Recent Developments in AdS/QCD

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Late 1960s: String Theory born from hadronic data.

Regge spectrum $M^2 \propto J$

s/t–channel duality

Regge behavior in small-angle scattering

Problem: power-law in large-angle scattering

Problem: Bjorken scaling

1973: Asymptotic Freedom (re)discovered.

QCD hailed as theory of strong interactions.

String Theory abandoned…

1974: String Theory reborn as theory of gravity.

etc.,etc.

Question: Why did string theory work at all in context of hadronic physics?

- Confinement: confining flux tubes act like strings.

  Hard to be precise without understanding of confinement.

- 't Hooft: Sum over Feynman diagrams (expansion in g) actually expansion in $\lambda = g^2 N$ and $1/N$

  $1/N$ *expansion* = *sum over string-like diagrams*

  i.e., like string pert. thy. in coupling $1/N$

- Wilson → Polyakov: Wilson loops are natural variables in gauge theory, like boundaries of strings.

Maybe 5d string theory = 4d gauge theory?

1997: Maldacena:

Gauge/String Correspondence

Large classes of four-dimensional gauge theories are **quantum-mechanically equivalent** to ten-dimensional string theories on a space of which five dimensions are compact, five are non-compact.

5d noncompact = 4d Minkowski space plus
an extra 'radial' coordinate r.

General Relation: Maldacena; Witten;Gubser,Klebanov,Polyakov

Gauge Theory			**Gravity Theory**
Ultraviolet $\mu \to \infty$			Top $r \to \infty$
Objects tend to *grow*	$\uparrow$	$\uparrow$	Objects tend to *fall*
	μ	r	
Infrared $\mu \to 0$			Bottom $r \to 0$

1997: Conformal Theories

Gauge Theory			**Gravity Theory**
Ultraviolet $\mu \to \infty$			Top $r \to \infty$
Scale – Invariant			Anti de Sitter space
	$\uparrow$	$\uparrow$	
Objects *grow forever*	μ	r	Objects *fall forever*
Infrared $\mu \to 0$			Horizon $r \to 0$

1998/2000: Confining Theories

Witten; Rey et al.

Polchinski+MJS; Klebanov+MJS

Gauge Theory	**Gravity Theory**
Ultraviolet $\mu \to \infty$	Top $r \to \infty$
Asympt. Scale – Invt.	Asympt. AdS
Objects grow to size Λ^{-1}	Objects fall to floor
Confinement $\mu = \Lambda$	Floor $r = r_{min}$

Hadrons:

Gauge Theory	**Gravity Theory**
Ultraviolet $\mu \to \infty$	Top $r \to \infty$
Hadrons	Cavity Modes
(mass splitting Λ)	(wavelength $\sim r_{min}$)
Confinement $\mu = \Lambda$	Floor $r = r_{min}$

Matching of parameters:

Gauge Theory	**Gravity Theory**
Ultraviolet $\mu \to \infty$	Top $r \to \infty$
$\lambda = g^2 N$	$R_{AdS}/\ell_s \propto \lambda^{1/4}$
N	$g_s = g^2 = \lambda/N$
Confinement $\mu = \Lambda$	Floor $r = r_{min}$

Perturbative?

Gauge Theory if $\lambda \ll 1$, String if $\lambda \gg 1$.

Gauge Theory	**Gravity Theory**
Ultraviolet $\mu \to \infty$	Top $r \to \infty$
Pert. : $\lambda \ll 1 \ll N$	Pert. : $1 \ll \lambda \ll N$
$\lambda = g^2 N$	$R_{AdS}/\ell_s \propto \lambda^{1/4}$
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QCD: $\lambda \ll 1$ in UV, runs to ~ 1 in IR.

String Theory never perturbative! Curvature large!

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So What Good Is This?

Theories with large λ are not QCD —

Cannot be used for numerical comparison with QCD

— *but*

- Unlike toy models of confinement, lattice gauge theory, they have correct kinematics

  (continuum 3+1 dims., UV scale-invt, IR discrete spectrum)
- They can be used to study aspects of nonperturbative physics involving physics on the light-cone, unstable hadrons, scattering and other dynamics — all largely inaccessible to weak-coupling methods, lattice theory
- The spectrum, form factors, etc. of low-spin hadrons are easily calculated.

In short, these are excellent *toy models* for QCD.

As with all toy models, care must be used in applying them and in extracting lessons.

- We do not expect to extract numbers for comparison with experiment.
- We expect the toy models will not always match QCD even qualitatively.
- But they may provide concrete insights into the workings of nonperturbative QCD and suggest new techniques applicable to QCD.

