Next-to-leading threshold resummation with time-like prescription

Giulia Ricciardi

Università di Napoli "Federico II", Italy

DPF2004, University of California, Riverside, CA, USA (27-31 Aug)

Plan of the talk

- **Jet processes at large x**
  - ♦ **Large logs at threshold x→1**
  - ♦ **Resumming into the form factor**
- **NLO log resumming**
  - ♦ **Calculation of form factors in perturbative QCD**
  - ♦ **Integration over the Landau pole**
  - ♦ **Form factor divergent at large x**
- **Improved prescription**
  - ♦ **Space-like to time-like QCD coupling: keeping trace**
  - ♦ **Regular form factors at large x**
  - ♦ **Great advantage in phenomenological calculations**

Large logs at threshold

In hard QCD processes (i.e. Drell-Yan, semi-inclusive heavy quark decays…) at threshold:

- ♦ **Large logs in (1-x) at x → 1**
- ♦ **Switching to Mellin transforms, large logs in N at N → ∞**

$$\sum_{m=0}^{2n} G_{nm}\, \alpha_s{}^n (\log N)^m$$

Physically:

- **observed final state at maximum energy**
- **real gluon emission is inhibited**
- **decays "loose" total inclusiveness**
  ⇒ soft and collinear singularities

Resumming into the form factor

- **Large logs exponentiate and factorize in the form factor $f_N(\alpha_S)$**
- **Reorganizing in series of λ:**

$$\lambda = \beta_0 \, \alpha_S(Q^2) \, \log N$$

$$f_N(\alpha_S) = \exp\left[L \, g_1(\lambda) + \sum_{n=0}^{\infty} \alpha_S^n \, g_{n+2}(\lambda) \right]$$

$$g_i(\lambda) = \sum_{n=1}^{\infty} g_{i,n} \lambda^n$$

- **Standard Fixed Log Approximation**
  - **Series in $\log N \, (\alpha_S \log N)^n$ LO**
  - **Series in $(\alpha_S \log N)^n$ NLO (truncating at g_2)**

….

Calculation of form factors in perturbative QCD

$$
\begin{aligned}
f_N(\alpha_S) \;=\; & \exp \int_0^1 dz \, \frac{z^{N-1}-1}{1-z} \left\{ \int_{Q^2(1-z)^2}^{Q^2(1-z)} \frac{dk_t^2}{k_t^2} \left[A_1\, \alpha_S(k_t^2) + A_2\, \alpha_S(k_t^2)^2 + \cdots \right] + \right. \\
& \left. + \; B_1\, \alpha_S(Q^2(1-z)) + \cdots + D_1\, \alpha_S(Q^2(1-z)^2) + \cdots \right\}
\end{aligned}
$$

Main problem:
$\alpha_S(k_t^2)$ integrated over gluon transverse momenta k_t from Q to 0:
hits the Landau pole
(at low k_t, large QCD coupling)

- **g_1, g_2 have branch cuts starting at $\lambda = ½$**
- **form factor divergent at large N (large x)**

Standard Prescription

- **At one loop α_S ⟶ effective coupling**

$$\alpha_S \quad \to \quad \tilde{\alpha}_S(k_t^2) = \frac{1}{2\pi i}\int_0^{k_t^2} ds \; \mathrm{Disc}_s \; \frac{1}{s\;\beta_0 \log\frac{-s}{\Lambda^2}}$$

$$\tilde{\alpha}_S(k_t^2) = \int_{-\pi}^{+\pi} \frac{d\varphi}{2\pi} \frac{1}{\beta_0\left[\log\frac{k_t^2}{\Lambda^2} + i\varphi\right]}$$

- **Usually the imaginary part is neglected**

$k_t \cong Q$ and $\log\frac{k_t^2}{\Lambda^2} \gg \pi$

$$\tilde{\alpha}_S(k_t^2) \quad \to \quad \frac{1}{\beta_0 \log\frac{k_t^2}{\Lambda^2}} \quad = \quad \alpha_S(k_t^2)$$

- **Questionable, since k_t is integrated up to 0**

Improved Prescription

- **We <u>do not neglect</u> the imaginary term**
  - **physically, we include effects related to the decay of gluon jets in the effective coupling constant**

