One-loop renormalization and improvement of the heavy-light currents with relativistic heavy and domain-wall light quarks

Norikazu Yamada RIKEN BNL Reserch Center
yamada@bnl.gov

in collaboration with

Sinya Aoki (Univ. of Tsukuba, RBRC)
Yoshinobu Kuramashi (Univ. of Tsukuba)

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Introduction

Lattice QCD : Model independent determination of nonperturbative quantities

$\Downarrow$

Weak matrix elements of heavy-light system

Uncertainties in heavy-light system on a lattice:

- † Discretization error $\propto (am_Q)^n \neq O(a^n)$
- † Chiral extrapolation $m_{\rm simulation} \gg m_{u/d}$
- † Renormalization of currents

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  $\Rightarrow$ Relativistic heavy quark action (RHQ)
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- † Chiral extrapolation $m_{\rm simulation} \gg m_{u/d}$
  $\Rightarrow$ Domain-wall light quark action (DWF)
- † Renormalization of currents
  $\Rightarrow$ This work
  One-loop renormalization and improvement of bilinear currents consisting of RHQ and DWF

Relativistic heavy quark

Aoki, Kuramashi and Tominaga (2003)

$$S_Q = \sum_x \Bigg[m_0 \bar{Q}(x)Q(x) + \bar{Q}(x)\gamma_0 D_0 Q(x) + \nu_Q \sum_i \bar{Q}(x)\gamma_i D_i Q(x) \\ - \frac{r_t a}{2}\bar{Q}(x) D_0^2 Q(x) - \frac{r_s a}{2}\sum_i \bar{Q}(x) D_i^2 Q(x) \\ - \frac{iga}{2} c_E \sum_i \bar{Q}(x)\sigma_{0i}F_{0i}Q(x) - \frac{iga}{4} c_B \sum_{i,j} \bar{Q}(x)\sigma_{ij}F_{ij}Q(x) \Bigg],$$

- the Fermilab action with a special choice of parameters [El-Khadra, Kronfeld and Mackenzie (1997)]
- ν_Q, r_s, c_E, c_B : determined to one-loop level in a mass dependent way. [Aoki, Kuramashi and Kayaba(2003)]
  $\Rightarrow$ no $O(\alpha_s(am_Q)^n)$ nor $O(\alpha_s(am_Q)^n a\vec{p})$ error in the action
- In $m_Q \to 0$, RHQ $\to$ the clover action, and the space-time rotational symmetry is restored.

Domain-wall fermion

Kaplan(1992), Shamir(1993), Furman and Shamir(1995)

$$
\begin{aligned}
S_{\rm DW} &= \sum_{x,y}\sum_{s,t}\bar\psi_s(x)D_{\rm DW}(x,s;y,t)\psi_t(y),\\
D_{\rm DW}(x,s;y,t) &= D_{\rm W}(x,y)\,\delta_{s,t}+\delta_{x,y}D_5(s,t),\\
D_{\rm W}(x,y) &= \sum_\mu \gamma_\mu D_\mu - \frac{a}{2}\sum_\mu D_\mu^2 - M_5\,\delta_{x,y},\\
D_5(s,t) &= \delta_{s,t}-P_L\,\delta_{s+1,t}-P_R\,\delta_{s-1,t}\\
&\quad +m_f\left[P_L\,\delta_{s,N_5}\delta_{1,t}+P_R\,\delta_{s,1}\delta_{N_5,t}\right],
\end{aligned}
$$

physical quark field : $\left\{\begin{array}{l} q(x) = P_L\psi_1(x)+P_R\psi_{N_5}(x)\\ \bar q(x) = \bar\psi_1(x)P_R+\bar\psi_{N_5}(x)P_L\end{array}\right.$,

$$P_{R/L} = (1\pm\gamma_5)/2,$$

$$1\le s,t\le N_5 \quad (N_5\to\infty)$$

- Exact chiral symmetry at the finite lattice spacing
- Leading scaling violation $\sim O(a^2)$ without any tuning
- The space-time rotational symmetry

Heavy-light currents (vector case)

- Define the lattice heavy-light vector current by

$$\bar{q}\,\gamma_\mu Q \quad \begin{cases} Q & : \text{Relativistic heavy quark} \\ q & : \text{Domain-wall light quark} \end{cases}$$

- To remove $O(\alpha_s (am_Q)^n a\vec{p})$ errors from the lattice current, all dimension four currents

$$\bar{q}\,\partial_\mu^-\,Q,\;\; \bar{q}\,\partial_\mu^+\,Q,\;\; (\vec{\partial}_i\bar{q})\gamma_i\gamma_\mu\,Q,\;\; \bar{q}\,\gamma_\mu\gamma_i(\vec{\partial}_i Q),$$

could mix under the renormalization.

† The equation of motion is used.

† The last two are the result of a violation of the space-time rotational symmetry.

$$\Downarrow$$

$V_\mu^{\rm latt,imp} \propto$ a linear combination of the above five currents.

Definition of improvement coefficients

e.g.) vector current :

$$V_\mu^{\rm latt,imp} = \bar{q}\gamma_\mu Q - g^2 c_{V_\mu}^{+}\{\bar{q}\partial_\mu^{-} Q\} - g^2 c_{V_\mu}^{-}\{\bar{q}\partial_\mu^{+} Q\}$$
$$-g^2 c_{V_\mu}^{L}\{\vec{\partial}_i \bar{q}\}\gamma_i\gamma_\mu Q - g^2 c_{V_\mu}^{H}\bar{q}\gamma_\mu\gamma_i\{\vec{\partial}_i Q\},$$
$$V_\mu^{\overline{\rm MS}} = Z_{V_\mu} V_\mu^{\rm latt,imp},$$

where

$$Z_{V_\mu} = \sqrt{\frac{Z_{Q,\rm latt}^{(0)}(m_Q)}{Z_w(M_5)}}\left[1 - g^2\Delta_{V_\mu}\right], \quad \begin{cases} Z_{Q,\rm latt}^{(0)}(m_Q)\text{: tree-level wave function} \\ Z_w(M_5)\text{: known to one-loop} \end{cases}$$

and $\partial_\mu^{\pm} = \overrightarrow{\partial}_\mu \pm \overleftarrow{\partial}_\mu$.

- Five coefficients Δ_{V_μ} and $c_{V_\mu}^{\pm,H,L}$ have to be determined in the following.
- Coefficients are obtained as functions of m_Q and M_5.

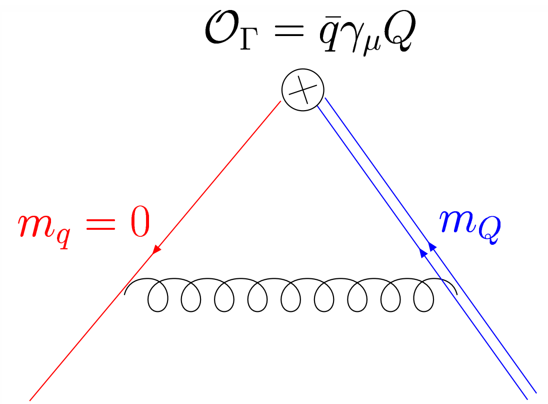
Properties of the coefficients

$$V_\mu^{\overline{\rm MS}} = Z_{V_\mu} V_\mu^{\rm latt,imp},$$
$$V_\mu^{\rm latt,imp} = \bar{q}\gamma_\mu Q - g^2 c_{V_\mu}^{+}\{\bar{q}\partial_\mu^{-} Q\} - g^2 c_{V_\mu}^{-}\{\bar{q}\partial_\mu^{+} Q\}$$
$$-g^2 c_{V_\mu}^{L}\{(\vec{\partial}_i \bar{q})\gamma_i\gamma_\mu Q\} - g^2 c_{V_\mu}^{H}\{\bar{q}\gamma_\mu\gamma_i(\vec{\partial}_i Q)\},$$