Blind calculation leads to useless information.

Must

- Understand where toy models differ from QCD to such a degree that their physics is qualitatively different and conclusions about QCD cannot be drawn.
- Understand where toy models and QCD share something essential, perhaps even universal to all gauge theories, or all confining gauge theories; here lessons about QCD might be obtained.

This process is well underway and I will tell you something of what we now know.

Aside:

These theories are not *merely* toy models for QCD!!

They are perfectly sensible four-dimensional field theories (with an equivalent description as a string theory.)

Models of these type can be *concrete* realizations of Randall-Sundrum compactifications and may show up at LHC and beyond.

Thus, they merit study in their own right.

Question: Consider high-energy $E \gg \Lambda$ fixed-large-angle $2 \to 2$ exclusive hadron scattering.

Ordinary QCD-like theory: $d\sigma/dt \sim E^{-n}$

String in flat 10-dimensions: $d\sigma/dt \sim e^{-E^2}$

For large-λ models = string theory on curved space [asymptotically AdS], which one applies?

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QCD-like power law!!!

Polchinski+MJS

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Power law at all λ! Even though stringy.

Follows from conformal field theory and factorization.

So here toy models match QCD, for a combination of kinematic, dynamical reasons.

Question: Consider deep inelastic scattering at high Q^2, moderate x.

Ordinary QCD-like theory: Bjorken scaling; hard scattering off valence partons.

String in flat 10-dimensions: Strings are soft, no hard scattering, no partons.

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Neither! Both!

Polchinski+MJS

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- DGLAP evolution (parton splitting) *rapid* at $\lambda \gg 1$: valence partons degrade, pdfs nonzero only for $x \gg 1$ even for moderate Q^2, no hard scattering off partons.

Lessons so far

- Aspects of QCD which follow from ultraviolet conformal invariance will also be true of large-λ gauge theories.
- Aspects of QCD which follow from properties of valence partons will not be true at large-λ, where "hadrons are all sea, partons are all wee."

Let's refine the last one...

Let's consider just one p.d.f. $f(x, Q^2)$.

Parton Model: $f(x, Q^2) = f(x)$

QCD: $f(x, Q^2) = f(x, Q_0^2) + \text{order}\,[\lambda \log(Q/Q_0)]$

More precisely...

$$M_n(Q^2) = \int_0^1 dx x^{n-1} f(x, Q^2)$$

$$\frac{dM_n}{dQ^2} = -\gamma_n \left(\alpha[Q^2]\right) M_n$$

where γ_n is the anomalous dimension of a twist-two operator of dimension $n+2$ and spin n, in QCD

$$\mathcal{O}_n = \bar{q}\gamma\partial\partial \ldots \partial q$$

$$\mathcal{O}_2 = T^{\mu\nu} \ , \ \gamma_2 = 0 \text{ (sum rule)}$$

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$\lambda \ll 1 \;\Rightarrow\; 0 = \gamma_2 < \gamma_n \sim \lambda$

(violations of Bjorken scaling are small in QCD, cause pdfs to move *slowly* to smaller x)

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$$\lambda \ll 1 \ \Rightarrow \ 0 = \gamma_2 < \gamma_n \sim \lambda \ll 1$$

$$\lambda \gg 1 \ \Rightarrow \ 0 = \gamma_2 < 1 \ll \gamma_n \sim \lambda^{1/4}$$

(violations of Bjorken scaling are large if $\lambda \gg 1$, cause pdfs to move *rapidly* to small x)

$$M_n(Q^2) = \int_0^1 dx x^{n-1} f(x, Q^2)$$

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(Crossover at $\lambda \sim 1$)

$$M_n(Q^2) = \int_0^1 dx x^{n-1} f(x, Q^2)$$

$$\frac{dM_n}{dQ^2} = -\gamma_n \left(\lambda[Q^2]\right) M_n$$

Nota Bene: For *all* λ , $0 = \gamma_2 < \gamma_n$.

This implies: *as* $Q^2 \to \infty$, *all partons become "wee"!!*

This large-Q^2 limit is well-known in QCD…

The large-λ theories just get there faster!

Qualitatively different, but not unfamiliar either.

Let's consider just one p.d.f. $f(x,Q^2)$.

