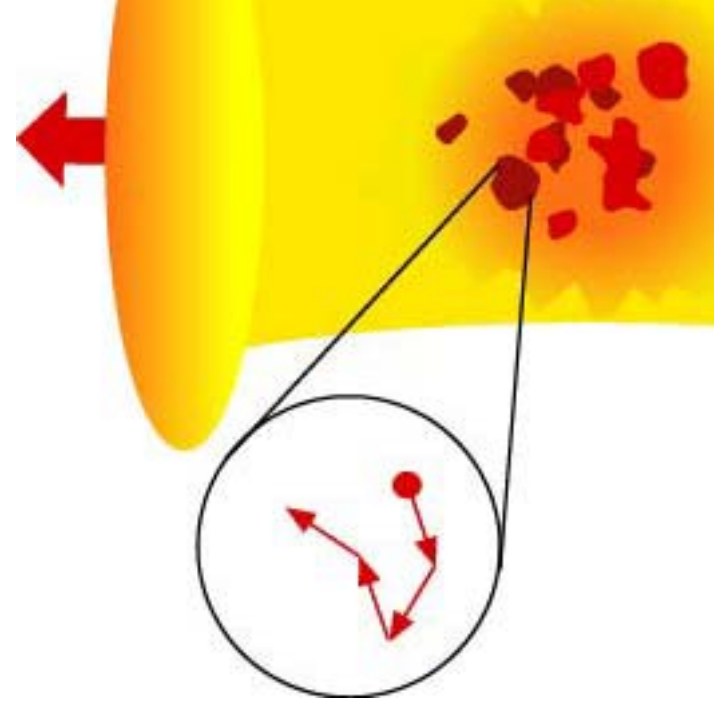
Boltzmann equation  $\Rightarrow$  
$$\langle p_t^2 \rangle = \langle p_t^2 \rangle_0 S + \langle p_t^2 \rangle_e (1-S)$$

Survival probability: 
$$S = e^{-N_{scat}} \approx (\tau_0 / \tau_F)^\alpha$$

$$\alpha = \sigma v_{rel} n_0 \tau_0$$

few collisions 
$$\langle p_t^2 \rangle_e - \langle p_t^2 \rangle_0 \sim \kappa$$

many collisions – saturates at 
$$\langle p_t^2 \rangle_e$$



# Nonequilibrium $\langle p_t \rangle$

more participants  $N \Rightarrow$  longer lifetime  $\Rightarrow$  smaller survival probability  $S$

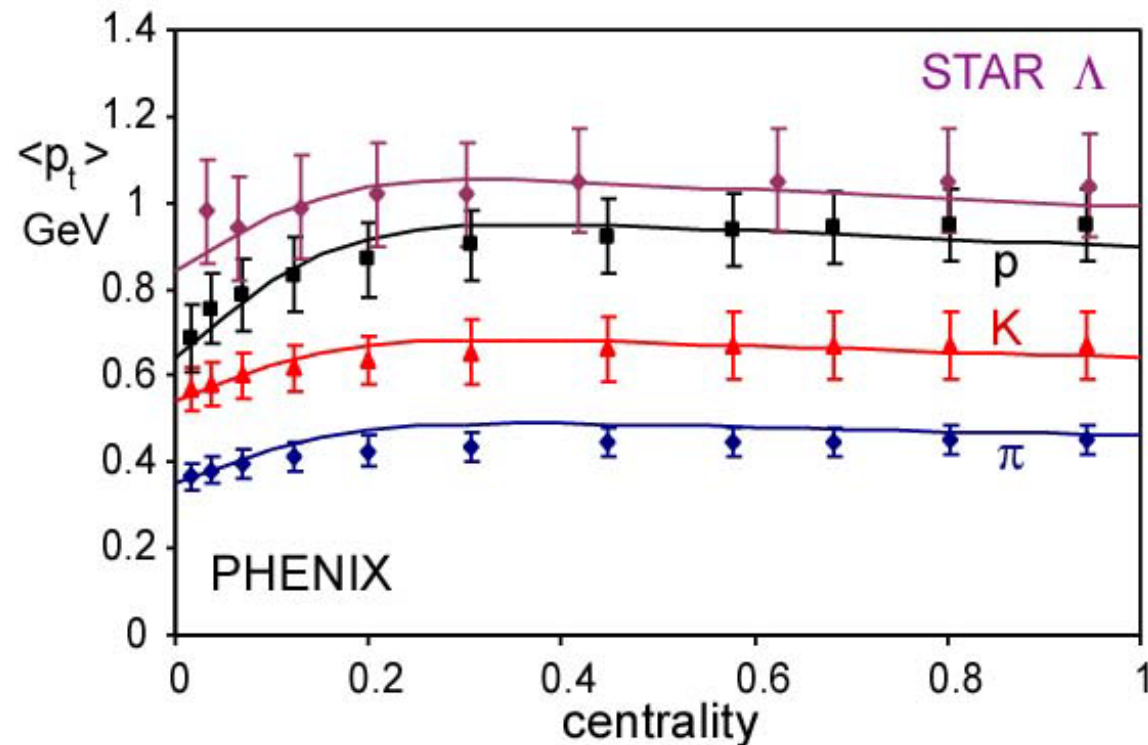
$$\langle p_t \rangle = \langle p_t \rangle_0 S + \langle p_t \rangle_e (1 - S)$$

