Covariant Light Front Approach for s-wave and p-wave Mesons

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Introduction

- Interest in even-parity charmed mesons has been revived by recent discoveries:
  two narrow resonances: D_{s0}^* (2317) (BABAR 03); D_{s1}(2460) (CLEO 03)
  two broad resonances: D_0^* (Belle 03,FOCUS 04), D_1 (Belle)

- The only systematic analysis for s- to p-wave transitions is the Isgur-Scora-Grinstein-Wise (ISGW) QM (ISGW89), which is nonrelativistic.

- Relativistic effect could be important at q^2=0 where final-state meson could be highly relativistic

- Light-front QM, which is the only relativistic QM, has been employed to obtain decay constants and weak form factors (Jaus 90,91,96; Ji,Chung,Cotanch 92, HYC,Cheung,Hwang 97).
  - So far it has been only applied to s- to s-wave meson transitions.
  - There exist some ambiguities in extracting the physical quantities (non-covariant).

- Covariant LFQM have been constructed
  - in (HYC,Cheung,Hwang,Zhang 98) within the framework of HQET
  - in (Jaus 1999) without using HQ limit
  - both apply to s- to s-wave meson transitions only

- We wish to extend the covariant LFQM to even-parity, p-wave mesons and study the corresponding Isgur-Wise functions.

Decay constants

- Mesons can be annihilated by vector, axial vector currents:

$$\langle 0|A_\mu|P(P')\rangle = if_P P'_\mu, \qquad \langle 0|V_\mu|V(P',\varepsilon')\rangle = M' f_V \varepsilon'_\mu,$$

$$\langle 0|V_\mu|S(P')\rangle = f_S P'_\mu, \qquad \langle 0|A_\mu|^{3(1)}A(P',\varepsilon')\rangle = M' f_{^{3(1)}A} \varepsilon'_\mu.$$

$^{2S+1}L_J = {}^3P_1, {}^1P_1$

- Two classes of constraints for decay constants:

(a) In SU(N) limit

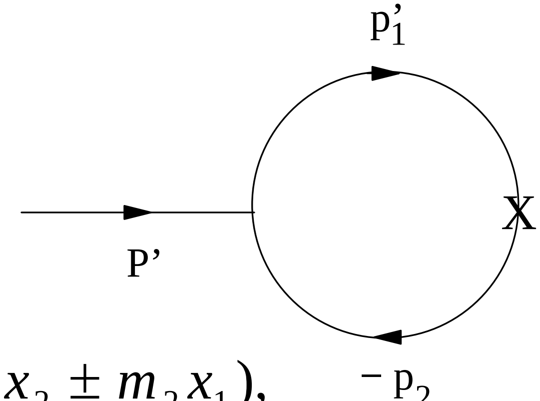
$$f_S = 0, \quad f_{^1A} = 0.$$

(b) In HQ limit

$$J^P_{j(light)} = (0^-_{1/2}, 1^-_{1/2}), \; (0^+_{1/2}, 1^+_{1/2}), \; (1^+_{3/2}, 2^+_{3/2})$$

$$f_V = f_P, \quad f_{A^{1/2}} \equiv \sqrt{\frac{1}{3}} f_{^1A} - \sqrt{\frac{2}{3}} f_{^3A} = f_S, \quad f_{A^{3/2}} \equiv \sqrt{\frac{2}{3}} f_{^1A} + \sqrt{\frac{1}{3}} f_{^3A} = 0,$$

