# Ginzburg-Landau Equation from SU(2) Gauge Field Theory

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# Ginzburg-Landau Lagrangian from pure SU(2)

**The aim of this talk is to show how the Ginzburg-Landau equations can be motivated from pure SU(2) Gauge Field and some approximate quantization techniques.**

The Lagrangian density of GL is

$$L = -\frac{1}{4} f_{\mu\nu} f^{\mu\nu} + \left(D_\mu \varphi^*\right)\left(D^\mu \varphi\right) - m^2 |\varphi|^2 - \lambda |\varphi|^4$$

**The U(1) gauge fields of GL will come from the Abelian Projection of a U(1) subgroup of SU(2).**

**The scalar fields will come from the coset fields of SU(2)/U(1) and the quantization ideas about expectation values of fields will be similar to the old ideas of Heisenberg:**


W. Heisenberg, *Introduction to the Unified Field Theory of Elementary Particles*, Max-Planck-Institute für Physik und Astrophysik (Interscience Publishers, 1966);
W. Heisenberg, *Nachr. Akad. Wiss. Göttingen* **8**, 111 (1953);
W. Heisenberg, *Z. Naturforsch*. **9a**, 292 (1954);
W. Heisenberg, *Z. Phys.* **144**, 1 (1956);
W. Heisenberg, *Nucl. Phys.* **4**, 532 (1957);
W. Heisenberg, *Rev. Mod. Phys*. **29**, 269 (1957).

**This is important since some models of confinement use the dual Meissner effect which comes from the dual GL equations**

## Work based on

(i) V. Dzhunushaliev and D. Singleton, *Phys. Rev. D* **65**, 125007 (2002)

(ii) V. Dzhunushaliev and D. Singleton, *Modern Physics Letters* **A18** (2003)

# Abelian projection

**Using Abelian projection methods of Kondo**
K.-I. Kondo, *Phys. Rev. D* **57**, 7467 (1998)
**we break up the SU(2) gauge fields into a U(1) part and a coset part as follows**

$$A_\mu = A^B_\mu T^B = a_\mu T^a + A^m_\mu T^m,$$

$$a_\mu \in U(1) \text{ and } A^m_\mu \in SU(2)/U(1),$$

$$F^B_{\mu\nu} T^B = F^3_{\mu\nu} T^3 + F^m_{\mu\nu} T^m$$

$$F_{\mu\nu} = f_{\mu\nu} + \Phi_{\mu\nu} \in U(1),$$

$$f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu \in U(1),$$

$$\Phi_{\mu\nu} = g\mathring{a}^{3mn} A^m_\mu A^n_\nu \in U(1),$$

$$F^m_{\mu\nu} = F^m_{\mu\nu} + G^m_{\mu\nu} \in SU(2)/U(1),$$

$$F^m_{\mu\nu} = \partial_\mu A^m_\nu - \partial_\nu A^m_\mu \in SU(2)/U(1)$$

**We will perform an approximate quantization on the Lagrangian with the following assumptions**

**1. coset fields are treated as stochastic**

$$\left\langle A_\mu^m(x) \right\rangle = 0 \ \text{ and } \ \left\langle A_\mu^m(x) A_\nu^n(x) \right\rangle \neq 0$$

**2. U(1) fields are classical and not correlated to the coset fields**

$$\left\langle f(a_\mu)\, g(A_\nu^m) \right\rangle = \left\langle f(a_\mu) \right\rangle \left\langle g(A_\mu^m) \right\rangle$$

$$= f(a_\mu) \left\langle g(A_\mu^m) \right\rangle$$

# Ginzburg-Landau Lagrangian from pure SU(2)

**(a) The expectation value of the SU(2) Lagrangian can be split as**

$$-\frac{1}{4}\left\langle F^{A}_{\mu\nu} F^{A\mu\nu} \right\rangle = -\frac{1}{4}\left\langle F^{3}_{\mu\nu} F^{3}_{\mu\nu} \right\rangle - \frac{1}{4}\left\langle F^{m}_{\mu\nu} F^{m}_{\mu\nu} \right\rangle$$

**(b) From assumptions 1 and 2 the first term becomes**

$$\left\langle f_{\mu\nu} f^{\mu\nu} \right\rangle + \left\langle f_{\mu\nu} \Phi^{\mu\nu} \right\rangle + \left\langle \Phi_{\mu\nu} f^{\mu\nu} \right\rangle + \left\langle \Phi_{\mu\nu} \Phi^{\mu\nu} \right\rangle$$

$$f_{\mu\nu} f^{\mu\nu} \qquad f_{\mu\nu}\left\langle \Phi^{\mu\nu} \right\rangle \qquad \left\langle \Phi\Phi \right\rangle$$

## Ginzburg-Landau Lagrangian from pure SU(2)

**(a) To deal with the terms involving <_> and <__> we assume**

$$\left\langle \Phi_{\mu\nu} \right\rangle = g\varepsilon^{3mn} \left\langle A_\mu^m A_\nu^n \right\rangle \qquad \left\langle A_\mu^m(y) A_\nu^n(x) \right\rangle = -\delta^{mn} \eta_{\mu\nu} G(y,x)$$