- In $m_Q \neq 0$ case:
  $\rightarrow$ no rotational symmetry
  $\rightarrow$ $\Delta_{V_0} \neq \Delta_{V_k}$, $c_{V_k}^{\pm,H,L} \neq c_{V_0}^{\pm,H,L}$, especially $c_{V_\mu}^{H,L} \neq 0$,
- In $m_Q = 0$ case:
  $\rightarrow$ the rotational symmetry restored
  $\rightarrow$ $\Delta_{V_0} = \Delta_{V_k}$, $c_{V_0}^{\pm,H,L} = c_{V_k}^{\pm,H,L}$, especially $c_{V_\mu}^{H,L} = 0$
- Eq. of motion $\rightarrow$ $c_{V_0}^{H,L} = 0$.
  Only two coefficients required for the improvement of V_0

Calculational method (vector)

Calculate the on-shell amplitude of the lattice current



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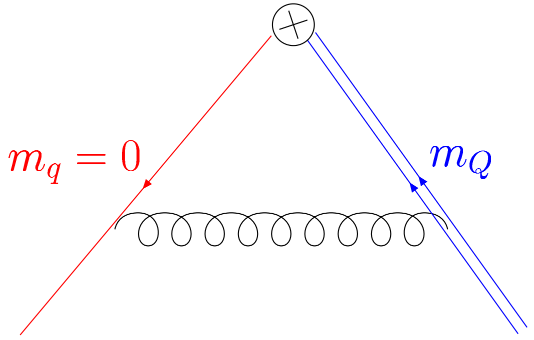


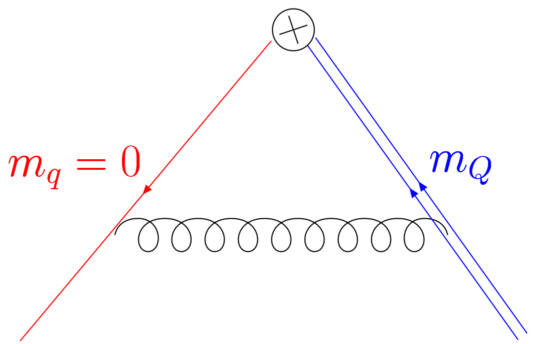
on the lattice

$$
\begin{aligned}
& X^{\rm latt}_\mu \, \bar{u}_q(q)\gamma_\mu u_Q(p) \\
& + Y^{\rm latt}_\mu \, (p_\mu + q_\mu)\bar{u}_q(q)u_Q(p) + Z^{\rm latt}_\mu \, (p_\mu - q_\mu)\bar{u}_q(q)u_Q(p) \\
& + R^{\rm latt}_\mu \, \bar{u}_q(q)\gamma_\mu \, (\gamma_i p_i)\, u_Q(p) + S^{\rm latt}_\mu \, \bar{u}_q(q)\, (\gamma_i q_i)\gamma_\mu u_Q(p) \\
& + O(a^2)
\end{aligned}
$$

Calculational method (vector)

Calculate the on-shell amplitude of the lattice current

$$\mathcal{O}_\Gamma = \bar{q}\gamma_\mu Q$$



on the lattice

$$X_\mu^{\rm latt}\,\bar{u}_q(q)\gamma_\mu u_Q(p)$$
$$+\,Y_\mu^{\rm latt}\,(p_\mu+q_\mu)\bar{u}_q(q)u_Q(p) + Z_\mu^{\rm latt}\,(p_\mu-q_\mu)\bar{u}_q(q)u_Q(p)$$
$$+\,R_\mu^{\rm latt}\,\bar{u}_q(q)\gamma_\mu\,(\gamma_i p_i)\,u_Q(p) + S_\mu^{\rm latt}\,\bar{u}_q(q)\,(\gamma_i q_i)\gamma_\mu u_Q(p)$$
$$+\,O(a^2)$$

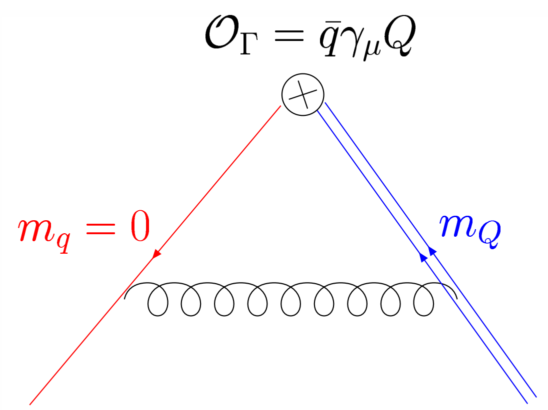
in the continuum

$$X_\mu^{{\rm cont},\overline{\rm MS}}\,\bar{u}_q(q)\gamma_\mu u_Q(p)$$
$$+\,Y_\mu^{\rm cont}\,(p_\mu+q_\mu)\bar{u}_q(q)u_Q(p) + Z_\mu^{\rm cont}\,(p_\mu-q_\mu)\bar{u}_q(q)u_Q(p)$$

The rotational symm.$\to$ $R_\mu^{\rm cont} = S_\mu^{\rm cont} = 0$

Calculational method (vector)

Calculate the on-shell amplitude of the lattice current



on the lattice

$$X_\mu^{\rm latt}\,\bar{u}_q(q)\gamma_\mu u_Q(p) + Y_\mu^{\rm latt}\,(p_\mu+q_\mu)\bar{u}_q(q)u_Q(p) + Z_\mu^{\rm latt}\,(p_\mu-q_\mu)\bar{u}_q(q)u_Q(p) + R_\mu^{\rm latt}\,\bar{u}_q(q)\gamma_\mu\,(\gamma_i p_i)\,u_Q(p) + S_\mu^{\rm latt}\,\bar{u}_q(q)\,(\gamma_i q_i)\gamma_\mu u_Q(p) + O(a^2)$$

in the continuum

$$X_\mu^{{\rm cont},\overline{\rm MS}}\,\bar{u}_q(q)\gamma_\mu u_Q(p) + Y_\mu^{\rm cont}\,(p_\mu+q_\mu)\bar{u}_q(q)u_Q(p) + Z_\mu^{\rm cont}\,(p_\mu-q_\mu)\bar{u}_q(q)u_Q(p)$$

The rotational symm.$\rightarrow$ $R_\mu^{\rm cont} = S_\mu^{\rm cont} = 0$

$\Delta_{V_\mu} \Leftarrow X_\mu^{\rm latt} - X_\mu^{{\rm cont},\overline{\rm MS}}$ and Z_Q, Z_q

$c_{V_\mu}^{\pm,H,L} \Leftarrow Y_\mu^{\rm latt} - Y_\mu^{\rm cont},\ Z_\mu^{\rm latt} - Z_\mu^{\rm cont},\ R_\mu^{\rm latt},\ S_\mu^{\rm latt}$

Numerical results

The numerical integrations were performed with

- 3 different gauge actions:
  plaquette, Iwasaki and DBW2
- 14 or more values of m_Q in $0.1 \leq m_Q \leq 6.0$ for each M_5
- 11 values of M_5 in $0.1 \leq M_5 \leq 1.9$

