# Towards a large-$N_C$ estimate of the $O(p^6)$ chiral low-energy constants

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# Outline

- Introduction: ChPT and $1/N_C$ basics
- "Matching" Strategy
- Explicit example to order $p^6$ (NNLO):
  - <VAP> Green Function
- Conclusions and outlook

Based on:

- VC, G. Ecker, M. Eidemuller, A. Pich, J. Portoles, Phys. Lett. B 596 (2004) 96
- VC, G. Ecker, M. Eidemuller, R. Kaiser, A. Pich, J. Portoles, in progress

# Chiral Perturbation Theory

- Low-energy Effective Theory of QCD, expansion around $m_{u,d,s} = 0$

- Degrees of freedom: $\pi$, K, $\eta$, Goldstone modes of S$\chi$SB

$$SU(3)_L \otimes SU(3)_R \Rightarrow SU(3)_{L+R=V}$$

- Low-Energy expansion: $p^{2n}$, $m^n$ ; $p^2/\Lambda^2 \sim 0.2$

$$\mathcal{L} = \sum_n \mathcal{L}_{2n}$$ *(To be used in **trees** and **loops**)*

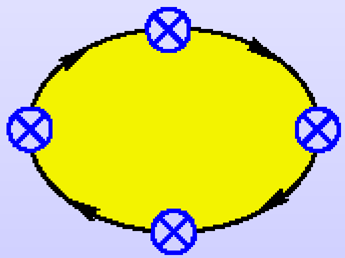
- Couplings: 2 at $O(p^2)$, 10 ($L_i$) at $O(p^4)$, 90 ($C_i$) at $O(p^6)$

Gasser-Leutwyler'84 Bijnens-Colangelo-Ecker'99

- $C_i$ are crucial input for low-energy precision physics (semileptonic K decays, $V_{us}$, $\pi \to \nu\, e\, \gamma$, … )

# The large-$N_C$ expansion

- $g_s^2 \sim O(1/N_C) \Rightarrow \alpha_s N_C$ fixed; $< T(J_1 \ldots J_n) > \sim N_C$



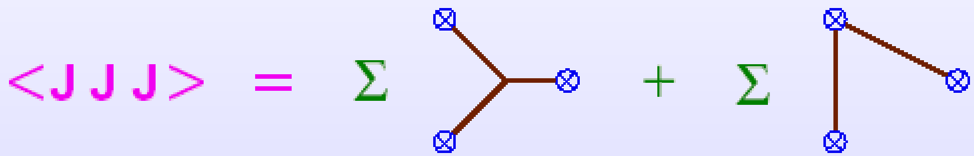
- Planar diagrams dominate
- Non-planar diagrams suppressed by $1/N^2$
- Internal quark loops suppressed by $1/N_C$

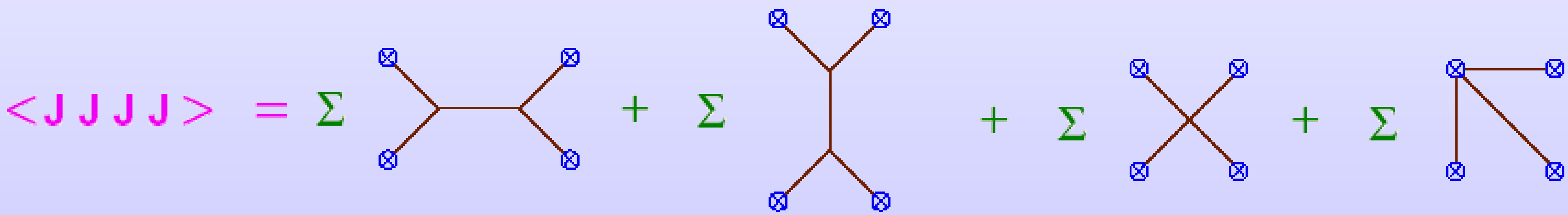
- Assume color confinement $\Rightarrow$ J |0> ~ |1 Meson>

$$\langle J(k)\; J(-k) \rangle = \sum_n \frac{f_n^2}{k^2 - M_n^2}$$

- **Infinite** number of **mesons** ($\sim \log k^2$)
- Counting: $f_n = \langle 0|J|n\rangle \sim \sqrt{N_C} \quad M_n \sim O(1)$
- Mesons are stable and non-interacting