- In the one-loop approximation we obtain:



$$f_{P,S} = \frac{N_C}{16\pi^3}\int dx\, d^2p'_\perp \frac{4h'_{P,S}}{x_1x_2(M'^2 - M_0'^2)}(m'_1x_2 \pm m_2x_1),$$

$$f_{V,{}^3A} = \pm\frac{N_C}{16\pi^3 M'}\int dx\, d^2p'_\perp \frac{h'_{V,{}^3A}}{x_1x_2(M'^2 - M_0'^2)}\left[x_1M_0'^2 - m'_1(m'_1 \mp m_2) - p_\perp'^2 + \frac{m'_1 \pm m_2}{w'_{V,{}^3A}}p_\perp'^2\right],$$

$$f_{{}^1A} = \frac{N_C}{16\pi^3 M'}\int dx\, d^2p'_\perp \frac{h'_{{}^1A}}{x_1x_2(M'^2 - M_0'^2)}\left(\frac{m'_1 - m_2}{w'_{{}^1A}}p_\perp'^2\right),$$

M_0:kinetic mass
h: vertex functions
$x_{1,2}$: momentum frac. of quark, antiquark.

- f(S,^{3}A) ~ f(P,V) with $m_2 \to - m_2$

- It is easily seen that $f_S = 0,\ f_{{}^1A} = 0$ in the SU(N) limit ($m'_1=m_2$).

PVSAA

$^{2S+1}L_J$	$\beta_{u\bar d}$	$\beta_{s\bar u}$	$\beta_{c\bar u}$	$\beta_{c\bar s}$	$\beta_{b\bar u}$
$^1S_0$	0.3102	0.3864	0.4496	0.4945	0.5329
$^3S_1$	0.2632	0.2727	0.3814	0.3932	0.4764
$^3P_0$	β_{a_1}	$\beta_{K(^3P_1)}$	0.3305	0.3376	0.4253
$^3P_1$	0.2983	0.303	0.3305	0.3376	0.4253
$^1P_1$	β_{a_1}	$\beta_{K(^3P_1)}$	0.3305	0.3376	0.4253

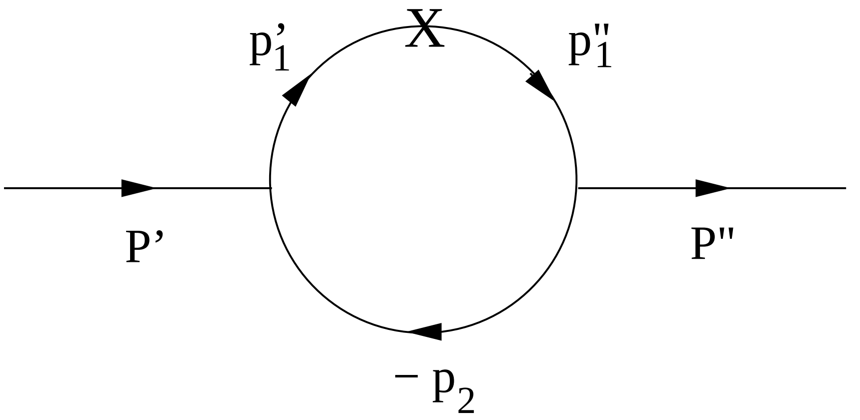
PVSAA

$^{2S+1}L_J$	$f_{u\bar d}$	$f_{s\bar u}$	$f_{c\bar u}$		$f_{c\bar s}$	$f_{b\bar u}$
$^1S_0$	(131)	(160)	(200)	<	(230)	(180)
$^3S_1$	(216)	(210)	(220)		(230)	(180)
$^3P_0$	0	21	86	>	71	112
$^3P_1$	(−203)	−186	−127		−121	−123
$^1P_1$	0	11	45		38	68
$P_1^{1/2}$	–	–	130		122	140
$P_1^{3/2}$	–	–	−36		−38	−15

- $f_{K_0^*(1430)} = 21\,\mathrm{MeV}$ is close to finite energy sum-rule result.
- $f_{D_s} > f_D,\ f_{D_{s0}^*} < f_{D_0^*}$ due to the different relative signs.
- Small $f_{D_{s0}^*}$ is favorable from $B \to \overline{D} D_{s0}^*$ decay (Belle 03).
- Consistent with SU(N) and HQ expectations.

Form Factors

- Form factors are calculated in one-loop approximation.