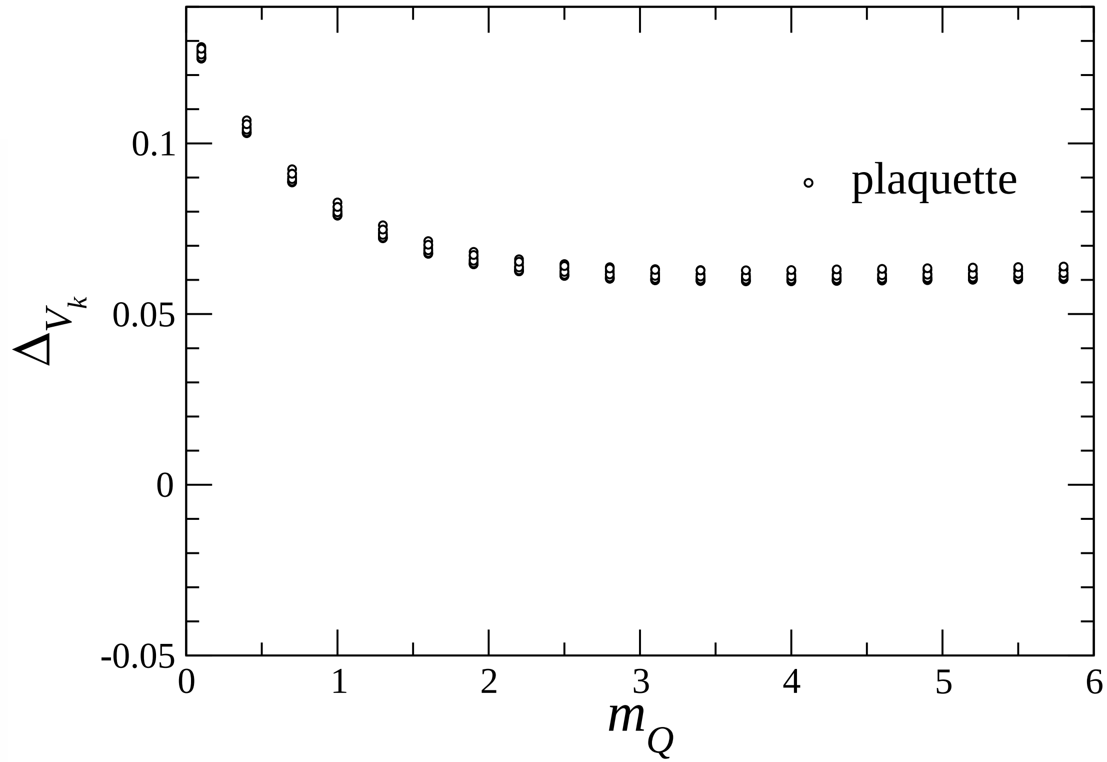
mainly at Riken Super Combined Cluster System at Riken.

In the following, the results of Δ_{V_μ} and $c_{V_\mu}^{\pm,H,L}$ are given as a function of m_Q and M_5.

Δ_{V_k} and Δ_{V_0}

$$Z_{V_\mu} = \sqrt{\frac{Z^{(0)}_{Q,\mathrm{latt}}(m_Q)}{Z_w(M_5)}} \left[1 - g^2 \Delta_{V_\mu}\right]$$

- The different point corresponds to the different M_5.
  $\rightarrow M_5$ dependence is relatively small.



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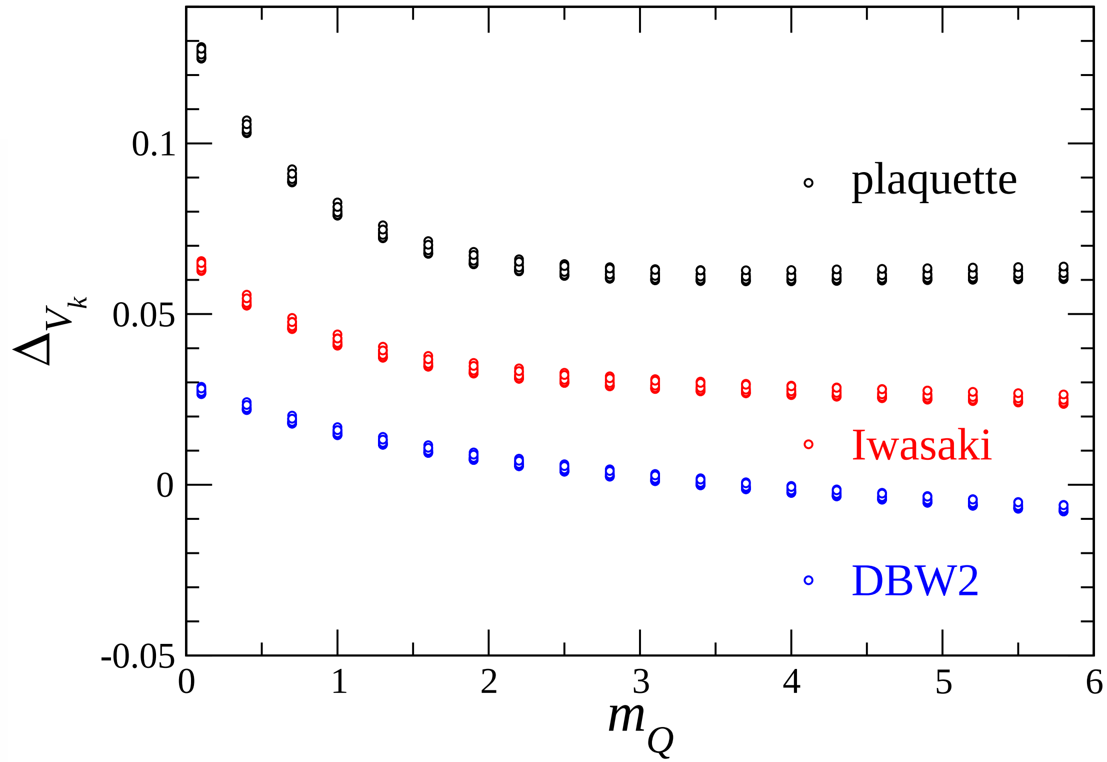