S. Gavin, Phys.Rev.Lett. 92 (2004) 162301

▶ centrality dependence

$$\tau_F \propto N^x, \alpha \propto N^y$$

$$\tau_F(0) = 6 \text{ fm}, \tau_0 = 1 \text{ fm}, T_0 = 450 \text{ MeV}$$
$$\alpha(0) = 4, \gamma = 0.15, x = 1, y = 1/2$$



# Nonequilibrium Dynamic Fluctuations

Boltzmann equation with Langevin noise  $\Rightarrow$  phase-space correlations  $\Rightarrow$  dynamic fluctuations

simple limits:

independent initial and near-local equilibrium correlations  $\langle \delta p_{t_1} \delta p_{t_2} \rangle = \langle \delta p_{t_1} \delta p_{t_2} \rangle_0 S^2 + \langle \delta p_{t_1} \delta p_{t_2} \rangle_{le} (1 - S^2)$

**same** centrality dependence of  $S$

# Initial Fluctuations

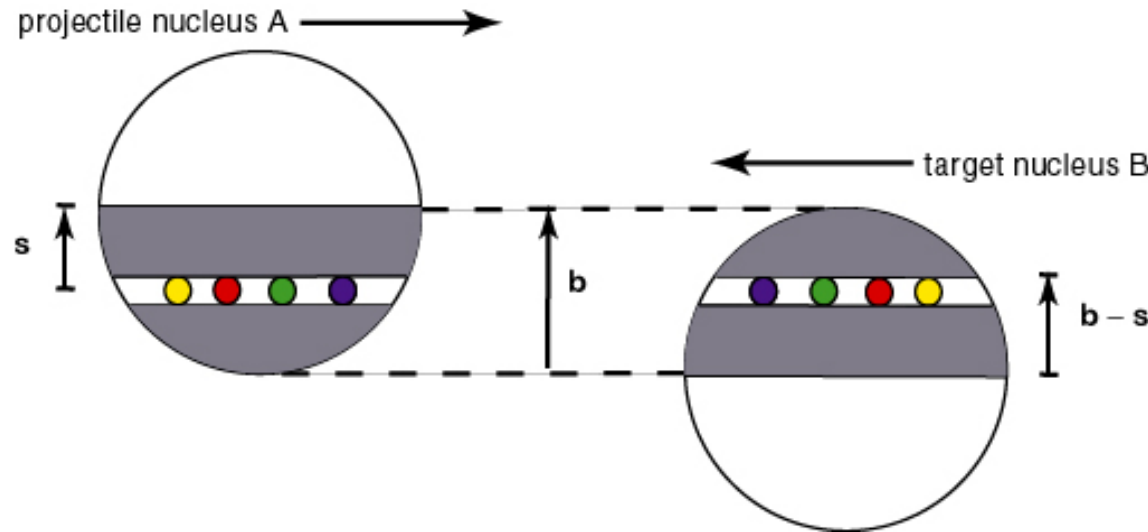
**AA collision:**

**$M$  participant nucleons**

→ independent strings

multiplicity  $\propto M$

variance  $R_{AA} \propto M^{-1}$



$$\langle \delta p_{t1} \delta p_{t2} \rangle \equiv \frac{1}{\langle N(N-1) \rangle} \left\langle \sum_{i \neq j} \delta p_{ti} \delta p_{tj} \right\rangle$$