Parton Model: $f(x,Q^2) = f(x)$

QCD: $f(x,Q^2) = f(x,Q_0^2) + \text{order}\,[\lambda \log(Q/Q_0)]$

Except at $x \ll 1$, $f(x,Q^2)$ tiny for $Q^2 \gg\gg\gg\gg \Lambda_{QCD}^2$

Large λ: $f(x,Q^2) = f(x,Q_0^2) + \text{order}\left[(Q/Q_0)^{\lambda^{1/4}}\right]$

Except at $x \ll 1$, $f(x,Q^2)$ tiny for $Q^2 \gg \Lambda^2$

What are some implications?

- High-Q^2 behavior of a form factor: for large class of hadrons, get similar power law, similar coefficient, for any λ.
- Drell-Yan: QCD gives many high-energy muon pairs in $p\bar{p}$ collisions, large–λ theories give none.
- Expect no high-energy central jets in $p\bar{p}$ collisions at large λ.

Does this mean we cannot study jets or Drell-Yan?

Not necessarily. It does mean we have to ask the right question in order to get the correct insights or useful information.

Electromagnetic Form Factors of Hadrons

Probe hadron $|H\rangle$ using a $U(1)$ current:

$$\langle H|J^\mu_{em}(q)|H\rangle \Rightarrow F(q^2)$$

Calculation in gravity theory yields:

Polchinski+Susskind; Hong,Yoon, MJS

$F(q^2) \to 1$ as $q^2 \to 0$ (Charge Conservation)

$F(q^2) \to q^{-2\Delta+2}$ as $q^2 \to \infty$ (Conformal Symmetry)

$$F(q^2) = \sum_n \frac{f_n \, g_{nHH}}{q^2 + m_n^2} \qquad \text{(Large } N\text{)}$$

Son+Stephanov

$$F(q^2) = \sum_n \frac{f_n \; g_{nHH}}{q^2 + m_n^2} \qquad\qquad (\text{Large } N)$$

where the spin-one hadrons $|n\rangle$ are created by the current

$$J_\mu|0\rangle = [q^2 g_{\mu\nu} - q_\mu q_\nu] f_n |n, \mu\rangle$$

and g_{nHH} is the coupling of these hadrons to H.

In QCD it is often approximated

$$F_\pi(q^2) \approx \frac{f_\rho \; g_{\rho\pi\pi}}{q^2 + m_\rho^2}$$

from which it follows

$$g_{\rho\pi\pi} \approx \frac{m_\rho^2}{f_\rho}$$

$$F(q^2) = \sum_n \frac{f_n\; g_{nHH}}{q^2 + m_n^2}$$

Curiously, in nature $g_{\rho\pi\pi} \approx g_{\rho NN}$.

"Vector–" or "ρ–dominance" conjecture:

$$F_H(q^2) \approx \frac{f_\rho\; g_{\rho HH}}{q^2 + m_\rho^2} + \ldots$$

for any $|H\rangle$ at small q^2.

(Dots indicate contributions of $\rho', \rho'', \cdots$.)

Since $F(0) = 1$, it follows that

$$g_{\rho HH} \approx \frac{m_\rho^2}{f_\rho}$$

the ρ's diagonal couplings are *universal*

i.e., independent of H

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Sakurai: universality — ρ as a gauge boson. . . ?

In AdS/QCD, the entire *set* of spin-one mesons, $\rho, \rho', \rho'', \ldots$, are cavity-modes of a *five-dimensional* gauge boson.

Question: Is ρ–dominance $\Rightarrow$ ρ–universality true in AdS/QCD?

ρ–dominance $\times$ not generically true

but despite this,

ρ–universality $\checkmark$ approximately true, to order 1

Hong,Yoon,MJS

Why?

Quasi-universality does *not* follow from conformal invariance or kinematics.

Follows from structure of calculation in gravity theory.

- ρ-universality is generically approximately true for all large–λ theories.
- ρ coupling is generically close to special value m_ρ^2/f_ρ.
- Proof uses fact that ρ is lightest state created by conserved current (lowest cavity mode of gauge boson.)
- Only other state with quasi-universal couplings is the lowest-lying spin-two glueball (created by $T^{\mu\nu}$)
- *Apparent* ρ–dominance follows [in particular, fit to ρ pole will work well] even when ρ–dominance [other poles negligible] is false.

Proof uses fact that it is lightest state created by conserved current (lowest cavity mode of gauge boson.)

Can this be translated into a QCD 'Proof' valid at all λ?

Apparent ρ–dominance follows [in particular, fit to ρ pole will work well] even when ρ–dominance [other poles negligible] is false.

Does this explain the data?

This appears to be a bit of universal physics — something true of QCD that is also true of other very different theories, but which does not follow from any known theorems.

Possible universal features of gauge theories:

- Anything following from ultraviolet conformal invariance.
- Approximate universality of ρ couplings.