**(b) Thus the cross terms vanish**

**(c) To deal with the quartic term we take**

$$\left\langle \Phi_{\mu\nu} \Phi^{\mu\nu} \right\rangle = g \begin{pmatrix} \left\langle A_\mu^1 A_\nu^2 A^{1\mu} A^{2\nu} \right\rangle + \left\langle A_\mu^2 A_\nu^1 A^{2\mu} A^{1\nu} \right\rangle \\ - \left\langle A_\mu^2 A_\nu^1 A^{1\mu} A^{2\nu} \right\rangle - \left\langle A_\mu^1 A_\nu^2 A^{2\mu} A^{1\nu} \right\rangle \end{pmatrix}$$

# Ginzburg-Landau Lagrangian from pure SU(2)

**(c) To deal with the quartic term we take**

$$\left\langle A_\alpha^m(x_1) A_\beta^n(x_2) A_\mu^p(x_3) A_\nu^q(x_4) \right\rangle = \delta^{mp}\delta^{nq}\eta_{\alpha\mu}\eta_{\beta\nu} G(x_1,x_3) G(x_2,x_4) + \delta^{mq}\delta^{np}\eta_{\alpha\nu}\eta_{\beta\mu} G(x_1,x_4) G(x_2,x_3) + \delta^{mn}\delta^{pq}\eta_{\alpha\beta}\eta_{\mu\nu} G(x_1,x_2) G(x_3,x_4)$$

$$\left\langle \Phi_{\mu\nu}(x) \Phi^{\mu\nu}(x) \right\rangle \approx 24 g^2 G^2(x,x)$$

**(d) Thus from the U(1) part we get**

$$\left\langle f_{\mu\nu} f^{\mu\nu} \right\rangle = f_{\mu\nu} f^{\mu\nu}$$

## Ginzburg-Landau Lagrangian from pure SU(2)

**(a) To deal with the coset part we look at**

$$\left\langle \mathsf{F}^{\,m}_{\mu\nu}(y)\,\mathsf{F}^{\,m\mu\nu}(x)\right\rangle\Big|_{y=x} =$$

$$\left(\left[\partial_{\mu y}A^m_\nu(y) - \mathsf{g}a_\mu(y)\varepsilon^{3mn}A^n_\nu(y)\right] - \left[\partial_{\nu y}A^m_\mu(y) - \mathsf{g}a_\nu(y)\varepsilon^{3mn}A^n_\mu(y)\right]\right)$$

$$\times\left(\left[\partial^\mu_x A^{m\nu}(x) - \mathsf{g}a^\mu(x)\varepsilon^{3mp}A^{p\nu}(x)\right] - \left[\partial^\nu_x A^{m\mu}(x) - \mathsf{g}a^\nu(x)\varepsilon^{3mp}A^{p\mu}(x)\right]\right)\Big|_{y=x}$$

This has many terms to deal with

**(b) One finds 4 terms like**

$$\left\langle \partial_{\mu y}A^m_\nu(y)\,\partial^\mu_x A^{m\nu}(x)\right\rangle\Big|_{y=x} = -8\,\partial_{\mu y}\partial^\mu_x \mathsf{G}(y,x)\Big|_{y=x}$$

# Ginzburg-Landau Lagrangian from pure SU(2)

**(c) One finds terms like**

$$\mathrm{g}^2 \varepsilon^{3mn} \varepsilon^{3mp} a_\mu(x) \langle A_\nu^n(x) A^{p\nu}(x) \rangle = -8\mathrm{g}^2 a_\mu a^\mu(x) \mathrm{G}(x,x)$$

**(d) Finally there are terms like which we approximate by**

$$\mathrm{g}\varepsilon^{3mn} a^\mu(x) \langle [\partial_{\mu y} A_\nu^m(y)] A^{n\nu}(x) \rangle \Big|_{y=x} = -\delta^{mn} \eta_{\alpha\beta} \partial_{\mu y} \mathrm{G}(y,x) -$$

$$i\varepsilon^{3mn} \eta_{\alpha\beta} \partial_{\mu y} \mathrm{P}(y,x)$$

**For the other cross term one finds similar result but with complex P**

# Ginzburg-Landau Lagrangian from pure SU(2)

**(a) Collecting all these results together gives**

$$\left\langle \mathrm{F}^{m}_{\mu\nu}\mathrm{F}^{m\mu\nu}\right\rangle = -20\begin{bmatrix}\partial_{\mu y}\partial^{\mu}_{x}\mathrm{G}(y,x)+\mathrm{g}^{2}a_{\mu}(x)a^{\mu}(x)\mathrm{G}(x,x)- \\ i\mathrm{g}a^{\mu}(x)\partial_{\mu x}\mathrm{P}^{*}(y,x)+i\mathrm{g}a^{\mu}(x)\partial_{\mu x}\mathrm{P}(y,x)\end{bmatrix}\Bigg|_{y=x}$$