$$\langle N(N-1) \rangle = \langle N \rangle^2 (1 + R_{AA})$$

$$\langle \delta p_{t1} \delta p_{t2} \rangle \approx \frac{2 \langle \delta p_{t1} \delta p_{t2} \rangle_{pp}}{M} \left( \frac{1 + R_{pp}}{1 + R_{AA}} \right)$$

ISR data  $\rightarrow \langle \delta p_{t1} \delta p_{t2} \rangle_{pp}$ ; HIJING  $R_{pp}, R_{AA}$

# Near Local Equilibrium Correlations

single particle phase space distribution:

$$f(p, x) = e^{-(E - \vec{p} \cdot \vec{v})/T(x)}, \quad \vec{v} \ll 1$$

particle correlations

$$P^e(x_1, p_1, x_2, p_2) = f(p_1, x_1) f(p_2, x_2) A$$

$$A(x_1, p_1, x_2, p_2) = r(x_1, x_2)$$

density correlations

# Fluctuations Near Equilibrium

**correlations** from non-uniformity – more likely to find particles near “hot spots” → spatial correlation function

$$\langle \delta p_{t1} \delta p_{t2} \rangle_e \propto \iint \delta T(x_1) \delta T(x_2) r(x_1, x_2) dx_1 dx_2$$

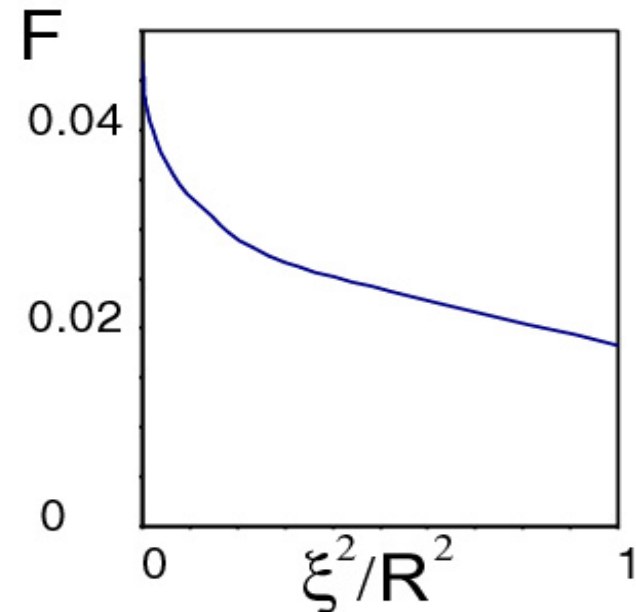
**assume:**

- ▶ longitudinal Bjorken expansion up to freeze out time
- ▶ independent longitudinal and transverse d.o.f.
- ▶  $\delta p_t \propto \delta T(x)$  independent of rapidity
- ▶ gaussian densities, correlation function

**obtain:**

$$\langle \delta p_{t1} \delta p_{t2} \rangle_e \approx \frac{\langle p_t \rangle^2 R_{AA}}{1 + R_{AA}} F(\xi / R_t)$$

