Describing recently discovered narrow states as quarkonia using a potential model

Stanley F. Radford
Department of Physics
Marietta College
and
Wayne W. Repko
Department of Physics and Astronomy
Michigan State University

MICHIGAN STATE UNIVERSITY

Outline

1. Introduction.
2. Overview of potential model approaches.
3. Results for a semi-relativistic model.
4. Conclusions

1. Introduction

With the recent experimental results for several expected states (η'_c and h_c) in the charmonium spectrum, and the discovery of a state (X(3872)), which could be a $^3D_2$ charmonium level, it seems an appropriate time to revisit the potential model interpretation of the $c\bar{c}$ spectrum. For such models, the challenges seem to be:

- Are potential models capable of describing the spin splitting in a quantitatively satisfactory way?

- Including the additional data, is it possible to determine the Lorentz properties of the phenomenological confining potential?

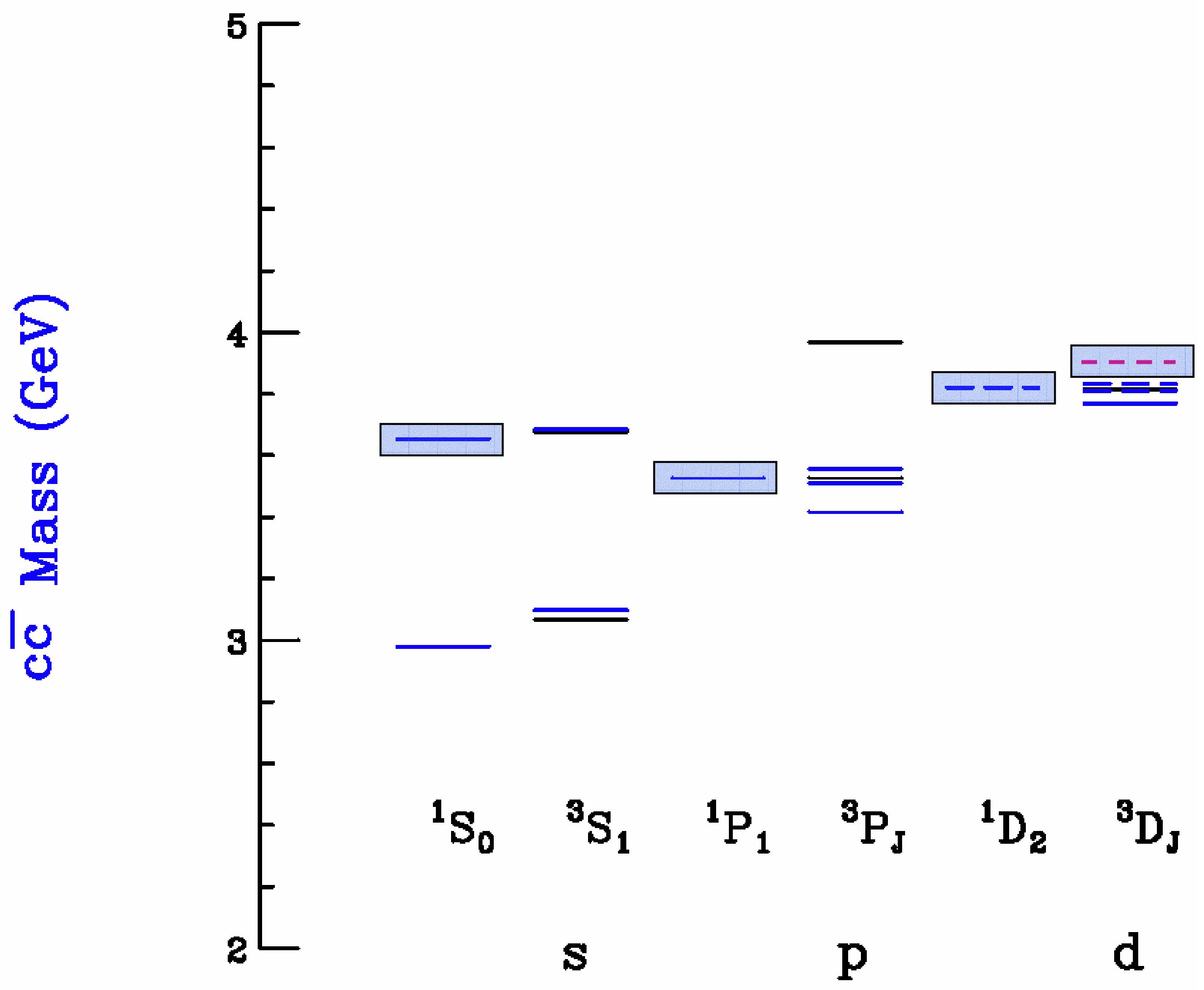
• How well are the leptonic and radiative decays predicted?