- For technical reason, form factors are first obtained in spacelike region ($q^2<0$). We fit them with

$$F(q^2) = \frac{F(0)}{1 - a(q^2/m^2_{B(D)}) + b(q^4/m^4_{B(D)})}$$

for B(D)→M transition and then analytically continue them to timelike region ($q^2>0$) .

- FF(P→S,^{3}A) ~ FF(P→P,V) with $m_1'' \to -m_1''$ (mass of final state quark)

Form Factors: numerical results (B → π …)

Form factors for $B \to \pi, \rho, a_0(1450), a_1(1260), b_1(1235), a_2(1320)$ transitions.

F	$F(0)$	$F(q^2_{\rm max})$	a	b
$F_1^{B\pi}$	0.25	1.16	1.73	0.95
$V^{B\rho}$	0.27	0.79	1.84	1.28
$A_1^{B\rho}$	0.22	0.53	0.95	0.21
$F_1^{Ba_0}$	0.26	0.68	1.57	0.70
A^{Ba_1}	0.25	0.76	1.51	0.64
$V_1^{Ba_1}$	0.37	0.42	0.29	0.14
A^{Bb_1}	0.10	0.23	1.92	1.62
$V_1^{Bb_1}$	0.18	0.36	1.03	0.32
h	0.008	0.015	2.20	2.30
b_+	−0.005	−0.011	1.95	1.80

F	$F(0)$	$F(q^2_{\rm max})$	a	b
$F_0^{B\pi}$	0.25	0.86	0.84	0.10
$A_0^{B\rho}$	0.28	0.76	1.73	1.20
$A_2^{B\rho}$	0.20	0.57	1.65	1.05
$F_0^{Ba_0}$	0.26	0.35	0.55	0.03
$V_0^{Ba_1}$	0.13	0.32	1.71	1.23
$V_2^{Ba_1}$	0.18	0.36	1.14	0.49
$V_0^{Bb_1}$	0.39	0.98	1.41	0.66
$V_2^{Bb_1}$	−0.03*	−0.15*	2.13*	2.39*
k	0.031	0.010	−2.47	2.47
b_-	0.0016	0.0011	−0.23	1.18

$$F^{Ba_{0,1}} \approx F^{B\pi,\rho}$$

$$F(q^2) = \frac{F(0)}{1 - a(q^2/m_B^2) + b(q^4/m_B^4)}$$

Form Factors: numerical results (B →K …)

Form factors for $B \to K, K^*, K_0^*(1430), K_{^1P_1}, K_{^3P_1}, K_2^*(1430)$ transitions.

F	$F(0)$	$F(q^2_{\max})$	a	b	F	$F(0)$	$F(q^2_{\max})$	a	b
F_1^{BK}	0.35	2.17	1.58	0.68	F_0^{BK}	0.35	0.80	0.71	0.04
V^{BK^*}	0.31	0.96	1.79	1.18	$A_0^{BK^*}$	0.31	0.87	1.68	1.08
$A_1^{BK^*}$	0.26	0.58	0.93	0.19	$A_2^{BK^*}$	0.24	0.70	1.63	0.98
$F_1^{BK_0^*}$	0.26	0.70	1.52	0.64	$F_0^{BK_0^*}$	0.26	0.33	0.44	0.05
$A^{BK_{^3P_1}}$	0.26	0.69	1.47	0.59	$V_0^{BK_{^3P_1}}$	0.14	0.31	1.62	1.14
$V_1^{BK_{^3P_1}}$	0.39	0.42	0.21	0.16	$V_2^{BK_{^3P_1}}$	0.17	0.30	1.02	0.45
$A^{BK_{^1P_1}}$	0.11	0.25	1.88	1.53	$V_0^{BK_{^1P_1}}$	0.41	0.99	1.40	0.64
$V_1^{BK_{^1P_1}}$	0.19	0.35	0.96	0.30	$V_2^{BK_{^1P_1}}$	-0.05^*	-0.16^*	1.78^*	2.12^*
h	0.008	0.018	2.17	2.22	k	0.015	0.004	-3.70	1.78
b_+	-0.006	-0.013	1.96	1.79	b_-	0.002	0.002	0.38	0.92

start to show deviation,
due to $m_s \to - m_s$

$$F(q^2) = \frac{F(0)}{1 - a(q^2/m_B^2) + b(q^4/m_B^4)}$$

Form Factors: numerical results (B→D …)