transverse size  $R_t$ , correlation length  $\xi$



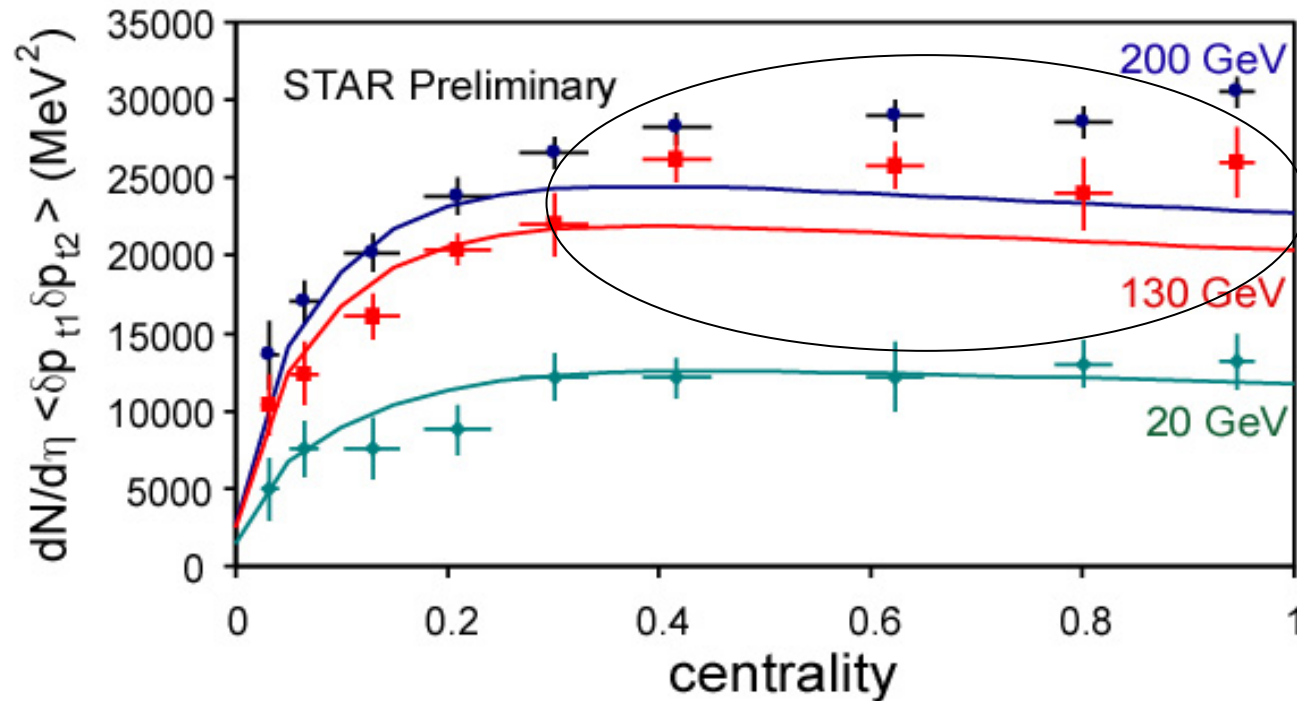
# Nonequilibrium Fluctuations

S. Gavin, Phys.Rev.Lett. 92 (2004) 162301

**initial fluctuations** –  
wounded nucleons

**near equilibrium**  
correlation length  
 $\xi \sim 1 \text{ fm}; R \propto N^{1/2}$

**survival probability**  
 $S$  vs. centrality  
from  $\langle p_t \rangle$  data



energy  $\rightarrow$  multiplicity  $N \rightarrow$  scattering  $\alpha \propto N$ , number variance  $R_{AA} \propto N^{-1}$