Δ_{V_k} and Δ_{V_0}

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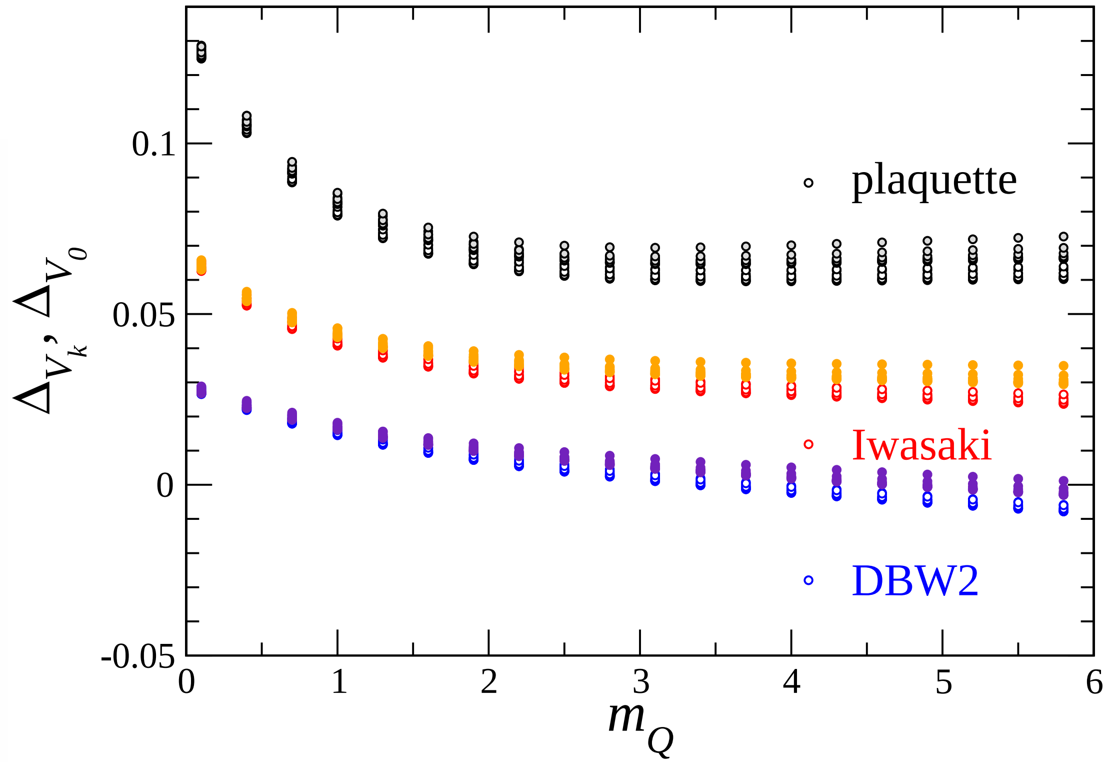
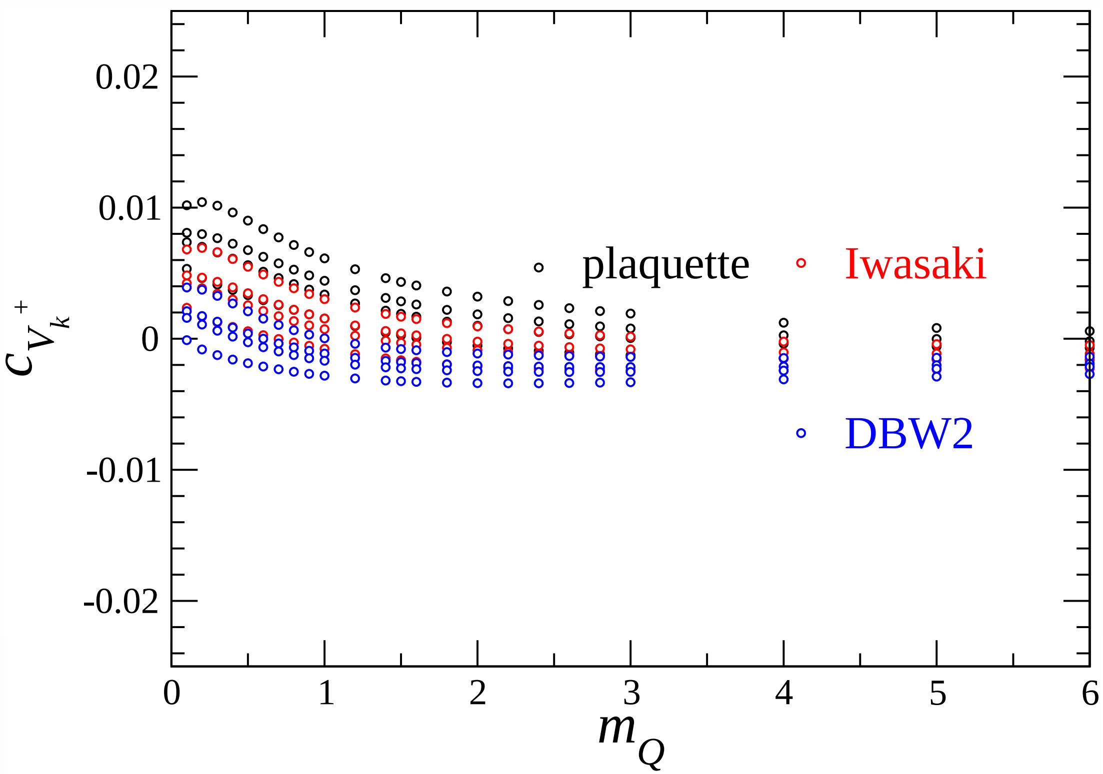
- The different point corresponds to the different M_5.
  $\rightarrow M_5$ dependence is relatively small.
- $\Delta_{V_k} = \Delta_{V_0}$ at $m_Q = 0$.



$c^+_{V_k}$ and $c^+_{V_0}$

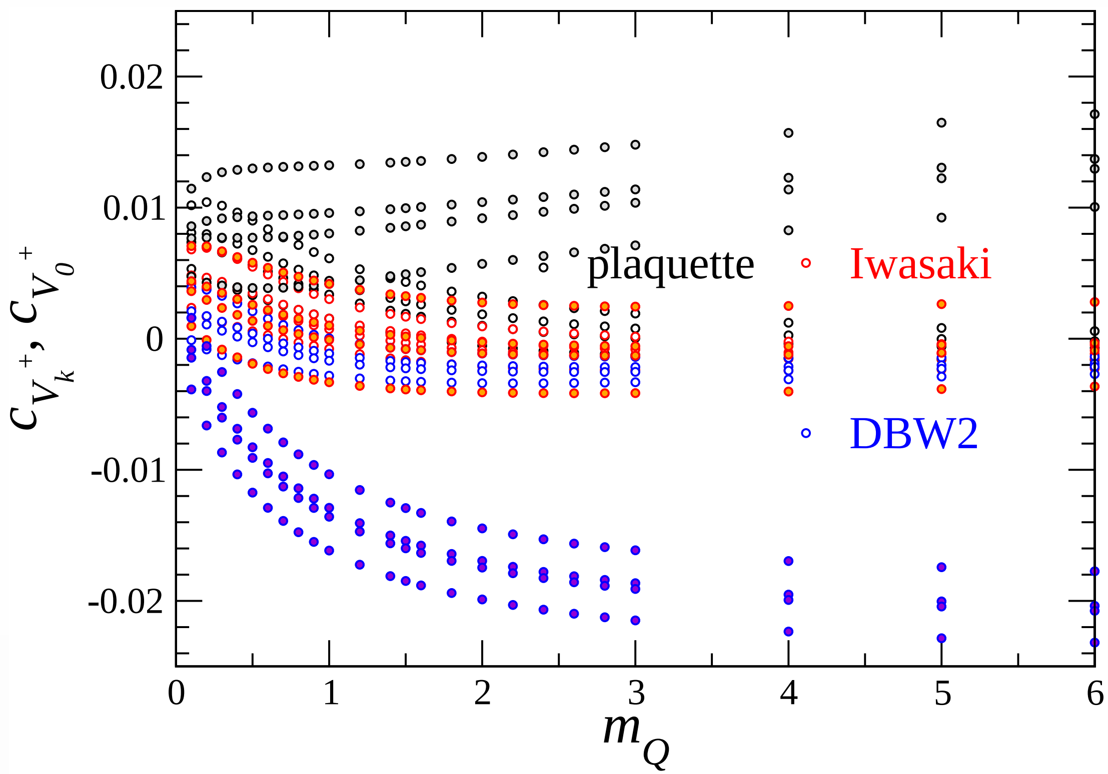
$$
\begin{aligned}
V_\mu^{\rm latt,imp} &= \bar{q}\gamma_\mu Q - g^2 c^+_{V_\mu}\{\bar{q}\partial^-_\mu Q\} - g^2 c^-_{V_\mu}\{\bar{q}\partial^+_\mu Q\} \\
&\quad - g^2 c^L_{V_\mu}\{(\vec{\partial}_i \bar{q})\gamma_i\gamma_\mu Q\} - g^2 c^H_{V_\mu}\{\bar{q}\gamma_\mu\gamma_i(\vec{\partial}_i Q)\},
\end{aligned}
$$