Form factors for $B \to D, D^*, D_0^*, D_1^{1/2}, D_1^{3/2}, D_2^*$ transitions. For the purpose of comparing with heavy quark symmetry, the form factors $u_\pm, c_\pm, \ell, q$ are also shown.

F	$F(0)$	$F(q^2_{\rm max})$	a	b	F	$F(0)$	$F(q^2_{\rm max})$	a	b
F_1^{BD}	0.67	1.22	1.25	0.39	F_0^{BD}	0.67	0.92	0.65	0.00
V^{BD^*}	0.75	1.32	1.29	0.45	$A_0^{BD^*}$	0.64	1.17	1.30	0.31
$A_1^{BD^*}$	0.63	0.83	0.65	0.02	$A_2^{BD^*}$	0.61	0.95	1.14	0.52
$F_1^{BD_0^*}$	0.24	0.34	1.03	0.27	$F_0^{BD_0^*}$	0.24	0.20	−0.49	0.35
$A^{BD_1^{1/2}}$	−0.12	−0.14	0.71	0.18	$V_0^{BD_1^{1/2}}$	0.08	0.13	1.28	−0.29
$V_1^{BD_1^{1/2}}$	−0.19	−0.13	−1.25	0.97	$V_2^{BD_1^{1/2}}$	−0.12	−0.14	0.67	0.20
$A^{BD_1^{3/2}}$	0.23	0.33	1.17	0.39	$V_0^{BD_1^{3/2}}$	0.47	0.70	1.17	0.03
$V_1^{BD_1^{3/2}}$	0.55	0.51	−0.19	0.27	$V_2^{BD_1^{3/2}}$	−0.09*	−0.17*	2.14*	4.21*
u_+	−0.24	−0.34	1.03	0.27	u_-	0.31	0.42	0.86	0.20
$\ell_{1/2}$	0.56	0.38	−1.25	0.97	$q_{1/2}$	0.041	0.050	0.71	0.18
$c_+^{1/2}$	−0.042	−0.050	0.67	0.20	$c_-^{1/2}$	0.045	0.055	0.71	0.20
$\ell_{3/2}$	−1.56	−1.45	−0.19	0.27	$q_{3/2}$	−0.079	−0.114	1.17	0.39
$c_+^{3/2}$	−0.032*	−0.061*	2.14*	4.21*	$c_-^{3/2}$	−0.027	−0.026	0.03	0.45
h	0.015	0.024	1.67	1.20	k	0.79	1.12	1.29	0.93
b_+	−0.013	−0.021	1.68	0.98	b_-	0.011	0.016	1.50	0.91

- It is non-trivial to have correct signs & relations consistent with HQS

Form Factors: comparison (s to s-wave)

Form factors of $D \to \pi, \rho, K, K^*$, $B \to \pi, \rho, K, K^*$, $B \to D, D^*$ transitions at $q^2 = 0$ in various models.