**FIND:** partial thermalization describes trends.

Deviation in central collisions at highest energies – **JETS/and or FLOW?**

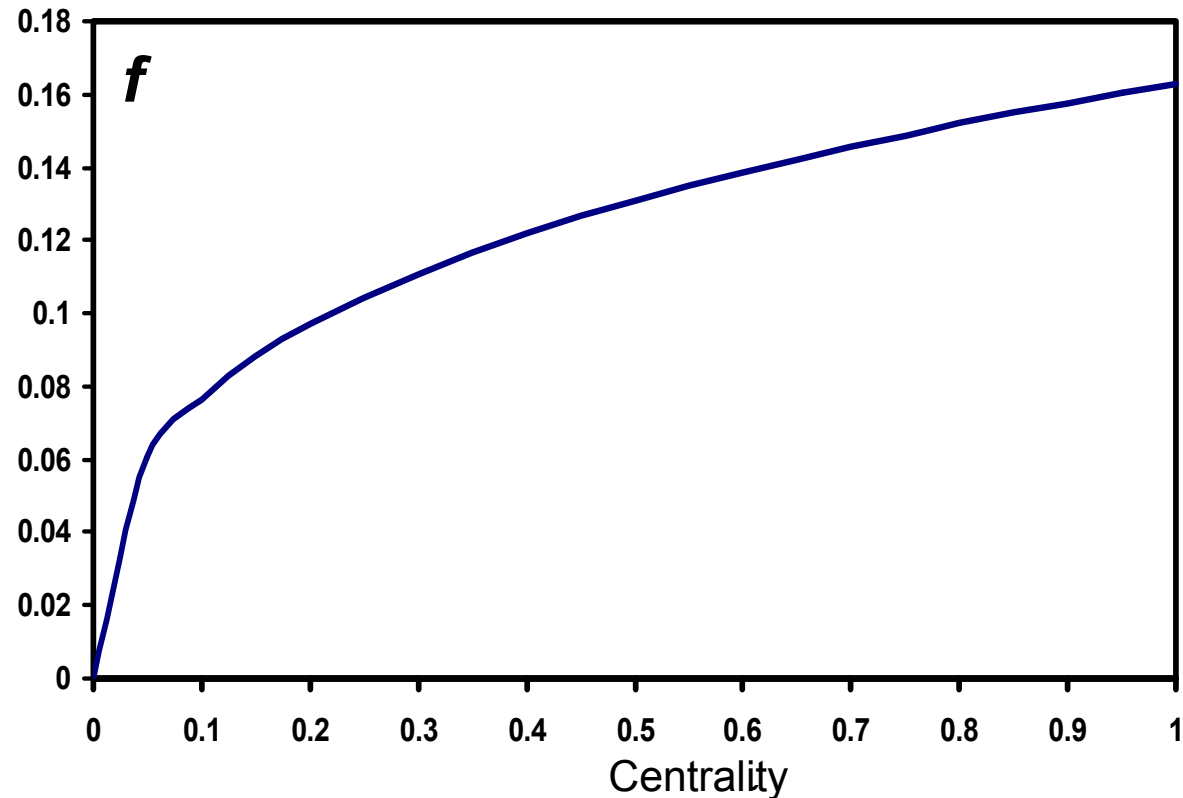
# Effect of Jets

$$f = \frac{\text{hard production}}{\text{soft production}}$$

D. Kharzeev, M. Nardi, Phys. Lett. B507 (2001) 121-128

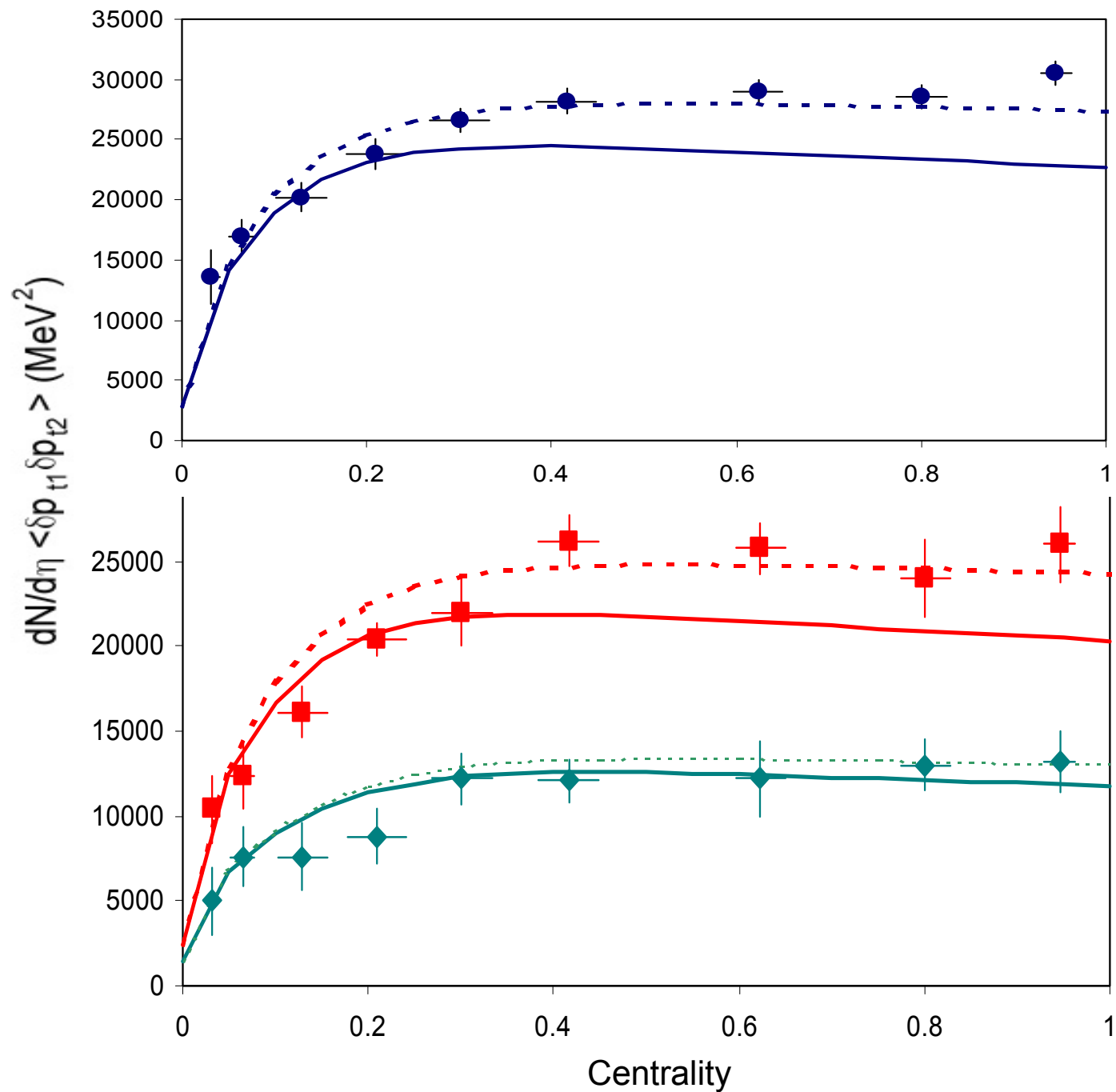
$$\Delta p \propto k_h n(x) \Delta x$$

Xin-Nian Wang. arXiv:nucl-th/045029



$$\delta T(x) \propto \delta n(x) \text{ "work in progress"}$$

$$\langle \delta p_{t1} \delta p_{t2} \rangle \approx \left[ 1 - f^2 \left( 1 - \left( \frac{k_h}{k_s} \right)^2 \right) \right] \langle \delta p_{t1} \delta p_{t2} \rangle_{no \ jets}$$