$c^{+}_{V_k}$ and $c^{+}_{V_0}$

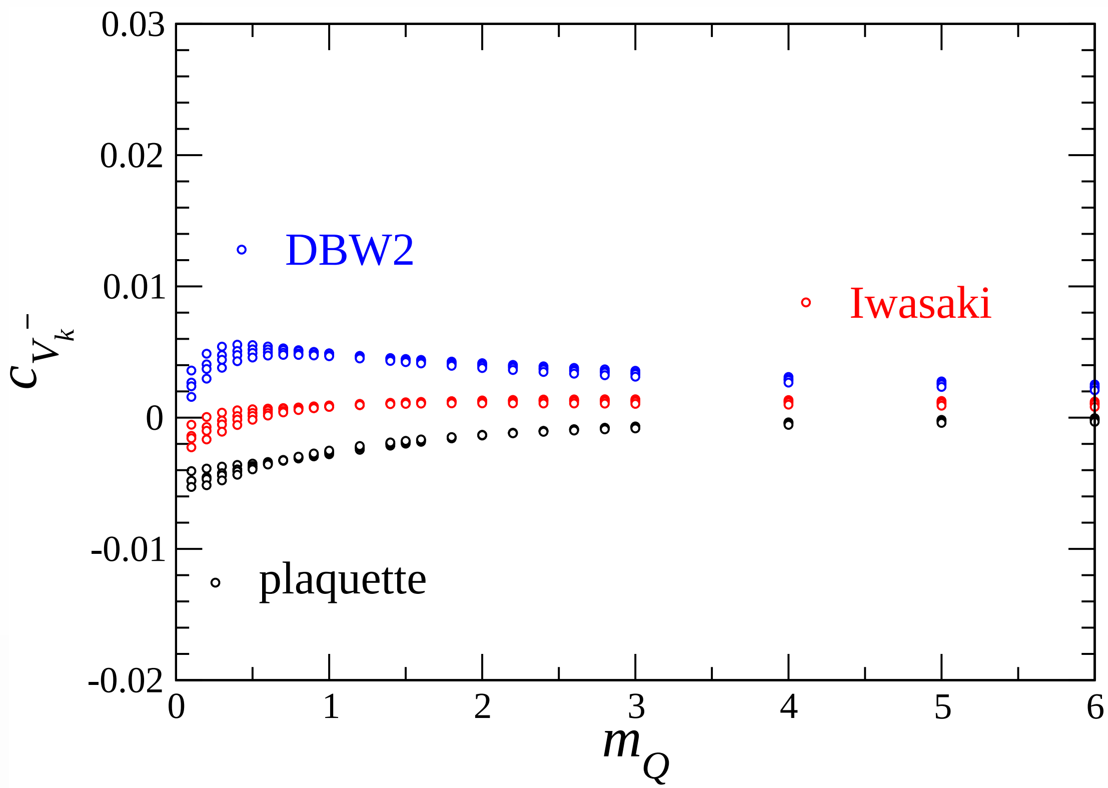
$$V_\mu^{\rm latt,imp} = \bar{q}\gamma_\mu Q - g^2 c^{+}_{V_\mu}\{\bar{q}\partial^{-}_\mu Q\} - g^2 c^{-}_{V_\mu}\{\bar{q}\partial^{+}_\mu Q\} - g^2 c^{L}_{V_\mu}\{(\vec{\partial}_i\bar{q})\gamma_i\gamma_\mu Q\} - g^2 c^{H}_{V_\mu}\{\bar{q}\gamma_\mu\gamma_i(\vec{\partial}_i Q)\},$$

- $c^{+}_{V_k} = c^{+}_{V_0}$ at $m_Q = 0$



$c^-_{V_k}$ and $c^-_{V_0}$

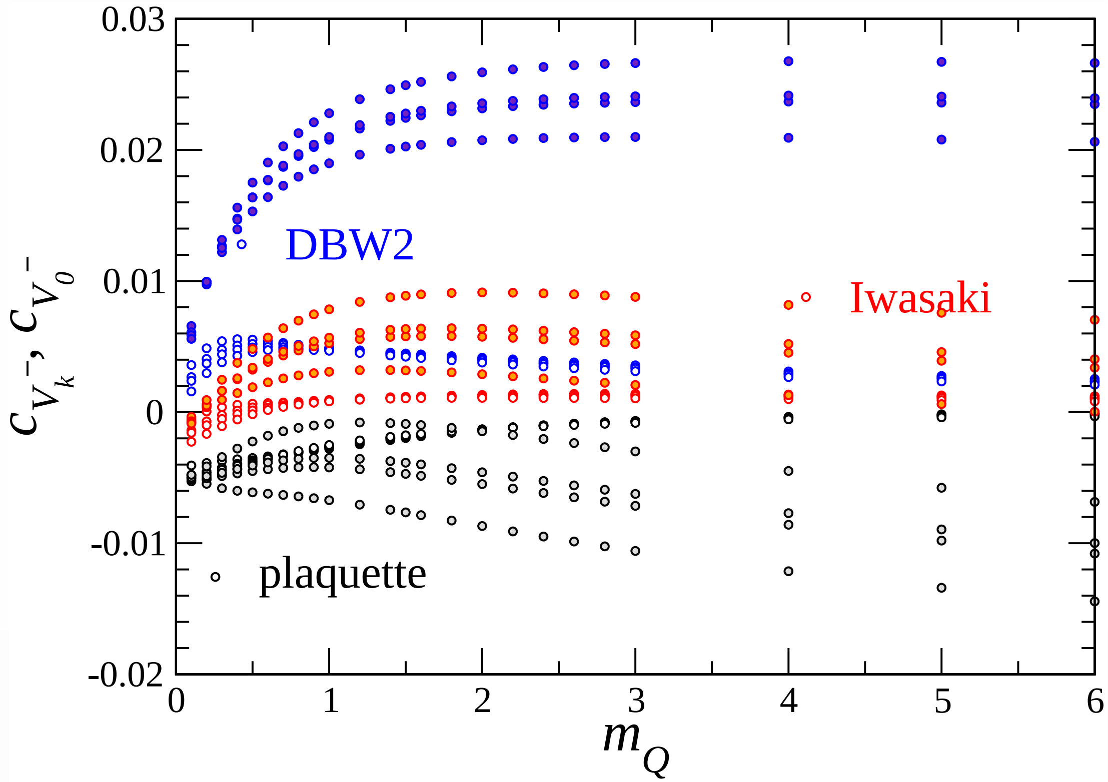
$$V_\mu^{\text{latt,imp}} = \bar{q}\gamma_\mu Q - g^2 c^+_{V_\mu}\{\bar{q}\partial^-_\mu Q\} - g^2 c^-_{V_\mu}\{\bar{q}\partial^+_\mu Q\} - g^2 c^L_{V_\mu}\{(\vec{\partial}_i\bar{q})\gamma_i\gamma_\mu Q\} - g^2 c^H_{V_\mu}\{\bar{q}\gamma_\mu\gamma_i(\vec{\partial}_i Q)\},$$



$c^-_{V_k}$ and $c^-_{V_0}$

$$V_\mu^{\rm latt,imp} = \bar{q}\gamma_\mu Q - g^2 c^+_{V_\mu}\{\bar{q}\partial^-_\mu Q\} - g^2 c^-_{V_\mu}\{\bar{q}\partial^+_\mu Q\}$$
$$- g^2 c^L_{V_\mu}\{(\vec{\partial}_i \bar{q})\gamma_i\gamma_\mu Q\} - g^2 c^H_{V_\mu}\{\bar{q}\gamma_\mu\gamma_i(\vec{\partial}_i Q)\},$$