• Under what circumstances can the state at 3872 be interpreted as a charmonium level?

Here, we will attempt to answer these questions using a potential model which includes the v^2/c^2 and all one-loop corrections to the short distance potential supplemented with a linear phenomenological confining potential and its v^2/c^2 corrections.

2. Overview of potential model approaches.

The data to be understood are illustrated by



Predictions for spin averaged levels can be obtained from a simple Hamiltonian of the form

$$H = \frac{\vec{p}^{\,2}}{2\mu} + V(r),$$

where the most notable choice for V(r) is the Cornell potential

$$V(r) = Ar - \frac{4}{3}\frac{\alpha_S}{r}.$$

The use of this simple potential, with the inclusion of continuum effects, has been remarkably successful in efforts to identify charmonium states which could be accessible to experiment. (Eichten, Lane and Quigg)

Spin effects can be included to order v^2/c^2 in a straightforward way (Pumplin, WWR & Sato, Schnitzer) to obtain a Hamiltonian of the form

$$H = \frac{\vec{p}^{\,2}}{2\mu} + V(r) + V_{HF} + V_{LS} + V_{TEN} + V_{SI}$$

where V_{SI} consists of spin-independent terms including the kinetic energy correction. Recently, Barnes has used this Hamiltonian without the spin-independent term to show that one can obtain a reasonable description of charmonium spectrum and decays.

To proceed beyond this level requires the inclusion of the one loop QCD corrections to the short distance potential (Gupta & SR). In general, these are messy (Gupta, SR &WWR). To get a sense of what happens when one loop corrections are included, the hyperfine potential V_{HF} evolves as

$$V_{HF} = \frac{32\pi\alpha_S}{9m^2}\vec{S}_1\cdot\vec{S}_2\delta(\vec{r})$$

$$V'_{HF} = \frac{32\pi\alpha_S}{9m^2}\vec{S}_1\cdot\vec{S}_2\left[1-\frac{\alpha_S}{12\pi}\left(26+9\ln 2\right)\right]\delta(\vec{r})$$

$$+\frac{32\pi\alpha_S}{9m^2}\vec{S}_1\cdot\vec{S}_2\left\{-\frac{\alpha_S}{24\pi^2}(33-2n_f)\nabla^2\left[\frac{\ln(\mu r)+\gamma_E}{r}\right]+\frac{21\alpha_S}{16\pi^2}\nabla^2\left[\frac{\ln(mr)+\gamma_E}{r}\right]\right\}$$

To obtain the eigenvalues and wavefunctions for these complicated potentials it is convenient to use a variational approach. Specifically, we use trial wave functions of the form

$$\psi_\ell^m(\vec{r}) = \sum_{n=1}^{N} C_n \left(r/R\right)^{n+\ell-1} e^{-(r/R)^\beta} Y_\ell^m(\Omega),$$

with β=1,2. The coefficients C_n are determined by the variational technique of minimizing

$$E = \frac{\langle \psi | H | \psi \rangle}{\langle \psi | \psi \rangle}$$

with respect to the C_n's. This results in a linear eigenvalue equation and is equivalent to solving the Schrödinger equation.

The resulting radial wave functions are orthogonal and the eigenvalues λ_n are upper bounds on the true energies E_n for every n, i.e. $E_n \leq \lambda_n$, $n = 1, \cdots, N$. In practice, for N ≥ 10, the lowest 3-4 eigenvalues are stable. This can be seen in a comparison of results for the Cornell potential,

	ELQ	Variational
A (GeV^2)	0.177	0.179
α_S	0.457	0.448
m_C (GeV)	1.84	1.92
$\langle 1s \rangle$ (MeV)	3067	3067
$\langle 1p \rangle$ (MeV)	3526	3526
$\langle 2s \rangle$ (MeV)	3678	3678
$\langle 1d \rangle$ (MeV)	3815	3814
$\langle 2p \rangle$ (MeV)	3968	3966
$\langle 1f \rangle$ (MeV)	4054	4052

3. Results for a semi-relativistic model.

In what follows, we have included the kinetic energy corrections by using a Hamiltonian of the form

$$H = 2\sqrt{\vec{p}^2 + m^2} + Ar - \frac{4\alpha_S}{3r}\left[1 - \frac{3\alpha_S}{2\pi} + \frac{\alpha_S}{6\pi}(33 - 2n_f)[\ln(\mu r) + \gamma_E]\right] + V_L + V_S.$$

V_L contains the scalar and vector order v^2/c^2 corrections to Ar and V_S includes all v^2/c^2 and one-loop QCD corrections to the short distance potential. Two versions of the model are examined