Any others?

- Regge phenomena in scattering and DIS
- Regge spectrum of high-spin states
- Anomalous dimensions of high-spin operators

Question: Consider high-energy $E \gg \Lambda$ fixed-large-angle $2 \to 2$ exclusive hadron scattering.

Ordinary QCD-like theory: $\mathbf{d\sigma/dt \sim E^{-n}}$ $\surd$

String in flat 10-dimensions: $d\sigma/dt \sim e^{-E^2}$ $\times$

Power law at all λ! Even though stringy.

Question: Consider high-energy $E \gg \Lambda$ fixed *small*-angle $2 \to 2$ exclusive hadron scattering.

Ordinary QCD-like theory: no prediction.

perturbative 'Pomeron'?

String in flat 10-dimensions: $d\sigma/dt \sim s^t \sim E^{-E^2\theta^2}$

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perturbative 'Pomeron'?

String in flat 10-dimensions: $d\sigma/dt \sim s^t \sim E^{-E^2\theta^2}$

Data: agrees with string result for small $|t|$, complicated otherwise.

Large-λ theory: agrees with string for small $|t|$, predicts power-law fall-off otherwise.

So Regge behavior apparently true at all λ.

What about small-x physics in DIS?

Perturbative ‘hard Pomeron’ predicts

$$f(x,Q^2) \sim x^{1-\alpha_0}$$

where $\alpha_0 \sim 1 + 4\lambda/\pi$ is the hard Pomeron “intercept”.

Large-λ theory predicts

$$f(x,Q^2) \sim x^{1-\alpha_0} Q^{-2\Delta+2}$$

where $\alpha_0 = 2 - \text{order}(\lambda^{-1/2})$ is the corresponding intercept of the “graviton trajectory”, Δ is the dimension of the operator creating the hadron.

Can we understand similarity/difference?

$$f(x, Q^2) \sim x^{1-\alpha_0} Q^{-2\Delta+2}$$

- QCD: $\alpha_0 \sim 1 + 4\lambda/\pi$, $\Delta = 1$
- $\lambda \gg 1$: $\alpha_0 = 2 - \text{order}(\lambda^{-1/2})$, $\Delta = \dim \mathcal{O}_H$

  where $\mathcal{O}|0\rangle = |H\rangle$

Pomeron *is* graviton trajectory, gives $1 - \alpha_0$ for any λ.

α_0 depends on λ, varies from 1 to 2.

$Q^{-2\Delta+2}$?

$\lambda \ll 1$, Pomeron strikes valence partons ($\Delta = 1$);

$\lambda \gg 1$, no partons except at *ultra*-small x, so Pomeron strikes entire hadron.

So, properly interpreting the nonuniversal parts of the calculation, we see there is a universal core.

Similarly, if we consider the leading twist-two operators of spin n, dimension $n+2$ GKP,Kruczenski

$$\bar{q}\gamma\partial\partial\ldots\partial q$$

and their anomalous dimensions γ_n as $n \to \infty$, we find

$$\gamma_n \sim f(\lambda)\log n$$

where the function is not universal

$$f(\lambda) \sim \lambda \quad (\lambda \ll 1 \ll n)$$

$$f(\lambda) \sim \sqrt{\lambda} \quad (n \gg \lambda \gg 1)$$

but apparently the $\log n$ behavior is universal.

Where is this leading?

- There are many unanswered questions about nonperturbative QCD beyond the reach of the lattice, other methods.

- Our understanding of the large–λ models is gradually deepening.

Exclusive hard scattering (power laws and Regge physics)
Deep inelastic scattering (wee partons and small-x physics)
Form factors and hadron couplings

I have omitted a number of other developments for lack of time.
(Sum rules, finite temperature, chiral symmetry breaking)

- We are approaching the point where we can ask intelligent questions in ways such that the answers may realistically be useful, or at least instructive.
- The situation is becoming ripe for advances.

But:

- To fully study QCD we must understand $\lambda \sim 1$ gauge theories = string theory on strongly curved spaces.
- Here the progress has been very limited; a new breakthrough is needed.

Stay Tuned . . .

2003, Karch/Katz: $N \to \infty$, N_f finite simple.

Gauge Theory	**Gravity Theory**
Ultraviolet $\mu \to \infty$	Top $r \to \infty$
Asympt. Scale – Invt.	Asympt. AdS
Quark of mass m	Defect $r \geq (m/\Lambda) r_{min}$
Confinement $\mu = \Lambda$	Floor $r = r_{min}$

Quarkonium:

Quark-antiquark bound states (low spin)