**(b) Next we replace G and P by a complex scalar field**

$$\mathrm{G}(y,x)=\mathrm{P}(y,x)=\frac{1}{5}\varphi^{*}(y)\varphi(x)$$

**(c) Putting these results for the coset part together gives**

$$-4\left|\partial_{\mu}\varphi - i\mathrm{g}a_{\mu}\varphi\right|^{2}$$

# Ginzburg-Landau Lagrangian from pure SU(2)

**(d) Putting everything together gives**

$$\langle \mathcal{L} \rangle = -\frac{1}{4}\langle F^{A}_{\mu\nu} F^{A\mu\nu} \rangle = -\frac{1}{4} f_{\mu\nu} f^{\mu\nu} + \left(D_\mu \varphi^*\right)\left(D^\mu \varphi\right) - \frac{6g^2}{25}|\varphi|^4$$

U(1) coset

**This is the massless GL equation. To have proper symmetry breaking we need a tachyonic mass term.**

# Mass term via Ghost Condensate

**(a) In References**

D. Dudal and H. Verschelde, “On ghost condensation and Abelian dominance in the maximal Abelian gauge”, hep-th/0209025

V. E. R. Lemes, M. S. Sarandy and S. P. Sorella, “Ghost number dynamical symmetry breaking in Yang-Mills theories in the maximal Abelian gauge", hep-th/0206251

**it was shown that the off-diagonal gluons for SU(2) developed a tachyonic mass via ghost/anti-ghost condensate**

## Mass term via Ghost Condensate

**(b) The gauge fixing and ghost parts can be written as**

$$L_{GF} + L_{FP} = -\frac{1}{2\alpha}\left(D_{\mu}^{mn} A^{\mu n}\right)^2 + i\overline{C}^m D_{\mu}^{mp} D^{\mu pn} C^n - i g^2 \varepsilon^{mq} \varepsilon^{pn} \overline{C}^m C^n A^{\mu p} A_{\mu}^q$$

$$+ \frac{\alpha}{4} g^2 \varepsilon^{mn} \varepsilon^{pq} \overline{C}^m \overline{C}^n C^p C^q$$

$$D_{\mu}^{mn} = \partial_{\mu} \delta^{mn} - g \varepsilon^{mn} a_{\mu}$$

**For the quartic ghost interaction one can introduce an auxiliary field** $\psi$

$$\frac{\alpha}{4} g^2 \varepsilon^{mn} \varepsilon^{pq} \overline{C}^m \overline{C}^n C^p C^q \rightarrow -\frac{1}{2\alpha g^2} \varnothing^2 - i\psi \varepsilon^{mn} \overline{C}^m C^n$$

# Mass term via Ghost Condensate

**(c) One can collect together kinetic energy terms for ghost field and interaction terms for the auxiliary field to arrive at**

$$\mathcal{L}_{ghost} = i\overline{C}^m \partial_\mu \partial^\mu C^m - \frac{1}{2\alpha g^2}\psi^2 - i\psi\varepsilon^{mn}\overline{C}^m C^n$$

**(d) One can apply the Coleman-Weinberg mechanism to this and find that $\psi$ develops a radiatively induced symmetry breaking form which has a non-zero vacuum expectation value of**

$$\psi = \pm\upsilon = \pm\overline{\mu}^2 \exp\left(1 - \frac{8\pi}{\alpha \overline{g}^2 \overline{\mu}^2}\right)$$

# Mass term via Ghost Condensate

**(a) From this it is found by investigating the ghost propagator that ghost/anti-ghost condensation occurs.**

$$\left\langle i\overline{C}^{m} C^{m} \right\rangle = -\frac{\upsilon}{16\pi} < 0$$

**(b) Then the ghost/anti-ghost/gauge field/gauge field interaction term from the GF+ghost Lagrangian becomes**

$$i\mathrm{g}^2 \varepsilon^{mn} \varepsilon^{pq} \overline{C}^{m} C^{n} A^{\mu p} A^{q}_{\mu} \rightarrow \frac{1}{2}\mathrm{g}^2 \left\langle i\overline{C}^{m} C^{m} \right\rangle \left\langle A^{n}_{\mu} A^{\mu n} \right\rangle$$

$$= \frac{1}{2}\mathrm{g}^2 \left( -\frac{\upsilon}{16\pi} \right) \left( -8\mathrm{G} \right) = \frac{\upsilon \mathrm{g}^2}{20\pi} \varphi^{*} \varphi$$

**(c) Putting this together with the quartic term now gives**

$$V_{\varphi} = -\frac{\upsilon g^{2}}{20\pi}\left|\varphi\right|^{2} + \frac{6g^{2}}{25}\left|\varphi\right|^{4}$$

**(d) This is the massive GL equation which gives symmetry breaking and gives Nielsen-Olesen flux tubes**

# Conclusions

(i) Split SU(2) fields into diagonal (ordered phase) and non-diagonal parts (disordered phase).

(ii) Using some approximation for the expectation values of the diagonal fields we arrive at massless GL equation.

(iii) Using ghost/anti-ghost condensate ideas we arrive at an effective massive GL equation with off-diagonal fields playing the role of the scalar field.

(iv) The dual version of this would give the dual Meissner effect and dual Nielsen-Olesen flux tubes which are thought to be important to confinement.

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