- **Performing the exact integration**

$$\boxed{\tilde{\alpha}_S(k_t^2) = \frac{1}{\beta_0}\left[\frac{1}{2} - \frac{1}{\pi}\arctan\frac{\log\frac{k_t^2}{\Lambda^2}}{\pi}\right]}$$

  - **The effective coupling is positive definite, monotonically decreasing and has a finite limit at zero momentum**
  - **It reproduces the standard coupling at $k_t^2 \gg \Lambda^2$, but it does not contain the infrared pole at $k_t^2 = \Lambda^2$**

Improved Prescription

Calculation of improved form factors in perturbative QCD

$$f_N(\alpha_S) = \exp \int_0^1 dz \, \frac{z^{N-1} - 1}{1 - z} \left\{ \int_{Q^2(1-z)^2}^{Q^2(1-z)} \frac{dk_t^2}{k_t^2} A_1 \, \tilde{\alpha}_S(k_t^2) + B_1 \, \tilde{\alpha}_S(Q^2(1-z)) + D_1 \, \tilde{\alpha}_S(Q^2(1-z)^2) \right\}$$

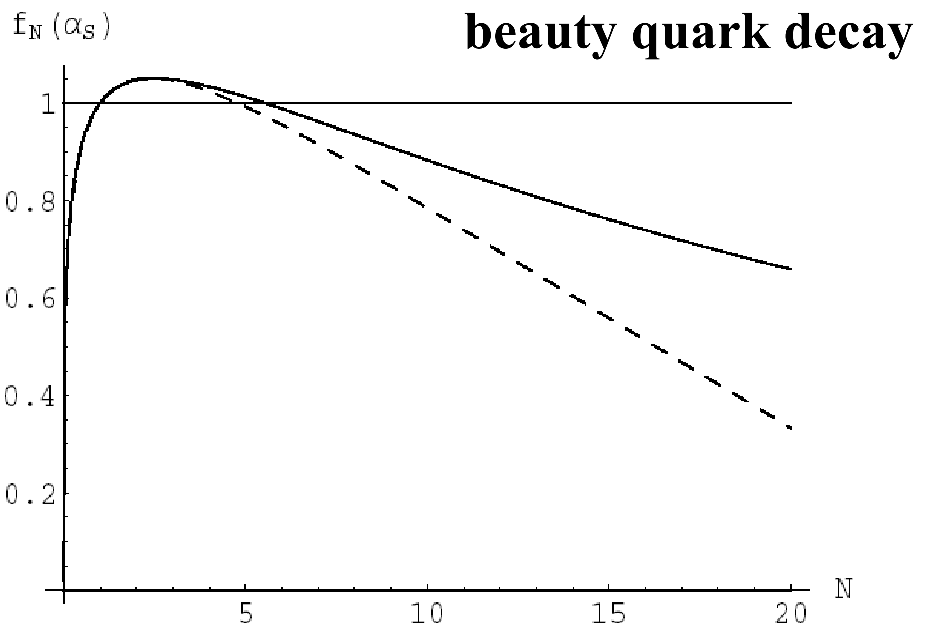
- **Consistent integration to low gluon transverse momenta k_t:**
  at low k_t, effective coupling does not diverge
- **Consistent Fixed Log Approximation:**
  subleading logs are suppressed by powers of $\tilde{\alpha}_S$, which is always small

- **g_1, g_2 are analytical in all the λ range:**
  no branch cuts
- **form factor analytical in all N (or x) range, including large values**

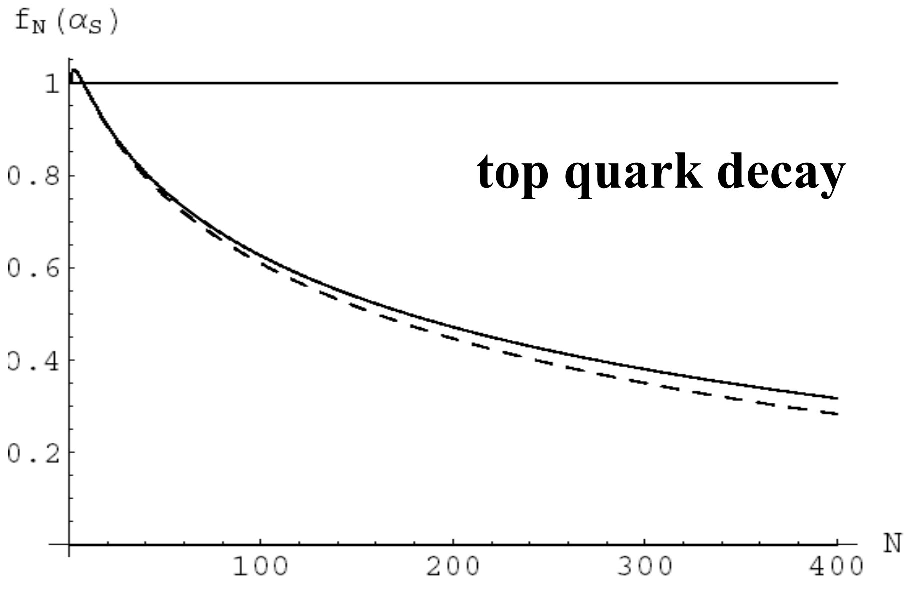
Examples of comparison between improved and standard form factors