Model	$F_{1,0}^{D\pi}(0)$	$A_0^{D\rho}(0)$	$A_1^{D\rho}(0)$	$A_2^{D\rho}(0)$	$V^{D\rho}(0)$	$F_{1,0}^{DK}(0)$	$A_0^{DK^*}(0)$	$A_1^{DK^*}(0)$	$A_2^{DK^*}(0)$	$V^{DK^*}(0)$
This work	0.67	0.64	0.58	0.48	0.86	0.78	0.69	0.65	0.57	0.94
MS	0.69	0.66	0.59	0.49	0.90	0.78	0.76	0.66	0.49	1.03
QSR	0.5	0.6	0.5	0.4	1.0	0.6	0.4	0.5	0.6	1.1
BSW	0.69	0.67	0.78	0.92	1.23	0.76	0.73	0.88	1.15	1.23
Model	$F_{1,0}^{B\pi}(0)$	$A_0^{B\rho}(0)$	$A_1^{B\rho}(0)$	$A_2^{B\rho}(0)$	$V^{B\rho}(0)$	$F_{1,0}^{BK}(0)$	$A_0^{BK^*}(0)$	$A_1^{BK^*}(0)$	$A_2^{BK^*}(0)$	$V^{BK^*}(0)$
This work	0.25	0.28	0.22	0.20	0.27	0.35	0.31	0.26	0.24	0.31
MS	0.29	0.29	0.26	0.24	0.31	0.36	0.45	0.36	0.32	0.44
LCSR	0.26	0.37	0.26	0.22	0.34	0.34	0.47	0.34	0.28	0.46
BSW	0.33	0.28	0.28	0.28	0.33	0.38	0.32	0.33	0.33	0.37

Model	$F_{1,0}^{BD}(0)$	$A_0^{BD^*}(0)$	$A_1^{BD^*}(0)$	$A_2^{BD^*}(0)$	$V^{BD^*}(0)$
This work	0.67	0.64	0.63	0.62	0.75
MS	0.67	0.69	0.66	0.62	0.76
BSW	0.69	0.62	0.65	0.69	0.71

- Basically our results agree with others

Data of $B^- \to \pi^- \pi^0$ and $B \to \rho\pi$ favor $F_0^{B\pi}(0) \sim 0.25$ and a small $A_0^{B\rho}(0)$

Form Factors: comparison (B to D**)

- Nonrelativistic treatment should be OK in the b to c transition.

Form factors of $B \to D^{**}$ transitions calculated in the ISGW2 model.

F	$F(0)$	$F(q^2_{\max})$	a	b	F	$F(0)$	$F(q^2_{\max})$	a	b
$F_1^{BD_0^*}$	0.18	0.24	0.28	0.25	$F_0^{BD_0^*}$	0.18	−0.008	–	–
$A^{BD_1^{1/2}}$	−0.16	−0.21	0.87	0.24	$V_0^{BD_1^{1/2}}$	0.18	0.23	0.89	0.25
$V_1^{BD_1^{1/2}}$	−0.19	0.006	–	–	$V_2^{BD_1^{1/2}}$	−0.18	−0.24	0.87	0.24
$A^{BD_1^{3/2}}$	0.16	0.19	0.46	0.065	$V_0^{BD_1^{3/2}}$	0.43	0.51	0.54	0.074
$V_1^{BD_1^{3/2}}$	0.40	0.32	−0.60	1.15	$V_2^{BD_1^{3/2}}$	−0.12	−0.19	1.45	0.83
u_+	−0.18	−0.24	0.88	0.25	u_-	0.46	0.62	0.87	0.25
$\ell_{1/2}$	0.54	−0.016	–	–	$q_{1/2}$	0.057	0.074	0.87	0.24
$c_+^{1/2}$	−0.064	−0.083	0.87	0.24	$c_-^{1/2}$	0.068	0.088	0.87	0.24
$\ell_{3/2}$	−1.15	−0.90	−0.60	1.15	$q_{3/2}$	−0.057	−0.066	0.46	0.065
$c_+^{3/2}$	−0.043	−0.066	1.45	0.83	$c_-^{3/2}$	−0.018	−0.013	0.23	5.38
h	0.011	0.014	0.86	0.23	k	0.60	0.68	0.40	0.68
b_+	−0.010	−0.013	0.86	0.23	b_-	0.010	0.013	0.86	0.23

- At low q^2, most FFs agree with ISGW2 calculation within 40%.
- q^2-dependence is different in general.

Form Factors: comparison (heavy to light)

- Using LCSR, Chernyak (01) obtained $F_{0,1}^{Ba_0(1450)}(0) = 0.46$ while we have 0.26.
- For B$\to$ a_1 FFs:

$B \to a_1(1260)$ transition form factors at $q^2 = 0$ in various models.