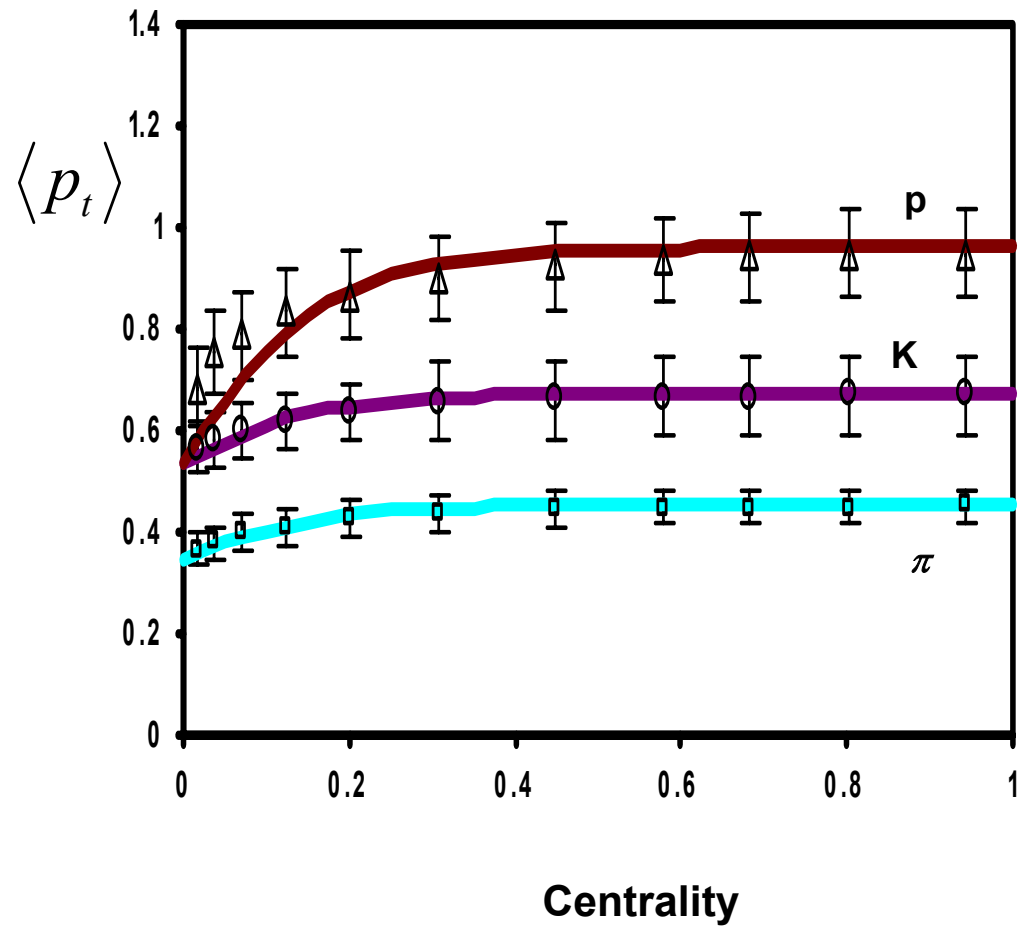


# Effect of Flow

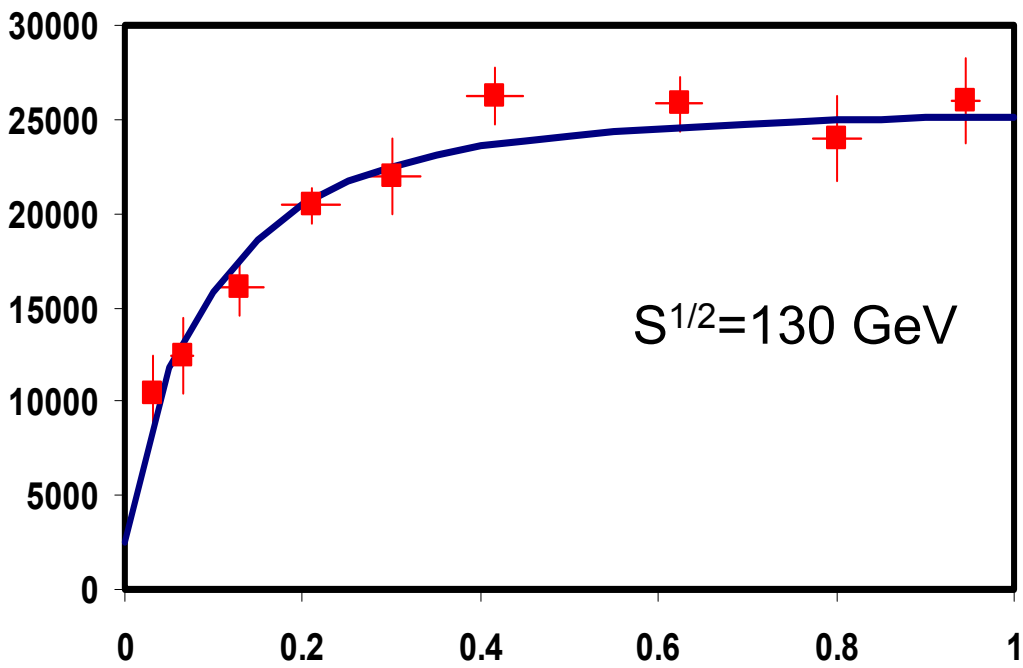
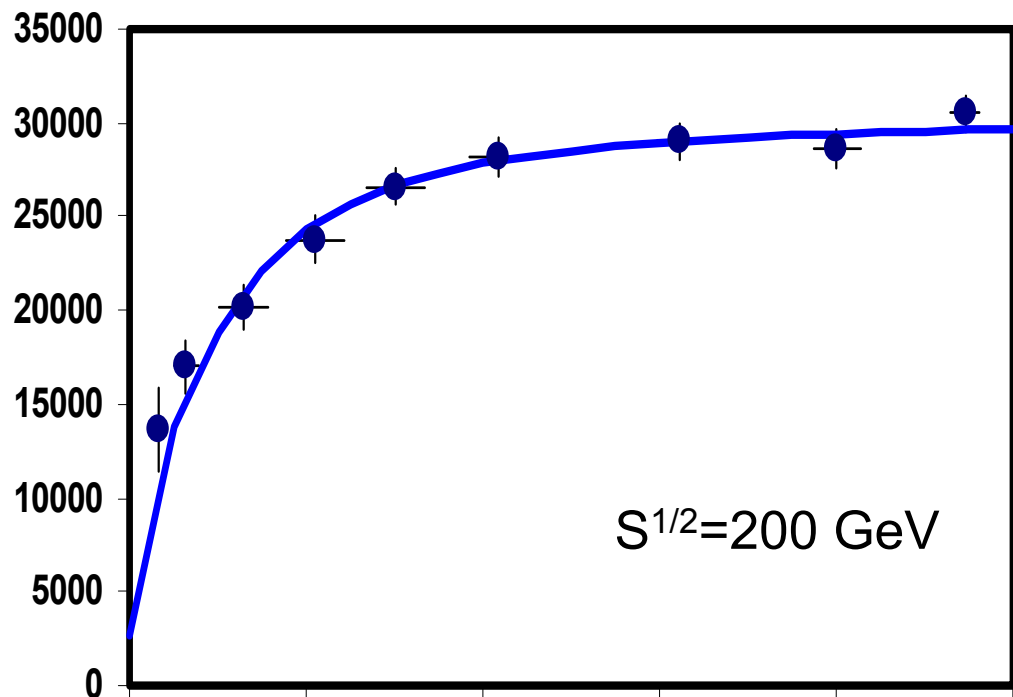
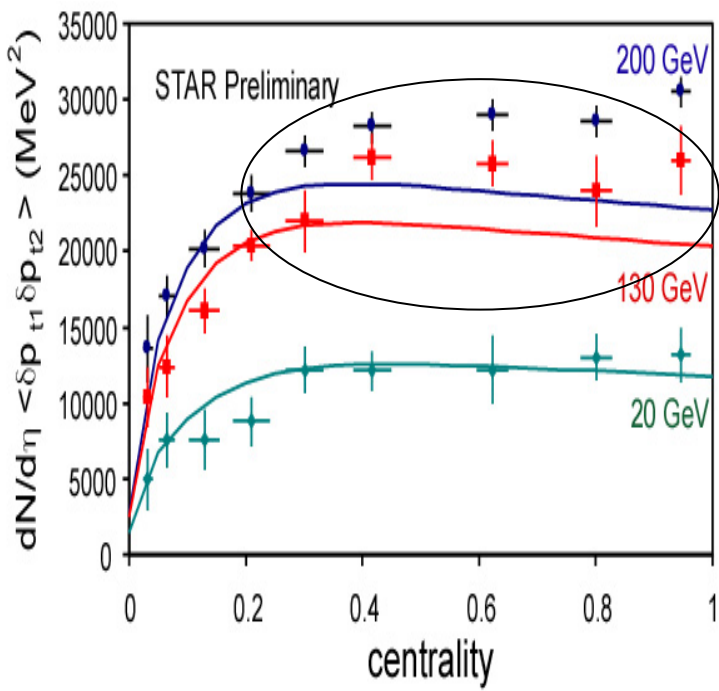
$$\langle \beta \rangle = 0.45 \pm 0.1$$

Olga Barannikova, nucl-ex/0403014

$$\langle p_t \rangle = \langle p_t \rangle_0 S + \langle p_t \rangle_e \left( \frac{1 + \beta}{1 - \beta} \right)^{\frac{1}{2}} (1 - S)$$



$$\langle \delta p_{t1} \delta p_{t2} \rangle_e = \left( \frac{1+\beta}{1-\beta} \right) \langle p_t \rangle^2 \frac{R_{AA}}{1+R_{AA}} F(\xi/R_t)$$



# Summary

- 1- Flow and jet can affect the fluctuation studies of  $\langle p_t \rangle$
- 2- Centrality dependence of the fluctuations, seems to be the same as the  $\langle p_t \rangle$