U. Aglietti, G. Ricciardi, hep-ph/0407225

In N space



beauty quark decay

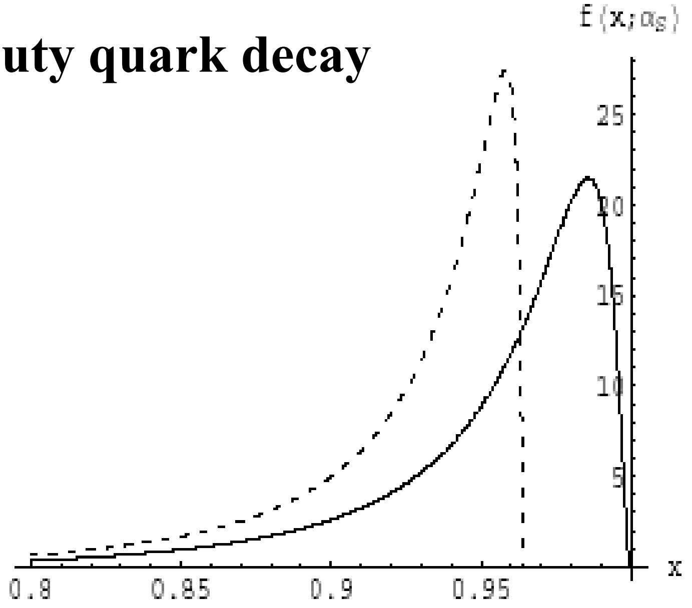


top quark decay

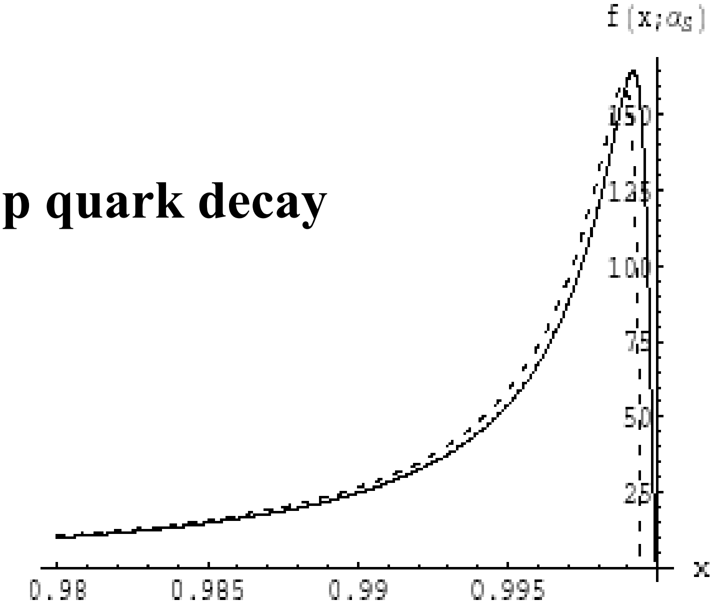
Examples of comparison between improved and standard form factors

In x space

beauty quark decay



top quark decay



U. Aglietti, G. Ricciardi, hep-ph/0407225

Conclusions

- We have an improved formula for threshold log resummation, where we include absorptive effects related to the decay of the gluon jets.

  Physically, the improved formula is not affected by the Landau pole because there is not enough resolution time to see Landau pole effects.

- Effective couplings turn out to be not singular in the infrared region and remain small in all the integration region.

  Form factors result well defined in the whole N or x space, which is also a great advantage in the calculation of differential distributions.