Model	$A^{Ba_1}(0)$	$V_0^{Ba_1}(0)$	$V_1^{Ba_1}(0)$	$V_2^{Ba_1}(0)$
This work	0.25	0.13	0.37	0.18
ISGW2	0.21	1.01	0.54	-0.05
CQM	0.09	1.20	1.32	0.34
QSR	-0.41 ± 0.06	-0.23 ± 0.05	-0.68 ± 0.08	-0.33 ± 0.03

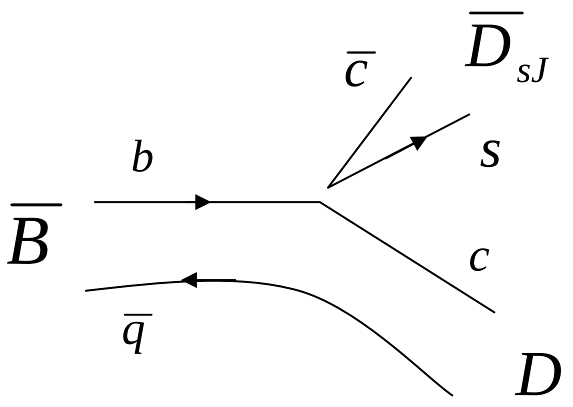
- $V_0^{Ba_1} \approx 1$ is unlikely, since we expect $V_{0,1}^{Ba_1} \approx A_{0,1}^{B\rho} = O(0.2)$
- It can be tested in $\overline{B}^0 \to a_1^+ \pi^-$. Recently BaBar reported

$$Br(B^0 \to a_1^+ \pi^-) = (42.6 \pm 4.2 \pm 4.1) \times 10^{-6}$$

not a test of the form factor V_0

Comparison with experiment

- From $B \to \overline{D} D_{sJ} (D_{sJ} \to D_s^{(*)} \pi^0)$ decays, we obtain



$$f_{D_{s0}^*(2317)} \approx 47 - 73\,\text{MeV}$$

$$f_{D_{s1}(2460)} \approx 110 - 190\,\text{MeV}$$

- Our predictions,

$$f_{D_{s0}^*(2317)} = 71\,\text{MeV},\; f_{D_{s1}(2460)} = 117\,\text{MeV}$$

are in agreement with data (recent BaBar results also seem to support this).

Comparison with experiment:

Compare with B→ D^{**} π decays:

Decay	This work	ISGW2	Expt
$B^- \to D_0^*(2308)^0\pi^-$	7.3×10^{-4}	4.8×10^{-4}	$(9.2 \pm 2.9) \times 10^{-4}$ [17]
$B^- \to D_1(2427)^0\pi^-$	4.6×10^{-4}	9.4×10^{-4}	$(7.5 \pm 1.7) \times 10^{-4}$ [17]
$B^- \to D_1'(2420)^0\pi^-$	1.1×10^{-3}	8.2×10^{-4}	$(9.3 \pm 1.4) \times 10^{-4}$ [17, 54]
			$(1.5 \pm 0.6) \times 10^{-3}$ [18]
$B^- \to D_2^*(2460)^0\pi^-$	1.0×10^{-3}	5.7×10^{-4}	$(7.4 \pm 0.8) \times 10^{-4}$ [17, 54]

	SCET	CLFQM	ISGW2	Neubert	Expt.
$\frac{Br(D_2^{*0}(2460)\pi^-)}{Br(D_1'(2420)\pi^-)}$	1	0.91	0.67	0.35	0.80±0.07 ±0.16 (BaBar) 0.77 ± 0.15 (Belle) 1.8 ±0.8 (CLEO)