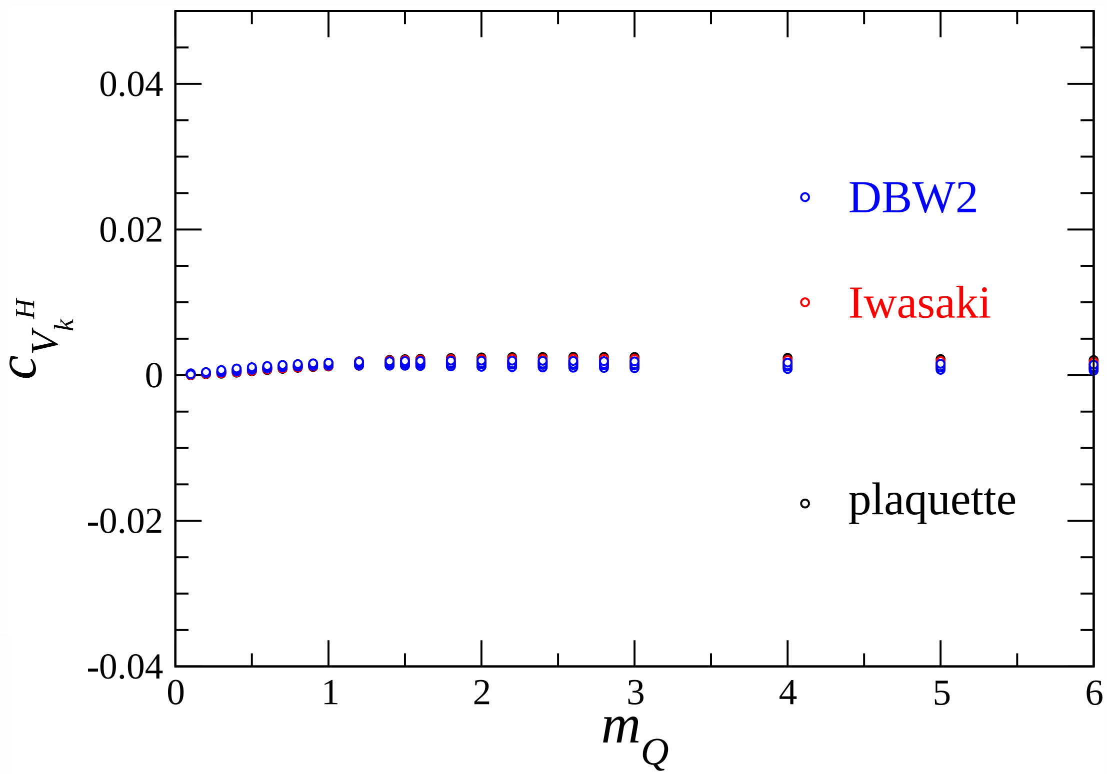
- $c^-_{V_k} = c^-_{V_0}$ at $m_Q = 0$



$c^H_{V_k}$ and $c^L_{V_k}$

$$V_\mu^{\mathrm{latt,imp}} = \bar{q}\gamma_\mu Q - g^2 c^+_{V_\mu}\{\bar{q}\partial^-_\mu Q\} - g^2 c^-_{V_\mu}\{\bar{q}\partial^+_\mu Q\}$$
$$-g^2 c^L_{V_\mu}\{(\vec{\partial}_i\bar{q})\gamma_i\gamma_\mu Q\} - g^2 c^H_{V_\mu}\{\bar{q}\gamma_\mu\gamma_i(\vec{\partial}_i Q)\},$$

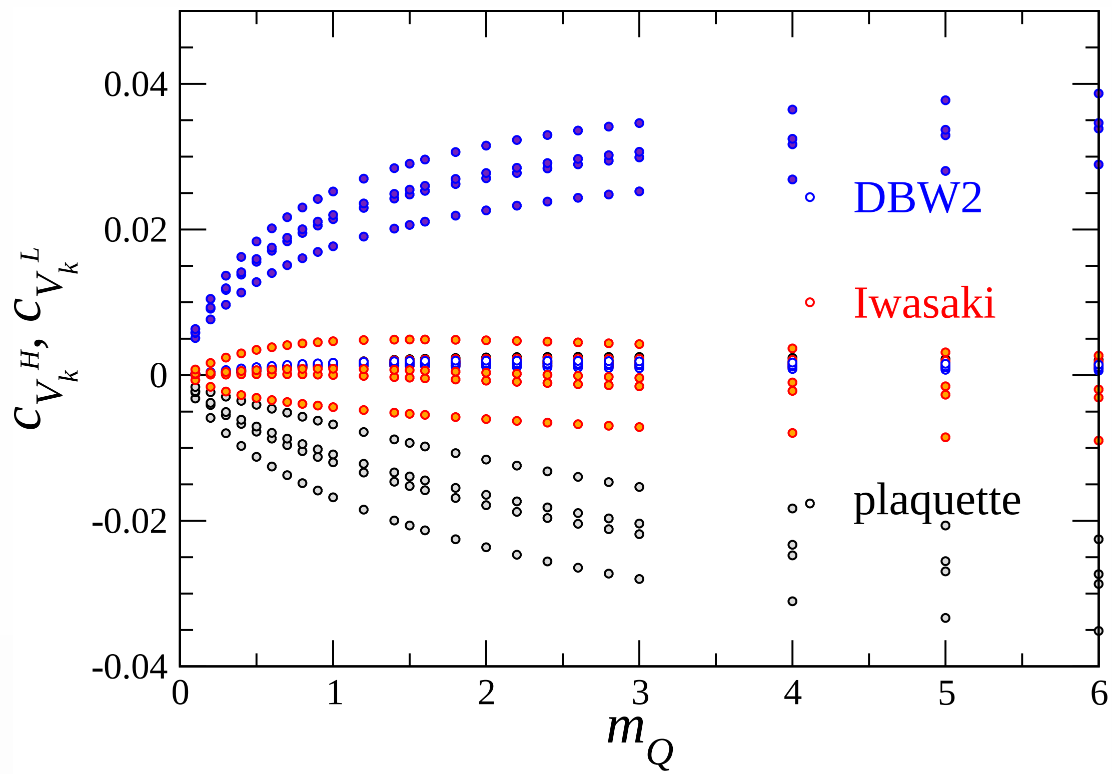
- $c^H_{V_k} = 0$ at $m_Q = 0$
  $c^H_{V_k} \approx 0$ in the whole range



$c_{V_k}^H$ and $c_{V_k}^L$

$$V_\mu^{\rm latt,imp} = \bar{q}\gamma_\mu Q - g^2 c_{V_\mu}^{+}\{\bar{q}\partial_\mu^{-}Q\} - g^2 c_{V_\mu}^{-}\{\bar{q}\partial_\mu^{+}Q\}$$
$$-g^2 c_{V_\mu}^{L}\{(\vec{\partial}_i\bar{q})\gamma_i\gamma_\mu Q\} - g^2 c_{V_\mu}^{H}\{\bar{q}\gamma_\mu\gamma_i(\vec{\partial}_i Q)\},$$

- $c_{V_k}^H = 0$ at $m_Q = 0$
  $c_{V_k}^H \approx 0$ in the whole range
- $c_{V_k}^L = 0$ at $m_Q = 0$



Benefit of chiral symmetry

Define the axial transformation of DWF by $\begin{cases} \delta\, q^i = i\,\frac{(\tau^3)^{ij}}{2}\,\gamma_5\, q^j \\ \delta\,\bar{q}^{\,i} = \bar{q}^{\,j}\, i\,\frac{(\tau^3)^{ji}}{2}\,\gamma_5 \end{cases}$,

Lattice W-T identity

$$\langle 0|\ \int_{\partial R} d\sigma_\mu(y) A_\mu^{3,\mathrm{ll}}(y)\, V_\nu^{\mathrm{hl}}(x)\, \mathcal{O}\ |0\rangle = -\frac{1}{2}\,\langle 0|\ A_\nu^{\mathrm{hl}}(x)\, \mathcal{O}\ |0\rangle$$

where $A_\mu^{3,\mathrm{ll}}(y)$: light-light conserved axil-vector current

$\mathcal{O}$: arbitrary operator

$$\{V_\nu^{\mathrm{hl}},\ A_\nu^{\mathrm{hl}}\} \quad = \quad \{\bar{q}\gamma_\nu Q,\ \bar{q}\gamma_\nu\gamma_5 Q\}$$

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where $A_\mu^{3,\mathrm{ll}}(y)$: light-light conserved axil-vector current

$\mathcal{O}$: arbitrary operator

$$\begin{aligned} \{V_\nu^{\mathrm{hl}},\ A_\nu^{\mathrm{hl}}\} \quad = \quad & \{\bar{q}\gamma_\nu Q,\ \bar{q}\gamma_\nu\gamma_5 Q\}, \\ & \{\bar{q}\partial_\mu^- Q,\ \bar{q}\partial_\mu^- \gamma_5 Q\}, \\ & \{\bar{q}\partial_\mu^+ Q,\ \bar{q}\partial_\mu^+ \gamma_5 Q\}, \\ & \{(\vec{\partial}_i \bar{q})\gamma_i\gamma_\mu Q,\ (\vec{\partial}_i \bar{q})\gamma_i\gamma_\mu\gamma_5 Q\}, \\ & \{\bar{q}\gamma_\mu\gamma_i(\vec{\partial}_i Q),\ -\bar{q}\gamma_\mu\gamma_5\gamma_i(\vec{\partial}_i Q)\} \end{aligned}$$

Benefit of chiral symmetry (contd.)