- Amplitudes are sums of pole terms (exchange of free mesons)





- Crossing + Unitarity ⇒ The pole terms come from the tree approximation to some mesonic Lagrangian (Witten)

# Matching strategy

- For a given $< T(J_1 \ldots J_n) >$ we know, to some degree:
  - low-momentum behavior (ChPT)
  - high-momentum behavior (OPE, asymptotic fall-off of form factors and scattering amplitudes)

- Construct hadronic interpolation based on large-$N_C$ using an appropriate** chiral-invariant hadronic lagrangian

- Constrain parameters by matching to QCD asymptotic behavior

- Focus on G.F.s that are order parameters of SχSB $\Leftrightarrow$ use **finite number of hadronic states**

# Approximations / Modeling

- Resonance content

**Include one resonance multiplet (SRA) per channel (V, A, S, P)**

- Lowest-lying states have largest impact on ChPT LECs
- SRA ⇔ QCD asymptotic behavior sets in at ~ 1.5 GeV
- Successful phenomenology at $O(p^4)$ [Ecker-Gasser-Leutwyler-Pich-deRafael '89]

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- Set of short distance constraints

- Highest priority to OPE constraints (crucial for applications to electro-weak LECs)
- Then, high momentum behavior of form factors involving on-shell Goldstone particles

# Explicit example: <VAP> Green Function

$$(\Pi_{VAP})^{abc}_{\mu\nu}(p,q) = \int d^4x \int d^4y \; e^{i(p\cdot x + q\cdot y)} \; \langle 0|T\{V^a_\mu(x) A^b_\nu(y) P^c(0)\}|0\rangle$$

$$V^a_\mu = \bar{\psi}\gamma_\mu \frac{\lambda^a}{2}\psi \qquad A^a_\mu = \bar{\psi}\gamma_\mu\gamma_5 \frac{\lambda^a}{2}\psi \qquad P^a = \bar{\psi} i\gamma_5 \frac{\lambda^a}{2}\psi \,.$$

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Chiral Ward Identities:

$$p^\mu (\Pi_{VAP})^{abc}_{\mu\nu}(p,q) = \langle \overline{\psi}\psi\rangle_0 f^{abc} \left[\frac{q_\nu}{q^2} - \frac{(p+q)_\nu}{(p+q)^2}\right]$$

$$q^\nu (\Pi_{VAP})^{abc}_{\mu\nu}(p,q) = \langle \overline{\psi}\psi\rangle_0 f^{abc} \frac{(p+q)_\mu}{(p+q)^2}$$

$$\begin{aligned}(\Pi_{VAP})^{abc}_{\mu\nu}(p,q) &= f^{abc}\Big\{ \langle\overline{\psi}\psi\rangle_0 \left[\frac{(p+2q)_\mu q_\nu}{q^2(p+q)^2} - \frac{g_{\mu\nu}}{(p+q)^2}\right] \\ &+ P_{\mu\nu}(p,q)\, \mathcal{F}(p^2,q^2,(p+q)^2) + Q_{\mu\nu}(p,q)\, \mathcal{G}(p^2,q^2,(p+q)^2)\Big\}\end{aligned}$$

$$P_{\mu\nu}(p,q) = q_\mu p_\nu - (p\cdot q) g_{\mu\nu} \qquad Q_{\mu\nu}(p,q) = p^2 q_\mu q_\nu + q^2 p_\mu p_\nu - (p\cdot q) p_\mu q_\nu - p^2 q^2 g_{\mu\nu}$$