- $V_L + V_S$ treated as a perturbation
- All terms treated nonperturbatively

In either case, all n=1, 2 and 3 s, p, and d levels were calculated. The potential parameters and the quark mass were determined by fitting the 1^1S_0, 1^3S_1, 1^3P_J, 2^1S_0, 2^3S_1, 1^3D_1, 3^3S_1, and 2^3D_1 levels to the data. The results are

	Pert	Non-pert	Expt		Pert	Non-pert	Expt
η_c	2985	2981	2979.7 ± 1.5	η_c'	3599	3624	(3637.7 ± 4.4)
J/ψ	3096.9	3096.9	3096.87 ± 0.04	ψ'	3686	3686	3686.0 ± 0.1
χ_0	3418.4	3415.8	3415.1 ± 0.8	χ_0'	3849	3872	
χ_1	3510.2	3510.4	3510.51 ± 0.12	χ_1'	3946	3951	
χ_2	3556.5	3556.3	3556.18 ± 0.17	χ_2'	3999	3996	
h_c	3527	3524	(3526.21 ± 0.25)	h_c'	3966	3966	
1^3D_1	3809	3790	3770 ± 2.5	2^3D_1	4174	4157	4160 ± 20
1^3D_2	3827	3826	3872 ± 1.0	2^3D_2	4198	4201	
1^3D_3	3831	3845		2^3D_3	4209	4223	
1^1D_2	3824	3825	3836 ± 13.0	2^1D_2	4199	4202	

The resulting parameters, leptonic, M_1 and E_1 widths are

	Pert	Non-pert
A	0.168 GeV^2	0.175 GeV^2
α_S	0.331	0.361
m_c	1.41 GeV	1.49 GeV
μ	2.32 GeV	1.07 GeV
f_V	0.00	0.18

$\Gamma_{e\bar{e}}$(keV)	Pert	Non-pert	Expt
J/ψ	10.7	2.7	5.40 ± 0.17
ψ'	6.3	1.7	2.12 ± 0.12

$\Gamma_\gamma(M1)$ (keV)	TH	EX
$J/\psi \to \gamma\eta_c$	2.78	1.2 ± 0.3
$\psi' \to \gamma\eta'_c$	0.45	
$\psi' \to \gamma\eta_c$	0.63	0.8 ± 0.2
$\eta'_c \to \gamma J/\psi$	1.04	
${}^3D_2(3872) \to \gamma {}^1D_2$	0.20	

$\Gamma_\gamma(E1)$ (keV)	TH	EX
$\chi_0 \to \gamma J/\psi$	169	119 ± 17
$\chi_1 \to \gamma J/\psi$	357	288 ± 51
$\chi_2 \to \gamma J/\psi$	468	426 ± 48
$h_c \to \gamma\eta_c$	670	
$\psi' \to \gamma\chi_0$	22	24.2 ± 2.5
$\psi' \to \gamma\chi_1$	33	23.6 ± 2.7
$\psi' \to \gamma\chi_2$	29	24.2 ± 2.5
$\eta'_c \to \gamma h_c$	22	
$\psi(3770) \to \gamma\chi_0$	291	320 ± 100
$\psi(3770) \to \gamma\chi_1$	125	280 ± 100
$\psi(3770) \to \gamma\chi_2$	5.6	≤ 330
$1^3D_2(3826) \to \gamma\chi_1$	314	
$1^3D_2(3826) \to \gamma\chi_2$	76.3	
$1^3D_2(3872) \to \gamma\chi_1$	459	
$1^3D_2(3872) \to \gamma\chi_2$	119	

4. Conclusions

- The semi-relativistic model provides a quantitatively good description of the charmonium spectrum. Of the states included in the fit, only the $^3D_1(3770)$ is poorly described.
- The Lorentz structure of the confining potential is interesting in that a perturbative treatment of the spin-dependent interactions always favors a pure scalar confining potential, while treating the spin terms non-perturbatively favors a scalar-vector mixture – 18% vector.
- The calculated E_1 decays compare favorably with experiment. Transitions between $J/\psi,\chi$ and ψ' appear to be dominated by spin rather than open channel effects.

- Based on the model considered here, the X(3872) cannot be explained solely in terms of a charmonium $^3D_2$ state described by a potential. Spin effects alone can only separate the $^3D_2$ from the $^3D_1$ by 40 MeV or so, which suggests that the inclusion of open channel effects is essential if this identification is to be established.

- Further analysis of the charmonium system will include S – D mixing and open channel effects

- A similar treatment of the $b\bar{b}$ system fits the data well (Gupta, SR & WWR) and should make useful predictions

- The potential for unequal mass systems has also been calculated and can be used to investigate the D_S and B_S mesons (Gupta, SR & WWR)