Our predictions are in agreement with data

Heavy Quark Limit

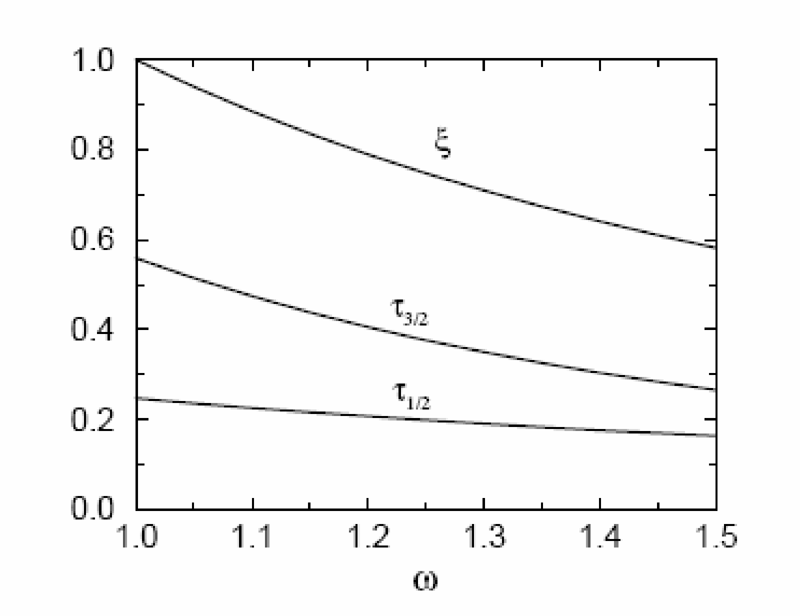
- We use both top-down and bottom up approach and we obtain consistent results.
  - Top-down: we re-derive (HYC,Cheung,Hwang,Zhang 98) Feynman rules.
  - Bottom-up: apply the "heavy quark mass → infinity" limit to our analytic results.

- Decay constant HQ relations checked.

$$f_V = f_P, \quad f_{A^{1/2}} \equiv \sqrt{\frac{1}{3}} f_{^1A} - \sqrt{\frac{2}{3}} f_{^3A} = f_S, \quad f_{A^{3/2}} \equiv \sqrt{\frac{2}{3}} f_{^1A} + \sqrt{\frac{1}{3}} f_{^3A} = 0,$$

- In HQ limit, FFs are related to some universal IW functions.
  - One IW function (ξ) for P to P,V transitions.
  - Two IW functions ($\tau_{1/2}$, $\tau_{3/2}$) for P to S, A transitions.

Isgur-Wise functions



CLF ⇒ $\tau_{1/2}(1)=0.31$, $\tau_{3/2}(1)=0.61$

in agreement with a recent lattice QCD calculation (Becirevic et al. 04): $\tau_{1/2}(1)=0.38\pm0.05$, $\tau_{3/2}(1)=0.58\pm0.08$

- Sum-rules due to Bjorken and Uraltsev are satisfied

$$Uraltsev: \quad \sum_n |\tau_{3/2}^{(n)}(1)|^2 - \sum_n |\tau_{1/2}^{(n)}(1)|^2 = \frac{1}{4}$$

$$Bjorken: \quad \rho^2 = \frac{1}{4} + \sum_n |\tau_{1/2}^{(n)}(1)|^2 + 2\sum_n |\tau_{3/2}^{(n)}|^2$$

Conclusion:

- A first RQM treatment of the decay constants and FFs for p-wave mesons.

- Comparing to the ISGW2 model (NRQM), B→D** transition form factors agree within 40% at $q^2=0$. Predictions on decay constants and B→D**π rates are in good agreement with data.

- The universal IW functions $\xi(\omega)$, $\tau_{1/2}(\omega)$ and $\tau_{3/2}(\omega)$ are obtained. Bjorken and Uraltsev sum rules for the IW functions are fairly satisfied.

- We apply the formalism to other processes, such as $B \to K^*\gamma$, $K_{1,2}\,\gamma$ decays obtaining ($T_1^{K^*\gamma}(0)=0.24$ in good agreement with data) and pentaquark decays.