On the other hand, from a continuum W-T identity,

$$\langle 0|\int_{\partial R} d\sigma_\mu(y) A_\mu^{3,\mathrm{ll}}(y)\, V_\nu^{\overline{\mathrm{MS}}}\, \mathcal{O}\, |0\rangle = -\frac{1}{2}\langle 0|\, A_\nu^{\overline{\mathrm{MS}}}\, \mathcal{O}\, |0\rangle$$

$$\begin{aligned} V_\mu^{\overline{\mathrm{MS}}} &= Z_{V_\mu}\Big[\bar{q}\gamma_\mu Q - g^2 c_{V_\mu}^{+}\{\bar{q}\partial_\mu^{-} Q\} - g^2 c_{V_\mu}^{-}\{\bar{q}\partial_\mu^{+} Q\} \\ &\qquad -g^2 c_{V_\mu}^{L}\{(\vec{\partial}_i\bar{q})\gamma_i\gamma_\mu Q\} - g^2 c_{V_\mu}^{H}\{\bar{q}\gamma_\mu\gamma_i(\vec{\partial}_i Q)\}\Big] \end{aligned}$$

$$\begin{aligned} A_\mu^{\overline{\mathrm{MS}}} &= Z_{V_\mu}\Big[\bar{q}\gamma_\mu\gamma_5 Q + g^2 c_{V_\mu}^{+}\{\bar{q}\partial_\mu^{-}\gamma_5 Q\} + g^2 c_{V_\mu}^{-}\{\bar{q}\partial_\mu^{+}\gamma_5 Q\} \\ &\qquad +g^2 c_{V_\mu}^{L}\{(\vec{\partial}_i\bar{q})\gamma_i\gamma_\mu\gamma_5 Q\} - g^2 c_{V_\mu}^{H}\{\bar{q}\gamma_\mu\gamma_5\gamma_i(\vec{\partial}_i Q)\}\Big], \end{aligned}$$

This relation holds

- if a conseved axial-vector current exists on the lattice,
- for arbitrary heavy quark action.

Improving PT series

- Mean field improvement [Lepage and Mackenzie (1993)]

Improving PT series

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$$Z_{V_\mu}^{\overline{\mathrm{MS}}-\mathrm{latt}} = \sqrt{\frac{Z_{Q,\mathrm{latt}}^{(0)}(m_Q)}{Z_w(M_5)}} \left[1 - g^2 \Delta_{V_\mu}(m_Q, M_5)\right]$$

$$\Rightarrow Z_{V_\mu}^{\overline{\mathrm{MS}}-\mathrm{latt,MF}} = u^{\mathrm{MC}} \sqrt{\frac{Z_{Q,\mathrm{latt}}^{(0)}(\tilde{m}_Q)}{Z_w^{\mathrm{MF}}(\tilde{M}_5)}} \left[1 - g^2 \Delta_{V_\mu}^{\mathrm{MF}}(\tilde{m}_Q, \tilde{M}_5)\right],$$

$\tilde{m}_Q$, $\tilde{M}_5$: the mean field tree level values of m_Q and M_5, respectively.

Improving PT series

- Mean field improvement [Lepage and Mackenzie (1993)]

$$Z_{V_\mu}^{\overline{\rm MS}-{\rm latt}} = \sqrt{\frac{Z_{Q,{\rm latt}}^{(0)}(m_Q)}{Z_w(M_5)}}\left[1 - g^2 \Delta_{V_\mu}(m_Q, M_5)\right]$$

$$\Rightarrow Z_{V_\mu}^{\overline{\rm MS}-{\rm latt},{\rm MF}} = u^{\rm MC}\sqrt{\frac{Z_{Q,{\rm latt}}^{(0)}(\tilde{m}_Q)}{Z_w^{\rm MF}(\tilde{M}_5)}}\left[1 - g^2 \Delta_{V_\mu}^{\rm MF}(\tilde{m}_Q, \tilde{M}_5)\right],$$

$\tilde{m}_Q$, $\tilde{M}_5$: the mean field tree level values of m_Q and M_5, respectively.

- Quasi-Nonperturbative + MF [Harada, *et al.* (2002) for h-l system]

Nonperturbative $Z_V^{\rm dw}$ $(= Z_{V_0}^{\rm dw} = Z_{V_k}^{\rm dw} = Z_A^{\rm dw})$ for the massless DW quarks is available, and includes Z_q and Z_w nonperturbatively.

Improving PT series

- Mean field improvement [Lepage and Mackenzie (1993)]

$$Z_{V_\mu}^{\overline{\rm MS}-{\rm latt}} = \sqrt{\frac{Z^{(0)}_{Q,{\rm latt}}(m_Q)}{Z_w(M_5)}}\,[1-g^2\Delta_{V_\mu}(m_Q,M_5)]$$

$$\Rightarrow Z_{V_\mu}^{\overline{\rm MS}-{\rm latt,MF}} = u^{\rm MC}\sqrt{\frac{Z^{(0)}_{Q,{\rm latt}}(\tilde m_Q)}{Z_w^{\rm MF}(\tilde M_5)}}\left[1-g^2\Delta^{\rm MF}_{V_\mu}(\tilde m_Q,\tilde M_5)\right],$$

$\tilde m_Q$, $\tilde M_5$: the mean field tree level values of m_Q and M_5, respectively.

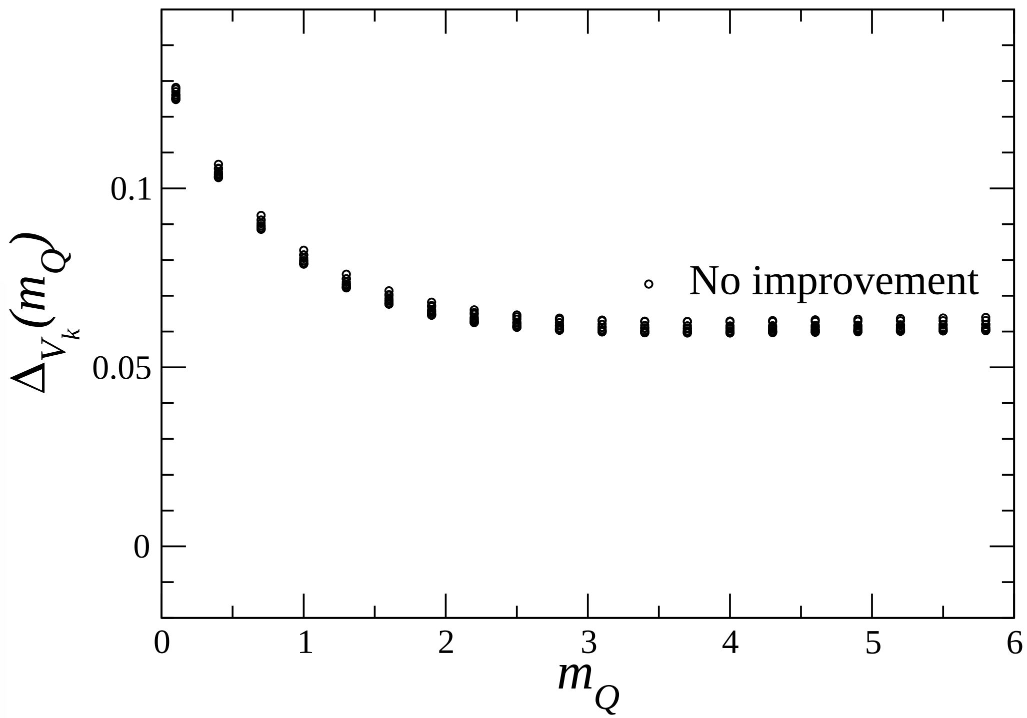
- Quasi-Nonperturbative + MF [Harada, *et al.* (2002) for h-l system]