# Long- and short-distance behavior of <VAP>

Moussallam '97; Knecht-Nyffeler '01

- Long-distance *(r = p + q)*

$$\mathcal{F}^{\rm CHPT}(p^2,q^2,r^2) = \frac{4\langle\overline{\psi}\psi\rangle_0}{F^2r^2}\Big[L_9+L_{10}+(C_{78}-5/2C_{88}-C_{89}+3C_{90})\,p^2 + (C_{78}-2C_{87}+1/2C_{88})\,q^2+(C_{78}+4C_{82}-1/2C_{88})\,r^2\Big]+\mathcal{O}(p^8)$$

$$\mathcal{G}^{\rm CHPT}(p^2,q^2,r^2) = \frac{4\langle\overline{\psi}\psi\rangle_0}{F^2q^2r^2}\Big[L_9+(-C_{88}+C_{90})p^2+(2C_{78}-C_{89}+C_{90})q^2-2C_{90}r^2\Big]+\mathcal{O}(p^8)$$

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$$\left. + (C_{78} - 2C_{87} + 1/2 C_{88})\, q^2 + (C_{78} + 4C_{82} - 1/2 C_{88})\, r^2\right] + \mathcal{O}(p^8)$$

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- Short distance ( *x ~ y ~ 0*; *x ~ y* , *x ~ 0*, *y ~ 0* )

$$\lim_{\lambda\to\infty} \mathcal{F}((\lambda p)^2,(\lambda q)^2,(\lambda r)^2) = \frac{\langle\bar\psi\psi\rangle_0}{2\lambda^4}\,\frac{p^2-q^2-r^2}{p^2q^2r^2} + \mathcal{O}\left(\frac{1}{\lambda^6}\right)$$

$$\lim_{\lambda\to\infty} \mathcal{G}((\lambda p)^2,(\lambda q)^2,(\lambda r)^2) = -\frac{\langle\bar\psi\psi\rangle_0}{\lambda^6}\,\frac{1}{p^2q^2r^2} + \mathcal{O}\left(\frac{1}{\lambda^8}\right)$$

# Hadronic interpolation

- Chiral-invariant hadronic Lagrangian:

$$\mathcal{L} = \mathcal{L}_{(2)}^{\rm GB} + \left[\mathcal{L}_{(2)}^{R_i} + \mathcal{L}_{(4)}^{R_i}\right] + \mathcal{L}_{(2)}^{R_i R_j} + \mathcal{L}_{(0)}^{R_i R_j R_k}$$

- $\mathcal{L}_{(n)}^{[...]}$ : Chiral monimial of order $p^n$
- Include all terms contributing to $p^6$ LECs and $1/N_C$ topologies
- Use **antisymmetric tensor** formulation for spin 1 fields

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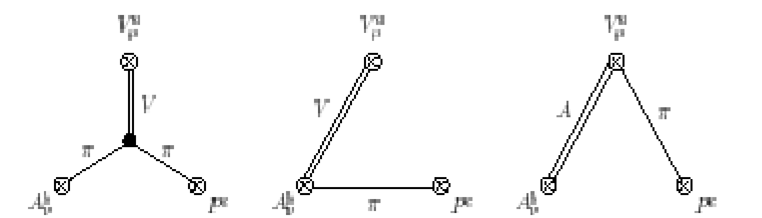
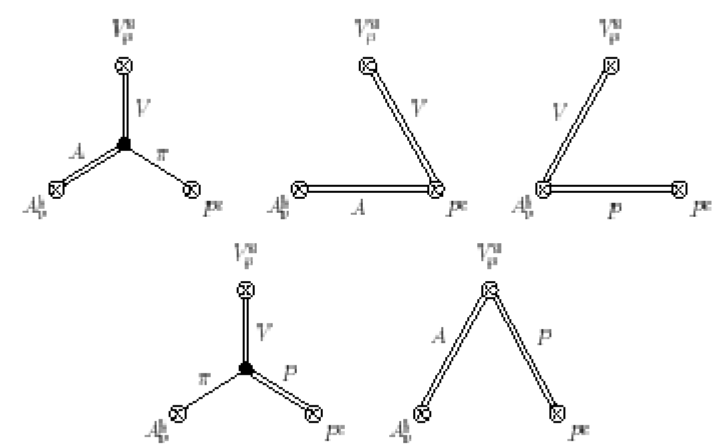
- Resonance Contributions to <VAP>:

$\mathcal{L}_{(2)}^{V,\ A,\ P}$ : Fixed by study of <J J>



$\mathcal{L}_{(2)}^{VA,\ VP,\ AP}$ : 6 couplings



- Result can be parameterized as (10 constants):

$$\mathcal{F}(p^2,q^2,r^2) = \frac{\langle\overline{\psi}\psi\rangle}{(p^2-M_V^2)(q^2-M_A^2)}\left[a_0 + \frac{b_1 + b_2 p^2 + b_3 q^2}{r^2} + \frac{c_1 + c_2 p^2 + c_3 q^2}{r^2 - M_P^2}\right]$$

$$\mathcal{G}(p^2,q^2,r^2) = \frac{\langle\overline{\psi}\psi\rangle}{(p^2-M_V^2)q^2}\left[\frac{d_1 + d_2 q^2}{r^2(q^2-M_A^2)} + \frac{f}{r^2 - M_P^2}\right]$$

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- <VAP> OPE conditions:

$$a_0 = -\frac{1}{2} \quad b_2 = 1+b_3 \quad d_2 = 2b_3 \quad c_2 = -\frac{1}{2} - b_3 \quad c_3 = c_2 \quad f = 2c_2$$

- Axial f.f. (<$\gamma$|A($t$)| $\pi$>): $G_A(t) = \frac{F_\pi^2}{M_V^2}\frac{b_1+b_3\,t}{M_A^2-t} \quad \Rightarrow \quad b_3 = 0$

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- <$\pi$|V($t$)| $\pi$> + <VV - AA>: $b_1=M_A^2-M_V^2$ $\qquad d_1=2M_A^2$

( Constraints incorporated in $\mathcal{L}_{(2)}^{V,\ A,\ P}$ )

- $c_1=-M_V^2c_2-M_A^2c_3$ (within our lagrangian framework)

# Results

- Matching determines resonance couplings and (upon 'integrating out' resonances) set of LECs entering <VAP>:

$$C_{78} = \frac{F_\pi^2(3M_A^2+4M_V^2)}{8M_V^4M_A^2} - \frac{F_\pi^2}{16M_V^2M_P^2} \qquad C_{82} = -\frac{F_\pi^2(4M_A^2+5M_V^2)}{32M_V^4M_A^2} - \frac{F_\pi^2}{32M_A^2M_P^2}$$

$$C_{87} = \frac{F_\pi^2(M_A^4+M_V^4+M_A^2M_V^2)}{8M_V^4M_A^4} \qquad C_{88} = -\frac{F_\pi^2}{4M_V^4} + \frac{F_\pi^2}{8M_V^2M_P^2}$$

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- $C_i$ differ from Knecht-Nyffeler'01 (different resonance content)

- Reliability of LECs ? Check size of sub-leading effects in $1/N_C$

$$\left[\frac{\delta C_i^{\text{Ren. Group}}}{C_i^{\text{RES}}}\right] < 0.30 \qquad \delta C_i^{\text{Ren. Group}} = |C_i(1\,\text{GeV}) - C_i(0.5\,\text{GeV})|$$

# Conclusion and outlook

- General strategy to match ChPT to QCD at large $N_C$

- Minimal resonance lagrangian is **consistent** with leading OPE for <VAP> and various GB form factors $(\langle\pi|V(t)|\pi\rangle,\ \langle\gamma|A(t)|\pi\rangle,\ \langle\pi\,\pi\,\pi\,|A(t)|\,0\rangle)$

- Matching $\Rightarrow$ determination of six LECs of $O(p^6)$.
  Sub-leading effects in $1/N_C$ (RG) are below 30%

- First exploration, part of a larger program
  - Full resonance (V,A,S,P) lagrangian contributing to $O(p^6)$
  - Study of additional 3-point functions (<PSP> and others)