Nonperturbative $Z_V^{\rm dw}$ $(= Z_{V_0}^{\rm dw} = Z_{V_k}^{\rm dw} = Z_A^{\rm dw})$ for the massless DW quarks is available, and includes Z_q and Z_w nonperturbatively.

$$\Rightarrow Z_{V_\mu}^{\overline{\rm MS}-{\rm latt,QNP+MF}} = \sqrt{u^{\rm MC}\, Z^{(0)}_{Q,{\rm latt}}(\tilde m_Q) Z_V^{\rm dw,NP}(M_5)}\left[1-g^2\Delta^{\rm QNP+MF}_{V_\mu}(\tilde m_Q,\tilde M_5)\right]$$

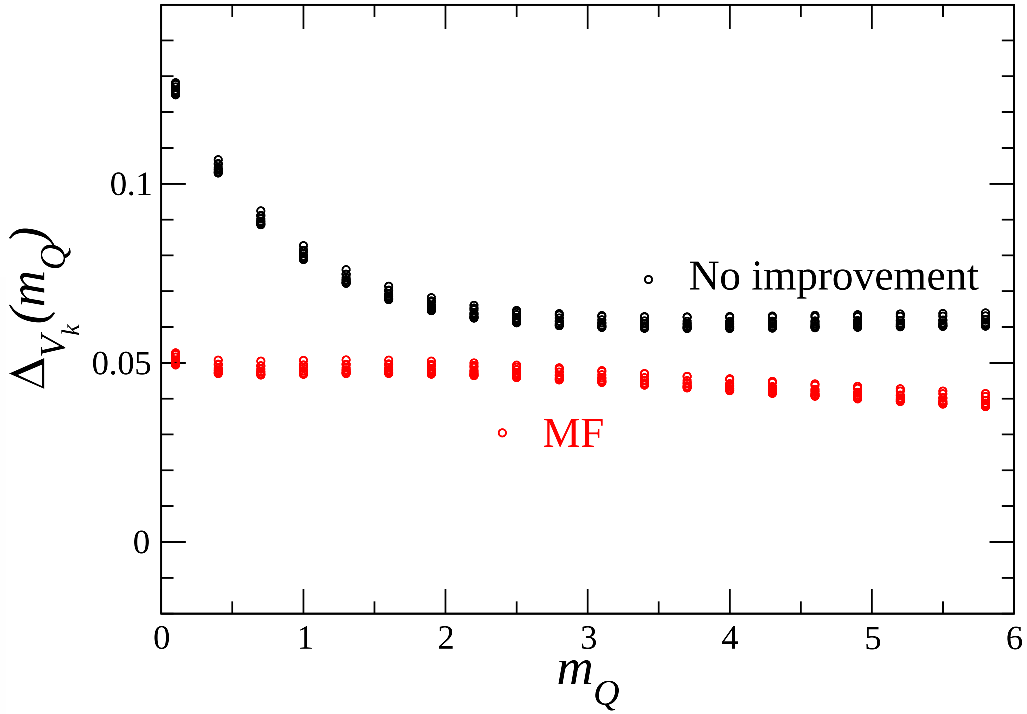
Improving PT series (contd.)

- No improved case Δ_{V_k}



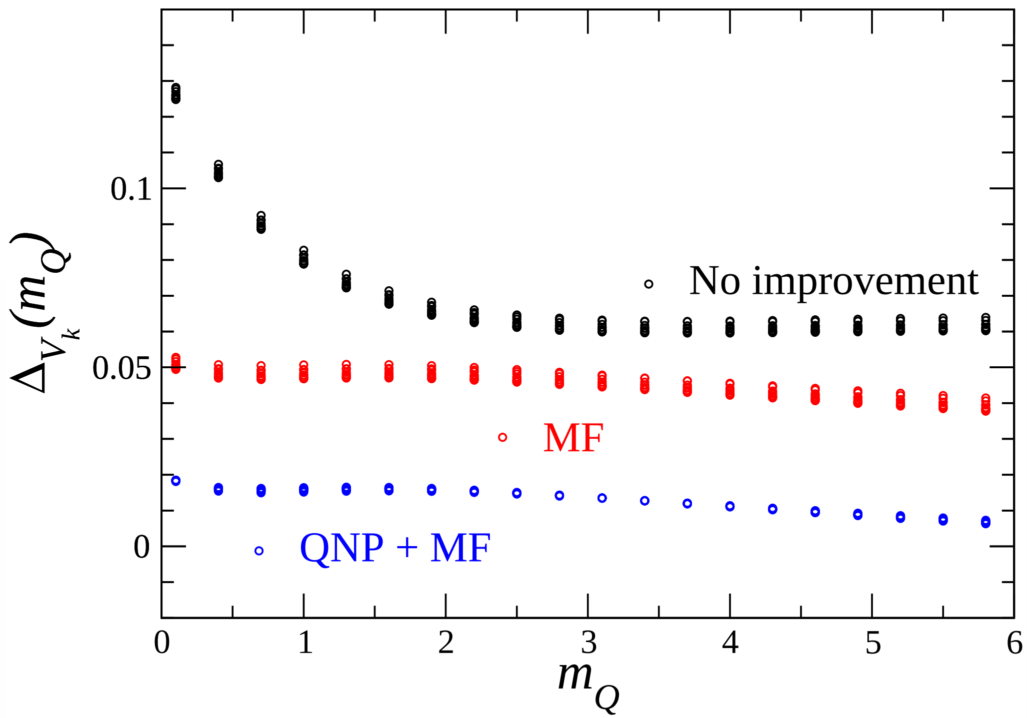
Improving PT series (contd.)

- No improved case Δ_{V_k}
- Mean field $\Delta_{V_k}^{\mathrm{MF}}$



Improving PT series (contd.)

- No improved case Δ_{V_k}
- Mean field $\Delta_{V_k}^{\mathrm{MF}}$
- QNP + MF $\Delta_{V_k}^{\mathrm{QNP+MF}}$



Conclusion

- One-loop determination of the $O(a)$ improvement coefficients for the heavy-light vector currents consisting of the relativistic heavy and the domain-wall light quarks.
  $\longrightarrow$ remove $O(\alpha_s(am_Q)^n)$ and $O(\alpha_s(am_Q)^n a\vec{p})$.
- Thanks to chiral symmetry, Z_{V_μ}=Z_{A_μ}, $c_{V_\mu}^{\pm,H,L} = c_{A_\mu}^{\pm,H,L}$.
  This property is very helpful for the test of the soft pion relation on the lattice.
- We exercise two ways of improving PT series, "MF" and "QNP + MF". "QNP + MF" seems to work well.